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THE
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STELLAR PARALLAXES,

DERIVED FROM PHOTOGRAPHS MADE WITH THE 40-INCH REFRACTOR OF THE YERKES OBSERVATORY,

By OLIVER J. LEE, ALFRED H. JOY AND GEORGES VAN BIESBROECK.

This is a summary of determinations of stellar parallax that have been made at the Yerkes Observatory in the last two years. Nine of these parallaxes were reported at San Francisco, in 1915. They are marked with an asterisk in the list below. The details of the observations and reductions will appear soon in Part 1 of Volume IV of the *Publications of the Yerkes Observatory*.

Star	α 1900	δ 1900	B.D. Number	Magn. and Spectrum	Proper-Motion	Relative Parallax	Probable Error	No. of Plates	Probable Error of One Plate	No. of Comp. Stars
	h m	° ′	°		″	″	″			
6 *Persei*	2 7	+50 36	+50 481	5.4 *G5*	0.4	−.006	±.010	15	±.025	5
β *Persei**	3 2	+40 34	+40 673	Var. *B8*	.01	+.020	.014	13	.035	3
β *G. C.* 2745	5 23	+54 36	+54 902	7.5 . .	.43	+.023	.008	12	.019	5
. . . .		. . . . . .	. . . . . .	9.2 . .		+.013	.005	12	.013	5
θ *Can. Maj.**	6 50	−11 55	−11 1681	4.5 *K5*	.18	+.012	.007	11	.016	5
A Oe 7846	7 18	+46 18	+46 1264	9.0 . .	.43	+.015	.005	12	.012	5
σ *Geminorum*	7 37	+29 8	+29 1590	4.3 *K*	.25	+.019	.013	11	.027	4
Lalande 15565*	7 54	+29 30	+29 1664	7.5 *G*	1.2	+.055	.013	10	.026	5
D'Agelet 1623*	8 49	+26 35	+26 1865	7.0 .	.43	+.012	.010	12	.025	3
W Ursæ Majoris	9 37	+56 25	+56 1400	Var. *G*		+.008	.010	14	.020	5
A Oe 10642*	10 7	+53 1	+53 1395	9.0 .	.70	+.069	.009	13	.020	5
Lalande 22667	11 59	+ 3 51	+ 3 2594	9 .	.55	+.011	.014	12	.028	5
Lalande 23223	12 20	− 3 40	− 3 3280	8.0 . .	1.34	+.051	.010	10	.018	5
Berlin B 5072	14 21	+24 7	+24 2733	9.0 .	1.4	+.070	.010	14	.022	6
Berlin B 5073	14 21	+24 7	. . . .	9.1 . .	1.4	+.065	.008	14	.018	6
A Oe 14318	15 5	−15 59	−15 4041	9.6 *G5*	3.69	+.025	.008	10	.015	6
A Oe 14322	15 5	−15 54	−15 4042	9.2 *G4*	3.67	+.061	.012	10	.025	6
Lalande 27744	15 9	− 0 58	− 0 2944	7.0	1.4	+.057	.007	14	.018	6
δ *Serpentis* prec.	15 30	+10 52	+11 2821	5.2 *A5*	.07	+.012	.004	10	.010	4
δ *Serpentis* fol.	. . . .	. . . . .	. . . .	4.2 *A5*	.07	+.004	.011	10	.023	4
ξ *Scorpii AB*	15 59	−11 6	−10 4237	4.8 *F8*	.03	+.049	.009	11	.020	5
ξ *Scorpii C*	. . . .	. . . .	. . . . . .	5.1	. . .	+.051	.006	11	.013	5
κ (appa) *Herculis* prec.	16 4	+17 19	+17 2964	5.3 *K*	.08	−.009	.008	12	.020	3
κ *Herculis* fol.	. . . .	. . . . .	+17 2965	6.5 . .	. . .	−.026	.008	12	.020	3
λ *Ophiuchi**	16 26	+ 2 12	+ 2 3118	3.8 *A*	.10	+.018	.003	13	.008	4
W. B. I 16° 906	16 50	− 8 9	+ 8 4352	9 .	1.27	+.131	.006	11	.013	6
ϵ *Herculis*	16 56	+31 4	+31 2947	3.9 *A*	.03	+.065	.012	12	.026	5

Star	α 1900	δ 1900	B.D. Number	Magn. and Spectrum	Proper-Motion	Relative Parallax	Probable Error	No. of Plates	Probable Error of One Plate	No. of Comp. Stars
	h m	° ′	°							
u Ophiuchi	17 11	+ 1 19	+ 1 3408	Var. *B8*		−.011	.004	12	.008	6
Anon	17 34	+18 36		9.1	1.33	+.138	.010	12	.017	5
BD 68° 946	17 37	+68 26	+68 946	9.1 *K*	1.33	+.217	.006	13	.015	5
68° 947			+68 947	8		+.005	.009	13	.023	5
Z Herculis	17 54	+15 9	+15 3311	Var. *F*	.09	+.028	.009	10	.016	6
RX Herculis	18 26	+12 33	+12 3557	Var. *A*	.	−.010	.016	12	.029	5
β *G. C.* 8679	18 33	− 3 17	− 3 4331	7	. .	+.030	.008	9	.016	4
Draconis 205	18 45	+49 19	+49 2871	6.5	.	+.012	.008	13	.020	5
β *G. C.* 9919 *D**	20 2	+35 30	+35 3956	8.9	. .	±.000	.012	12	.032	6
*A**		.	+35 3957	8.2 *O*	.	+.008	.008	12	.022	6
BD 43° 3571 .	20 17	+43 33	+43 3571	7.5 *O*	.	+.011	.010	12	.022	5
ε *Delphini*	20 28	+10 58	+10 4321	4.0 *B5*	.03	+.013	.008	9	.015	5
β *Cephei*	21 27	+70 7	+69 1173	3.3 *B1*	.01	−.016	.014	13	.035	6
κ *Pegasi*	21 40	+25 11	+24 4463	4.3 *F5*	.03	+.021	.011	14	.018	7
λ *Cephei*	22 8	+58 55	+58 2402	5.6 *O*	.	−.020	.006	11	.013	5
BD 29° 4753*	22 41	+29 54	+29 4753	7.2	.44	+.027	.011	14	.026	7

Yerkes Observatory, June, 1916.

OBSERVATIONS OF THE SATELLITE OF *NEPTUNE*,

By E. E. BARNARD.

The following measures of the satellite of *Neptune* were made with the large telescope and a magnifying power of 700 diameters, and are a continuation of those printed in *A. J.*, Vol. XXIX, p. 39. They have not been corrected for refraction. The closing in of its orbit by perspective brings the satellite nearer the planet at times, and this, if the seeing is bad, makes it more difficult to observe. In these observations it was about the 13th magnitude. The times are $6^h\,0^m$ slow of Greenwich Mean Time.

Measures of the Satellite

C. S. Time	P. A.	Dist.	Comp.	Remarks
d h m s	°			
1915 Apr. 7 9 12 32	148.46		7	Not well seen. Sky poor.
9 18 12		13.74	10	
11 7 39 25	103.61		5	
7 44 17		16.24	8	
22 8 21 19	316.21		6	Faint in haze.
8 26 50		14.96	8	
May 9 8 24 24	357.46		6	Faint. Seeing very bad.
8 32 40		11.43	10	
12 8 15 51	170.01		7	Faint in clouds.
8 22 30		11.47	8	
Oct. 21 15 50 12	311.09		5	13th magnitude. Seeing = 3.
15 56 2	. . .	15.65	8	Observation good.

Measures of the Satellite (Continued)

C. S. Time	P. A.	Dist.	Comp.	Remarks
d h m s	°	″		
1915 Oct. 26 15 7 40	6.71	...	11	Seen only momentarily.
15 17 14	...	11.01	11	Seeing very bad.
Nov. 2 16 7 36	298.78	...	5	Seeing poor.
16 14 41	...	16.18	8	
Dec. 4 14 16 51	136.09	...	5	Faint from thick sky but image very steady.
14 23 33	...	15.83	10	
18 14 44 22	356.98	...	6	Very difficult.
14 51 12	...	11.67	11	Seeing very bad.
1916 Jan. 1 17 59 17	224.66	...	5	Very difficult.
18 3 40	...	11.25	6	Planet very low.
5 10 55 26	342.59	...	7	Seen with difficulty.
11 2 36	...	13.66	10	Seeing very bad.
15 13 49 13	102.12	...	7	Very difficult. Seeing excessively bad.
13 54 26	...	16.30	9	
Feb. 2 10 46 14	90.25	...	7	Seeing poor but observation good.
10 51 55	...	15.03	8	
5 15 6 44	257.72	...	7	Faint and blurred. Seeing bad but observation good.
15 11 31	...	13.36	8	
9 12 47 48	174.21	...	5	Well seen.
12 52 7	...	11.84	8	
26 10 4 14	62.21	...	5	Well seen. 12½ or 13 magnitude.
10 9 22	...	11.93	8	
Mar. 4 9 0 36	332.87	...	5	Well seen though seeing poor.
9 5 47	...	13.57	8	
8 9 20 54	104.71	...	5	Seeing = 3.
9 29 51	...	16.56	8	
15 9 29 16	32.50	...	5	Faint in moonlight.
9 34 23	...	10.95	8	
18 10 28 48	203.96	...	5	Well seen.
10 36 35	...	10.92	8	
22 10 54 44	310.88	...	5	Seeing very bad.
10 58 53	...	15.72	8	
29 11 28 35	260.29	...	7	Fairly well seen. Sky thick but seeing good.
11 35 45	...	13.49	10	
Apr. 1 8 50 3	83.38	...	5	
8 54 35	...	13.71	8	
9 35 7	81.68	...	5	Seen only at long intervals through clouds.
9 49 39	...	13.71	8	
3 8 0 22	306.06		5	Observations stopped by clouds.
5 9 8 48	173.90	...	5	Faint. Seeing very bad.
9 13 37	...	11.05	8	
8 9 52 49	346.88		6	Seeing = 2.
9 58 1	...	11.85	8	
12 9 22 25	115.68	...	8	Very faint. Seeing excessively bad.
9 28 37	...	16.94	8	
19 9 31 4	56.68	...	7	Seeing very bad. Wind shaking telescope.
9 39 11	...	11.21	12	
May 3 9 4 5	277.93		5	

Measures of the Satellite (*Continued*)

C. S. Time	P. A.	Dist.	Comp.	Remarks
d h m s	°	″		
1916 May 3 6 7 19	. .	14.81	8	
6 8 46 11	96.19	. . .	5	Seeing = 3.
8 51 18		15.20	8	
10 8 8 32	202.41	. . .	3	Only feebly glimpsed. Seeing very bad.
Neptune and a faint star (15 mag.) north				
1915 Oct. 21 16 2 39	319.49		4	
16 9 28	. . .	26.66	7	
Neptune and a star north of the satellite and slightly less bright. Of the two, this is assumed to be the star.				
1916 Mar. 8 9 24 20	81.91	. . . .	5	
9 36 34	. .	13.80	10	

A small nebula was found while observing *Neptune*. Its position was referred to the planet.

Nebula — *Neptune*

1916 April 1^{d} 9^{h} 0^{m} $\Delta\alpha \cos\delta + 0' \, 18''.1$ (2)

$= \Delta\alpha + 1^{s}.28, \; \Delta\delta - 1' \, 5''.7$(2)

It was also referred to *Neptune* by position angle and distance.

1916 April 5^{d} 9^{h} 20^{m} P. A. 139°.9 (3). Dist. 104″.9 (3)

By using the positions of *Neptune* given in the *Nautical Almanac*, these give respectively the following places of the nebula:

1916.0 α 8^{h} 7^{m} 59^{s}.76, δ + 19° 53′ 15″.0

8 7 59 .65 + 19 53 16 .7

1916.0 α 8^{h} 7^{m} 59^{s}.70 δ + 19° 53′ 15″.9

It is of about 14½ magnitude, ½′ in diameter, round, a little brighter in the middle, with perhaps a very faint nucleus. This nebula is not in any of DREYER's lists.

Yerkes Observatory, Williams Bay, Wisconsin,
1916, July 28.

A NEW VARIABLE STAR,

1855.0 α 15^{h} 9^{m} 37^{s} ± δ − 5° 12′

BY E. E. BARNARD.

Professor E. C. PICKERING kindly informs me that this star, which was found on the Bruce plates of this Observatory, seems to be new and that it may be of the β *Lyræ* type. It is found on a number of the Harvard plates, where the variation is somewhat less than one magnitude. Visually its mean brightness is about 13 or 13½ magnitude, and the period is about 7.1 days. It is the second star (from the preceding end) of a gentle curve of four small stars which bends north, with the brightest star at the following end. The star is 21^{s} ± following and 6′-7′ north of BD − 5° 1044. A 13½ magnitude star follows it in the position:

1916.390 May 22 P. A. 87°.90, Dist. 133″.4 (1n.)

Yerkes Observatory, Williams Bay, Wisconsin, 1916, September 4.

SUNSPOT OBSERVATIONS,

MADE AT BERWYN, PENN., WITH A $4\frac{1}{2}$-INCH REFRACTOR,

By A. W. QUIMBY.

1916	Time	New Grs.	Total Grs.	Total Spots	Fac. Grs.	Def.
Jan. 1	8	–	3	6	1	poor
2	2	–	3	15	–	poor
3	8	1	4	24	–	fair
4	8	1	5	32	1	fair
5	11	–	5	27	1	fair
6	8	–	5	24	2	fair
7	2	–	5	13	3	fair
8	8	–	3	8	1	fair
9	8	–	3	3	–	poor
11	10	–	3	3	1	fair
13	4	–	2	2	–	fair
14	8	–	2	2	2	fair
15	8	–	2	2	1	poor
16	8	–	1	1	1	fair
17	4	–	1	1	1	poor
18	8	–	1	1	1	fair
19	4	1	2	2	2	fair
21	8	2	4	11	2	fair
23	8	–	3	10	3	fair
24	8	0	3	8	3	fair
25	11	1	3	11	2	fair
26	8	–	3	9	1	fair
27	10	1	4	9	1	fair
28	9	–	3	6	3	fair
29	8	–	3	5	2	fair
31	3	–	3	8	–	fair
Feb. 3	8	1	3	21	2	fair
4	9	2	4	16	3	fair
5	8	2	5	41	3	fair
6	8	–	3	27	2	fair
7	8	–	3	10	1	poor
8	8	1	5	50	2	fair
10	9	3	8	40	3	fair
11	3	–	5	12	1	poor
14	8	–	4	8	2	fair
15	8	–	4	7	2	fair
16	8	–	4	9	2	fair
17	11	–	4	13	2	fair
18	8	–	3	7	2	fair
19	8	–	3	10	2	fair
20	4	–	1	16	1	fair
22	8	–	1	16	1	fair
23	10	–	1	10	1	poor
26	8	2	2	2	1	fair
27	4	–	2	3	1	fair
28	8	–	2	5	1	fair
29	8	–	2	6	1	fair
Mar. 1	8	1	3	6	2	fair
3	8	1	4	47	2	fair
4	8	–	4	62	2	fair
5	4	1	5	50	3	fair
7	5	–	5	41	3	fair

1916	Time	New Grs.	Total Grs.	Total Spots	Fac. Grs.	Def.
Mar. 8	4	1	5	28	3	fair
9	8	–	4	18	3	fair
10	11	–	2	10		poor
11	12	–	3	20	2	fair
12	8	–	2	6	2	fair
13	8	–	2	2	2	fair
14	8	–		–	–	poor
16	8	1	1	2	–	poor
17	8	1	2	4	1	fair
18	8	–	2	4	1	fair
19	8	1	1	3	7	fair
20	8	–	3	8	1	fair
21	5	–	3	11	1	fair
23	7	–	2	18	1	fair
24	7	–	2	28	1	fair
25	7	1	4	31	1	fair
26	7	–	4	30	–	fair
27	7	–	4	28	1	fair
30	2	2	6	30	2	fair
31	4	–	6	32	2	fair
Apr. 1	9	–	5	24	2	fair
2	4	–	5	32	1	fair
5	7	1	6	40	3	fair
6	3	–	4	12	1	poor
7	7	–	4	10	1	fair
9	4	–	4	8	3	poor
10	4	1	5	34	4	good
11	7	–	4	24	2	fair
12	7	–	2	20	1	fair
13	7	1	3	32	2	fair
14	5	1	3	25	3	fair
15	7	–	2	20	2	fair
16	7	–	2	12	2	poor
17	7	–	2	8	1	poor
18	7	–	2	5	1	fair
19	7	1	2	3	2	fair
20	7	3	5	10	4	good
21	7	–	5	10	2	good
22	7	–	5	12	3	good
23	7	1	3	10	3	good
24	10	–	3	17	3	good
25	7	1	3	12	–	poor
26	7	–	3	24	2	fair
28	7	1	4	22	3	fair
29	7	3	7	35	6	fair
30	7	–	7	25	3	fair
May 1	5	–	5	36	3	fair
2	7	–	4	25	2	fair
3	7	–	3	31	2	fair
4	7	2	5	31	2	fair
5	7	–	3	26	2	fair
6	4	1	4	35	2	fair

1916	Time	New Grs.	Total Grs.	Total Spots	Fac. Grs.	Def.
May 7	4	–	4	26	2	fair
8	7		4	21	2	fair
9	7		3	7	2	fair
10	7		3	8	2	fair
11	4	1	4	9	3	fair
12	7	–	2	6	3	fair
13	7	1	2	4	1	poor
14	7	–	2	4	–	poor
15	7	–	2	4		fair
17	7	1	3	9	–	fair
18	7	1	4	11	–	fair
19	5	1	4	22	3	good
20	5	1	5	15	3	good
21	6	1	6	14	3	fair
22	7	–	5	14	3	fair
24	7	–	5	24	3	fair
25	6	–	5	35	3	fair
26	5	1	6	66	2	good
27	6	–	6	65	1	fair
28	7	–	6	49	1	fair
29	7	–	6	48	2	fair
30	6	–	5	24	2	fair
31	6	–	5	16	2	fair
June 1	6	–	4	9	4	fair
2	6	–	3	3	1	fair
3	6	3	4	9	2	fair
4	6	–	2	25	2	fair
5	6	–	2	25	2	fair
6	6	–	1	24	2	fair
7	6	1	1	1	2	fair
9	12	–	1	1	–	poor
11	5	–	1	5	1	fair
12	5	–	1	14	2	good
13	6	1	2	14	2	fair
15	9	–	2	24	2	fair
16	9	1	2	6	1	poor
17	6	–	2	6	1	fair
18	6	2	4	13	4	fair
19	6	2	6	24	4	fair
20	6	1	5	33	3	fair
21	8	–	6	30	3	fair
22	5	2	8	83	2	v. g.
23	6	–	8	72	2	fair
24	6	–	8	14	2	fair
25	5	–	8	66	3	fair
26	6	1	9	45	3	fair
27	10	–	8	43	3	fair
28	8	1	8	34	3	fair
29	5	–	7	22	2	fair
30	1	–	4	23	2	fair

NOTE ON Σ *Oe 14318–20* ($15^h\ 5^m$),

BY OLIVER J. LEE.

The earlier proper-motions (*Astronomical Journal*, *12*, 25, 1892) as well as ADAMS' recent radial velocities of these two stars (*Popular Astronomy*, *24*, 266, 1916) show a remarkable similarity for stars so widely separated — they are 5′ apart. The parallax and the proper-motion in R. A. have just been derived from plates taken with the 40-inch covering a period of 3.3 years. The parallaxes are $+0''.025 \pm 0''.008$ and $-0''.061 \pm 0''.012$, the proper-motions are $-0^s.0699$ and $-0^s.0692$ respectively. The large difference in relative parallax suggested that the proper-motions might also be different. PROF. J. G. PORTER has been kind enough to give me his latest values. They are: $-0^s.0696$ and $-0^s.0683$, $-3''.555$ and $-3''.540$. Total motions, $3''.693$ in $195°.7$ and $3''.675$ in $195°.6$ respectively. PROFESSOR PORTER'S values of course have by far the greater weight. The parallaxes are derived for both stars using the same set of six comparison stars. The parallaxes of the comparison stars have been determined. The sum of the products of each of these by its dependence was taken for each of the two stars, and was found to be $+0''.006$ for each. Hence the difference between the two parallaxes $0''.036$ cannot be attributed in any way to the parallaxes of the comparison stars although affected by different dependences for the two stars. In the *Publications of the Carnegie Institution*, No. 147, PROFESSOR RUSSELL gives the parallaxes of the two stars as $+0''.045 \pm 0''.022$ and $+0''.014 \pm 0''.023$. He has kindly informed me that a transposition of the two values had accidentally been made in transcribing so that the difference $0''.031$ is really in the same sense as my difference $0''.036$. The evidence is fairly consistent in indicating for these two stars a very wide separation in space with nearly identical proper-motions and radial velocities.

Yerkes Observatory, July, 1916.

SAN LUIS DECLINATIONS (*Second Paper*),

BY ARTHUR J. ROY.

The next step in the reduction of the San Luis Declinations in continuation of the results given in *A. J.* 686-687 involved the refraction factor necessary to adjust each series to the mean system indicated by the whole. Inasmuch as there are unknown constant differences between separate series, each series or sub-series was solved by itself. Naturally there are many sub-series with but trifling weight, particularly the morning and afternoon series, and even the mean of many is still rather uncertain. However, there is sufficient material when grouped with regard to the relation to sunset and sunrise to be convincing that there is a diurnal variation in the refraction. In the following exhibit, all the material has been included although in every group there are some sub-series which partly belong to an adjacent group. This is particularly true of the group at dawn, but even the afternoon group would sometimes extend till after dark in the winter, and in the summer the first half might begin before dark. The morning observations grouped under early sunrise and late sunrise had comparatively few overlapping series but probably a complete separation would increase the contrast between them.

In view of the possibility that preceding conditions in the morning might have an influence either through an effect on the upper atmosphere or through a restraint on the internal temperature due to a closed roof, this work was divided on the basis of presence or absence of preceding work. The absence was usually though not always due to clouds, but for whatever reason absent, the building would be closed for several hours preceding. The parts when regular work preceded are given in f' and g' and the isolated parts in f'' and g''. Except in one case, the agreement of the separate parts for each observer is much better than was to be expected, leading to the conclusion that no account need be taken of preceding conditions.

From this material, in some parts rather meager, it appears that the refraction at San Luis in the afternoon is well represented by the Pulkova tables, that about sunset the refraction decreases rapidly, that the decrease continues throughout the night, perhaps even for some appreciable time after sunrise, that the increase does not begin at dawn, that it is moderate for an hour or more after sunrise and then is quite rapid. The observations do not furnish the data to cover the culmination as very few were taken between 9 A. M. and 3 P. M.

Observer		*R.*		*V.*		*T.*	
		Wt.	Factor	Wt.	Factor	Wt.	Factor
(*a*)	Afternoon	213	1.0004	218	0.9975	197	0.9992
(*b*)	First half of night	1916	0.9931	2108	0.9945	2279	0.9919
(*c*)	Second half of night	484	0.9908	420	0.9928	682	0.9906
(*d*)	Just preceding dawn	90	0.9894	89	0.9907	277	0.9889
(*e*)	Dawn	30	0.9956	21	0.9919	58	0.9891
(*f*)	Early sunrise	69	0.9943	82	0.9933	48	0.9922
(*g*)	Late sunrise	91	1.0037	66	1.0009	27	0.9984
(*f′*)	(See text)	38	0.9943	61	0.9933	35	0.9937
(*f″*)		32	0.9943	21	0.9933	13	0.9886
(*g′*)		72	1.0036	51	1.0011	18	0.9983
(*g″*)		20	1.0040	13	1.0005	8	0.9985

OBSERVATIONS OF MINOR PLANETS,

MADE WITH THE 26-INCH EQUATORIAL OF THE U. S. NAVAL OBSERVATORY,

BY H. E. BURTON, ASSISTANT IN THE OBSERVATORY.

[Communicated by Captain J. A. Hoogewerff, U. S. Navy, Superintendent.]

Date Wash. M. T.	*	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$ α	log $p\Delta$ δ	Red. to App. Pl.	
				(433) *Eros*						
1914 h m s			m s	′ ″	h m s	° ′ ″			s	″
Aug. 19 14 37 1	1	14 , 4	−3 7.32	+0 7.4	0 20 3.99	+19 4 57.8	8.236	0.471	+3.29	+18.9
31 14 45 15	2	20 , 4	+1 24.56	+4 46.4	0 10 44.54	+21 50 7.5	9.188	0.429	+3.55	+20.8
Sept. 4 12 22 48	3	20 , 4	−4 40.67	+0 16.3	0 6 3.64	+22 35 27.1	9.024*n*	0.398	+3.60	+21.4
9 12 13 1	4	20 , 5	+0 51.63	−0 12.2	23 58 55.16	+23 24 59.2	8.845*n*	0.371	+3.64	+22.1
20 13 11 2	5	20 , 4	−4 51.01	−5 44.7	23 39 24.89	+24 30 40.5	9.278	0.374	+3.72	+24.2
29 12 12 43	6	20 , 5	+1 51.38	−0 52.5	23 21 53.13	+24 31 54.7	9.252	0.370	+3.70	+25.7
Oct. 20 9 55 36	7	20 , 4	−4 39.97	−0 16.2	22 49 33.07	+22 4 30.8	9.113	0.416	+3.46	+27.3
23 12 21 33	8	20, 5	+2 43.42	+0 14.6	22 46 51.19	+21 33 19.8	9.607	0.566	+3.39	+27.2
28 11 59 12	9	30 , 6	−3 26.21	−7 11.8	22 43 54.64	+20 43 5.2	9.605	0.576	+3.37	+27.5
Nov. 10 12 15 58	10	25 , 5	+2 12.97	−3 7.0	22 44 2.66	+18 45 8.8	9.663	0.658	+3.20	+27.2
18 10 24 14	11	10 , 2	−5 12.79	−5 33.3	22 49 13.95	+17 50 40.2	9.572	0.593	+3.21	+27.6
19 11 52 46	12	30 , 6	−1 12.11	−0 3.6	22 50 11.43	+17 44 37.9	9.664	0.668	+3.18	+27.4
Dec. 14 7 49 12	13	30 , 10	−0 31.57	−7 30.7	23 27 36.19	+16 48 43.8	9.354	0.546	+3.17	+28.0
1915										
Jan. 4 8 9 1	14	25 , 5	−0 40.97	+1 54.0	0 17 6.03	+17 48 19.4	9.503	0.564	+0.26	+ 8.9
9 7 46 18	15	25 , 5	+0 51.89	−3 59.0	0 30 46.69	+18 11 40.1	9.465	0.547	+0.25	+ 8.9
13 7 26 8	16	25 , 5	−0 52.64	+5 34.3	0 42 12.70	+18 31 57.4	9.424	0.531	+0.31	+ 9.2
Feb. 9 7 45 30	17	25 , 5	+2 13.90	+3 27.6	2 10 16.35	+20 52 21.3	9.522	0.570	+0.52	+10.3
27 8 10 28	18	30 , 6	−2 2.91	−1 34.2	3 17 35.64	+21 43 0.5	9.576	0.541	+0.75	+10.6
Mar. 3 8 59 8	19	25 , 5	−6 40.42	−1 14.3	3 33 20.32	+21 44 55.7	9.639	0.594	+0.81	+10.5
17 8 6 58	20	30 , 6	−1 53 93	+0 30.3	4 28 59.85	+21 18 16.3	9.569	0.543	+0.93	+ 9.5
26 9 34 59	21	29 , 6	+1 31.48	+6 39.8	5 5 33.50	+20 31 12.2	9.664	0.639	+1.02	+ 8.4
Apr. 8 9 7 33	22	25 , 5	−1 50.94	+0 49.8	5 57 32.15	+18 42 48.3	9.636	0.627	+1.17	+ 6.3
17 9 23 17	23	30 , 6	+2 3.66	−5 16.4	6 32 53.95	+17 1 35.3	9.646	0.653	+1.18	+ 4.6

Date Wash. M. T.		✱	Comp.	Δα	Δδ	App. α	App. δ	log pΔ α	log pΔ δ	Red. to App. Pl.	
					(796) [1914 *UH*]						
1914	h m s			m s		h m s	° ′ ″			s	″
Dec. 21	10 54 21	24	25 , 5	+2 29.89	−6 6.2	1 51 38.99	+25 15 41.6	9.557	0.466	+4.35	+31.6
27	9 19 54	25	25 , 5	−3 3.91	+1 53.5	1 54 10.30	+25 51 36.5	9.461	0.396	+4.35	+31.3
					(624) *Hector*						
1915											
Nov. 1	8 49 38	26	25 , 5	+3 3.64	+2 53.6	1 52 50.08	+31 29 31.9	9.273*n*	0.125	+4.88	+29.6
2	9 13 8	27	25 , 5	+3 34.03	−4 22.6	1 52 14.99	+31 27 27.4	9.408*n*	0.189	+4.88	+29.8
3	9 17 3	26	30 , 6	+1 52.62	−1 23.5	1 51 39.09	+31 25 15.0	9.377*n*	0.173	+4.88	+29.8

Mean Places of Comparison Stars for the Beginning of the Year

✱	α	δ	Authority	✱	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	0 23 8.02	+19 4 31.5	A.G. Berlin A. 104	15	0 29 54.55	+18 15 30.2	A.G. Berlin A. 158
2	0 9 16.43	+21 45 0.3	A.G. Berlin B. 35	16	0 43 5.03	+18 26 13.9	A.G. Berlin A. 211
3	0 10 40.71	+22 34 49.4	A.G. Beriln B. 45	17	2 8 1.93	+20 48 43.4	A.G. Berlin B. 661
4	23 57 59.89	+23 24 49.3	A.G. Berlin B. 9191	18	3 19 37.80	+21 44 24.1	A.G. Berlin B. 1009
5	23 44 12.18	+24 36 1.0	A.G. Berlin B. 9096	19	3 39 59.93	+21 45 59.5	A.G. Berlin B. 1124
6	23 19 58.05	+24 32 21.5	A.G. Berlin B. 8958	20	4 30 52.85	+21 17 36.5	A.G. Berlin B. 1469
7	22 54 9.58	+22 4 19.7	A.G. Berlin B. 8802	21	5 4 1.00	+20 24 24.0	A.G. Berlin B. 1651
8	22 44 4.38	+21 32 38.0	A.G. Berlin B. 8756	22	5 59 21.92	+18 41 52.2	A.G. Berlin A. 1880
9	22 47 17.48	+20 49 49.5	A.G. Berlin B. 8771	23	6 30 49.11	+17 6 47.1	A.G. Berlin A. 2250
10	22 41 46.49	+18 47 48.6	A.G. Berlin A. 9313	24	1 49 4.75	+25 21 16.2	A.G. Camb. Engl. 1028
11	22 54 23.53	+17 55 45.9	A.G. Berlin A. 9392	25	1 57 9.86	+25 49 11.7	A.G. Camb. Engl. 1101
12	22 51 20.36	+17 44 14.1	A.G. Berlin A. 9370	26	1 49 41.59	+31 26 8.7	A.G. Leiden 706
13	23 28 4.59	+16 55 46.5	A.G. Berlin A. 9605	27	1 48 36.08	+31 31 20.2	A.G. Leiden 702
14	0 17 46.74	+17 46 16.5	A.G. Berlin A. 75				

ENCKE'S COMET.

A cablegram received from Prof. ELIS STRÖMGREN announces the following observation of ENCKE'S Comet:

September 22.3796 G. M. T. R. A. $22^h\ 28^m\ 39^s.0$ Dec. $-7°\ 8'\ 55''$

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OBSERVATIONS OF COMETS,

By C. D. PERRINE.

MADE WITH THE 12-INCH REFRACTOR OF THE ARGENTINE NATIONAL OBSERVATORY, CORDOBA,

Greenwich Mean Time	*	No. of Comp.	☌ − * Δα	Δδ	☌'s Apparent α	δ	log pΔ α	δ	Red. to Ap. Pl. α	δ
Tuttle's Periodic Comet (1912 *b*)										
1912 d h m s			m s	′ ″	h m s	° ′ ″			s	″
Nov. 8 19 53 6	1	10 , 10	−2 38.17	−10 49.0	11 0 2.73	−23 51 18.2	*n*9.6879	*n*0.4445	+1.35	−0.4
9 19 41 7	2	10 , 10	−2 35.37	+ 8 32.5	11 3 13.20	−24 59 14.5	*n*9.7021	*n*0.4509	+1.34	−0.4
10 19 45 24	3	10 , 10	−2 27.56	− 8 7.4	11 6 28.29	−26 7 19.5	*n*9.7018	*n*0.4257	+1.34	−0.5
11 19 4 42	4	10 , 10	−0 33.95	− 6 18.6	11 9 38.95	−27 12 26.5	*n*9.7345	*n*0.4958	+1.34	−0.5
11 20 3 43	5	10*d*, 8	−0 3.96	+ 5 0.0	11 9 46.81	−27 15 11.0	*n*9.6870	*n*0.3645	+1.34	−0.4
15 19 48 38	6	10*d*, 8	−0 15.17	+ 5 56.5	11 23 7.62	−31 33 58.4	*n*9.7185	*n*0.3194	+1.31	−0.7
16 19 59 33	7	10 , 10	+1 26.02	+ 4 33.9	11 26 34.73	−32 36 57.1	*n*9.7118	*n*0.2583	+1.31	−0.7
21 19 35 45	8	10 , 10	+2 25.53	+ 5 52.6	11 44 7.97	−37 33 20.9	*n*9.7594	*n*0.2324	+1.30	−0.9
22 19 37 9	9	10 , 10	+0 59.00	− 0 53.9	11 47 46.29	−38 29 45.8	*n*9.7635	*n*0.2009	+1.30	−1.1
24 19 46 46	10	10 , 10	+1 15.66	+ 8 2.3	11 55 9.21	−40 19 27.6	*n*9.7653	*n*0.0884	+1.30	−1.4
25 20 0 32	11	10*d*, 8	−0 39.65	+ 0 14.5	11 58 54.95	−41 12 55.3	*n*9.7563	*n*9.9414	+1.33	−1.6
30 19 39 32	12	11*d*, 8	−0 8.71	−10 12.4	12 18 2.26	−45 20 29.6	*n*9.8062	*n*9.9309	+1.32	−2.1
Dec. 15 19 59 27	13	6 , 6	+1 19.13	− 3 19.4	13 20 25.26	−55 2 54.1	*n*9.8788	9.6828	+1.57	−4.3
19 19 47 2	14	. 4	. . . . . .	+ 8 59.5	. . . . . .	−56 59 50.2	. .	9.7576		−4.6
19 19 56 38	14	5*d*, . .	+0 9.76		13 37 57.14		*n*9.9057		+1.59	
22 19 55 16	15	8*d*, 6	−0 5.22	−12 12.0	13 51 17.31	−58 18 24.7	*n*9.9242	9.8019	+1.83	−4.9
1913										
Jan. 2 19 56 9	16	10 , 10	−2 31.82	− 7 44.2	14 40 33.36	−62 5 40.7	*n*9.9795	9.9470	−2.37	+9.8
7 19 0 54	17	10*d*, 8	+0 20.47	− 3 32.2	15 2 27.76	−63 21 53.8	*n*0.0378	*n*9.7597	−2.29	+8.7
8 19 36 30	18	10*d*, 8	+0 42.35	− 2 13.4	15 6 54.44	−63 35 59.1	*n*0.0208	9.6359	−2.29	+8.4
9 19 35 37	19	10*d*, 8	+0 44.60	− 4 13.8	15 11 13.86	−63 48 48.8	*n*0.0251	9.6209	−2.28	+8.2
11 19 26 51	20	10*d*, 6	−0 12.76	. . . . 0.0	15 19 42.50	−64 13 25.2	*n*0.0378	9.2870	−2.28	+7.7
12 19 29 26	21	10*d*, 8	−1 23.48	− 4 1.5	15 24 0.77	−64 25 13.2	*n*0.0394	9.4394	−2.29	+7.4
13 19 47 44	22	10*d*, 8	−0 5.47	+ 1 31.8	15 28 13.11	−64 36 31.3	*n*0.0290	9.8981	−2.27	+7.3
14 19 46 29	23	10*d*, 8	−1 13.61	+ 0 2.2	15 32 20.56	−64 46 55.7	*n*0.0330	9.8847	−2.27	+7.1
15 19 39 44	24	10*d*, 8	+0 18.72	+ 9 17.2	15 36 30.48	−64 57 21.0	*n*0.0411	9.7726	−2.26	+7.0
16 19 28 39	25	10*d*, 8	+0 37.57	+ 3 0.0	15 40 31.87	−65 7 7.5	*n*0.0517	9.4675	−2.23	+6.9
17 19 34 4	26	10*d*, 8	−0 35.35	− 1 7.3	15 44 34.64	−65 16 44.7	*n*0.0509	9.6610	−2.20	+6.6
18 19 45 57	27	10*d*, 8	−1 16.74	+ 3 54.6	15 48 38.94	−65 25 32.3	*n*0.0444	9.9024	−2.20	+6.5

Greenwich Mean Time	✱	No. of Comp.	☄ − ✱ Δα	☄ − ✱ Δδ	☄'s Apparent α	☄'s Apparent δ	log pΔ α	log pΔ δ	Red. to Ap. Pl. α	Red. to Ap. Pl. δ
Comet Zimmer – Giacobini (1913 e)										
1913 d h m s			m s	′ ″	h m s	° ′ ″			s	″
Oct. 26 12 19 16	28	10*d*, 8	+0 28.45	+ 5 35.9	18 56 18.38	− 7 24 40.5	9.5959	*n*0.5929	+2.27	−0.6
27 12 21 47	29	10 , 10	+0 54.73	− 5 25.7	19 1 8.07	− 8 20 30.6	9.5993	*n*0.5839	+2.30	−0.5
28 12 17 1	30	10 , 10	+0 40.62	+ 0 50.2	19 6 3.74	− 9 16 59.6	9.5916	*n*0.5713	+2.34	−0.5
29 12 36 5	31	10 , 10	−0 58.42	+ 0 30.9	19 11 11.05	−10 15 14.5	9.6190	*n*0.5701	+2.38	−0.3
29 12 55 1	32	8 , 8	−0 43.49	+ 3 39.4	19 11 15.19	−10 15 59.0	9.6423	*n*0.5807	+2.38	−0.3
30 12 32 59	33	10*d*, 8	−0 19.92	− 1 7.4	19 16 20.93	−11 13 25.0	9.6144	*n*0.5572	+2.41	−0.2
31 12 27 57	34	10*d*, 8	+0 26.29	− 6 33.5	19 21 37.79	−12 12 6.4	9.6068	*n*0.5418	+2.44	−0.1
Nov. 1 12 54 36	35	9 , 9	+1 3.52	− 2 56.6	19 27 8.55	−13 12 47.5	9.6418	*n*0.5489	+2.48	−0.1
2 12 35 24	36	10*d*, 8	−0 19.30	− 2 56.5	19 32 37.67	−14 12 4.4	9.6167	*n*0.5206	+2.53	+0.1
3 12 48 21	37	11 , 11	+0 57.33	− 2 41.7	19 38 21.06	−15 13 10.6	9.6338	*n*0.5181	+2.56	0.0
4 12 45 4	38	10*d*, 8	−0 3.86	− 1 52.0	19 44 8.52	−16 13 47.1	9.6292	*n*0.5002	+2.61	+0.2
5 12 29 24	39	10*d*, 8	+0 1.85	+ 2 36.7	19 50 0.60	−17 14 7.2	9.6058	*n*0.4673	+2.66	+0.2
6 12 12 51	40	10 , 10	+2 2.52	− 4 3.1	19 56 7.47	−18 15 41.2	9.6253	*n*0.4649	+2.69	+0.2
7 12 39 37	41	10*d*, 8	+0 12.80	+ 3 2.7	20 2 18.09	−19 16 34.2	9.6200	*n*0.4421	+2.75	+0.3
8 12 22 29	42	11*d*, 8	+0 10.04	− 7 35.0	20 8 32.98	−20 16 34.2	9.5908	*n*0.3973	+2.79	+0.5
9 12 28 38	43	10*d*, 8	+0 37.30	− 3 45.8	20 15 1.39	−21 17 10.5	9.6001	*n*0.3831	+2.83	+0.5
10 12 52 1	44	10*d*, 8	−0 30.70	− 1 59.6	20 21 41.75	−22 17 55.1	9.6358	*n*0.3982	+2.89	+0.8
11 12 47 8	45	10*d*, 8	+0 27.10	− 3 18.4	20 28 22.46	−23 16 48.9	9.6276	*n*0.3657	+2.93	+0.8
27 12 36 17	46	10*d*, 8	+0 6.58	+ 3 11.4	22 27 35.65	−35 26 24.2	9.5396	*n*9.0625	+3.62	+5.0
Comet Metcalf 1913 (b)										
Nov. 29 12 47 18	47	10*d*, 8	−0 16.65	− 1 14.6	20 52 10.99	−14 19 52.9	9.6630	*n*0.5527	+2.66	+5.5

Mean Places of Comparison Stars for 1912.0 and 1913.0.

✱	α	δ	Authority
	h m s	° ′ ″	
1	11 2 39.55	−23 16 28.8	*Argentine G. C.* 15203.
2	11 5 17.23	−25 7 16.6	*Argentine G. C.* 15286.
3	11 8 51.51	−25 59 11.6	*Argentine G. C.* 15351.
4	11 10 11.56	−27 6 7.4	*Cordoba B* 7412.
5	11 9 19.43	−27 20 10.6	*Cordoba B* 7405.
6	11 23 21.18	−31 39 51.2	*Cordoba B* 7538.
7	11 25 7.40	−32 11 30.3	*Cordoba C* 5769.
8	11 41 11.11	−37 39 12.6	*Argentine G. C.* 16089.
9	11 46 15.99	−38 28 50.8	Hawkins, *Cordoba M. C.* Mag. 8.8
10	11 53 52.25	−40 27 28.5	*Argentine G. C.* 16345.
11	11 59 33.27	−41 13 8.2	*Argentine G. C.* 16470.
12	12 18 9.65	−45 10 14.8	Hawkins, *Cordoba M. C.* Mag. 7.4.
13	13 19 1.56	−54 59 30.4	Hawkins, *Cordoba M. C.* Mag. 8.4.
14	13 37 45.79	−57 8 45.1	*Argentine G. C.* 18622.
15	13 51 20.70	−58 6 7.8	Hawkins, *Cordoba M. C.* Mag. 8.6.

*	α	δ	Authority
	h m s	° ′ ″	
16	14 43 7.55	−61 58 6.3	*Argentine G. C.* 20039.
17	15 2 9.58	−63 18 30.3	*Argentine G. C.* 20186.
18	15 6 11.38	−63 33 51.1	Hawkins, *Cordoba M. C.* Mag. 8.8.
19	15 10 31.54	−63 14 13.2	Hawkins, *Cordoba M. C.* Mag. 9.2.
20	15 19 57.54	−64 13 32.9	Boss *P. G. C.* 3920.
21	15 25 26.54	−64 21 19.1	Hawkins, *Cordoba M. C.* Mag. 8.9.
22	15 28 20.85	−64 38 10.4	Hawkins, *Cordoba M. C.* Mag. 9.
23	15 33 36.14	−64 47 5.0	*Argentine G. C.* 21171.
24	15 36 14.02	−65 6 15.2	Hawkins, *Cordoba M. C.* Mag. 8.
25	15 39 56.53	−65 10 11.4	Boss *P. G. C.* 3999.
26	15 45 12.19	−65 15 14.0	Hawkins, *Cordoba M. C.* Mag. 9.1.
27	15 49 57.88	−65 29 33.4	Hawkins, *Cordoba M. C.* Mag. 8.6.
28	18 55 47.66	− 7 30 15.8	*A. G. Wien-Ottakring* 6499.
29	19 0 11.04	− 8 15 4.4	*A. G. Wien-Ottakring* 6550.
30	19 5 20.78	− 9 17 49.3	*A. G. Wien-Ottakring* 6594.
31	19 12 7.09	−10 15 45.1	*A. G. Harvard* 6697.
32	19 11 56.30	−10 19 38.1	*A. G. Harvard* 6695.
33	19 16 38.44	−11 12 17.4	*A. G. Harvard* 6747.
34	19 21 9.06	−12 5 32.8	*A. G. Harvard* 6785.
35	19 26 2.55	−13 9 50.8	*A. G. Harvard* 6830.
36	19 32 54.44	−14 9 8.0	*A. G. Washington* 7369.
37	19 37 21.17	−15 10 28.9	*A. G. Washington* 7407.
38	19 44 9.77	−16 11 55.3	*A. G. Washington* 7452.
39	19 49 56.09	−17 16 44.1	*A. G. Washington* 7484.
40	19 54 2.26	−18 11 41.3	Hawkins, *Cordoba M. C.* Mag. 8.0.
41	20 2 2.54	−19 19 37.2	Hawkins, *Cordoba M. C.* Mag. 9.3.
42	20 8 20.15	−20 8 59.7	Hawkins, *Cordoba M. C.* Mag. 8.3.
43	20 14 21.26	−21 13 25.2	*Argentine G. C.* 27826.
44	20 22 9.56	−22 15 56.3	*Cordoba A.* 14188.
45	20 27 52.43	−23 13 31.3	Hawkins, *Cordoba M. C.* Mag. 9.6.
46	22 27 25.45	−35 29 40.6	*Argentine G. C.* 30723.
47	20 52 24.98	−14 18 43.8	Micrometer comparison with *48.
48	20 54 30.20	−14 28 23.0	*A. G. Washington*, 7899.

NOTES

Comet Tuttle

1912 Nov. 8. Comet 9th mag., 5′ diam. Very faint stellar nucleus.
9. Comet $8\frac{1}{2}$ mag. Central condensation decidedly brighter and easier to observe than last night.
10. Comet a little fainter than Nov. 9.
15. Comet 9th mag. Nucleus just discernible.
16. Comet 9th mag. Faint nucleus.
21. Comet 9th mag. Nucleus 12th mag. Diam. of comet 4′ ±.
24. Comet 9th mag. Fairly easy in moonlight.
30. Comet 9th mag. Fully as bright as last observation. Easy to measure in moonlight. Diam. 4′.
Dec. 15. Observation stopped by clouds.
19. Observation stopped by clouds. Comet 10th mag.
22. Comet near * 10 mag. which probably influences measures. Comet $10\frac{1}{2}$ or 11 mag.

Comet TUTTLE (*Continued*)

1913 Jan. 2. Comet $10\frac{1}{2}$ or 11 mag. Diam. 3′. Faint condensation.
7. Comet $10\frac{1}{2}$ or 11 mag. Diam. 3′. Central condensation but no nucleus.
8. Comet 11 mag.
9. Comet 11 mag. Diam. 2′.
11. Comet 11 mag. Difficult to measure in δ, too close to star.
12. Comet $11\frac{1}{2}$ mag. Very faint and difficult to measure.
13. Comet $11\frac{1}{2}$ - 12 mag. Very faint and difficult to measure.
14. Comet $11\frac{1}{2}$ mag.
15. Comet 12 mag. Very faint and difficult to measure.
16. Comet $11\frac{1}{2}$ or 12 mag. 1′ - 2′ diam. Comet easier to measure because in region freer of stars.
17. Comet $11\frac{1}{2}$ or 12 mag.
18. Comet $11\frac{1}{2}$ mag. Difficult.

Comet ZIMMER – GIACOBINI

1913 Oct. 28. Comet 9th mag. Head of comet 2′ diam. Tail 10′ long.
29. Comet 9th mag. Head of comet 2′ diam. Tail 10′ long, fan shaped. Nucleus fully 10 mag.
30. Comet same brightness as last night.
31. Comet $8\frac{1}{2}$ mag. Nucleus fully 10th mag. Head not over 2′ diam. Tail 8′ or 10′ long.
Nov. 4. Comet easy in moonlight.
6. Comet fully 9th mag. and easy in moonlight. Sharp nucleus 10th mag. Nebulosity extends N. E. but faint.
8. Comet easy notwithstanding moonlight. Rather sharp nucleus 10th mag.
9. Comet about $8\frac{1}{2}$ mag. Nucleus 10 or $10\frac{1}{2}$ mag.
10. Comet easy in moonlight.
27. Comet 8th mag. Nucleus 10th mag. Short tail n. f.

Observatorio Nacional Argentino, Cordoba, May 31, 1916.

OBSERVATIONS OF COMETS,

MADE WITH THE 26-INCH EQUATORIAL OF THE NAVAL OBSERVATORY.

[Communicated by Captain J. A. HOOGEWERFF, U. S. Navy, Superintendent.]

Date Wash. M. T.	✱	Comp.	Δα	Δδ	App. α	App. δ	log p Δ α	log p Δ δ	Red. to App. Pl. α	Red. to App. Pl. δ
				1914 *c* (NEUJMIN)						
1914 July 28 10 31 9 (h m s)	1	30 , 6	−3 21.58 (m s)	+1 40.7 (′ ″)	17 38 24.31 (h m s)	− 6 43 28.1 (° ′ ″)	9.179	0.794	+3.38 (s)	− 6.2* (″)
				1914 *e* (CAMPBELL)						
1914 Oct. 1 11 2 18	2	20 , 1	+5 26.02	+2 23.9	22 18 13.26	−26 19 1.7	9.074	0.897	+4.44	+17.3†
10 9 27 55	3	20 , 5	+2 1.21	+3 0.0	22 5 36.94	− 9 43 50.2	8.880	0.817	+3.84	+17.0†
18 9 21 20	1	20 , 1	+6 16.12	+2 1.1	21 51 45.11	− 2 19 58.1	9.192	0.760	+3.51	+17.3†
23 10 1 30	5	20 , 1	−3 31.11	−7 7.2	21 17 52.05	+ 0 38 51.9	9.428	0.735	+3.44	+18.9†
Nov. 10 6 27 21	6	5 , 1	+2 11.18	−4 34.4	21 47 25.80	+ 6 51 41.9	7.768*n*	0.666	+3.05	+19.9†
10 8 36 55	7	25 , 5	−3 59.08	+1 53.1	21 17 27.14	+ 6 53 2.3	9.381	0.678	+3.09	+20.3†
19 9 57 12	8	50 , 10	−1 19.18	−2 1.4	21 51 22.62	+ 8 51 32.6	9.602	0.690	+2.93	+20.8†

Date	Wash. M. T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ α	log pΔ δ	Red. to App. Pl. α	Red. to App. Pl. δ
					1915 *a* (Mellish)						
1915	h m s			m s	′ ″	h m s	° ′ ″			s	″
Feb. 17	16 43 41	9	30 , 6	−3 9.18	−1 35.7	17 12 12.23	+ 2 36 55.1	9.467*n*	0.720	+0.51	−16.9†
19	16 18 15	10	30 , 6	−3 21.19	+2 54.1	17 14 43.43	+ 2 27 58.5	9.509*n*	0.723	+0.58	−16.9†
26	17 19 55	11	25 , 5	−3 53.77	+1 11.9	17 23 37.58	+ 1 55 15.0	9.282*n*	0.723	+0.77	−17.1†
Mar. 9	15 23 30	12	30 , 6	−1 47.63	−5 41.4	17 37 22.19	+ 1 0 30.6	9.521*n*	0.734	+1.01	−17.0†
18	15 14 42	13	30 , 6	−4 7.80	+3 10.2	17 48 36.44	+ 0 7 32.5	9.191*n*	0.739	+1.23	−16.5†
31	13 21 36	14	30 , 6	+3 13.44	−5 58.9	18 4 40.71	− 1 33 50.2	9.607*n*	0.747	+1.58	−15.5†
Apr. 7	13 55 20	15	25 , 5	−3 14.28	−1 18.6	18 13 28.67	− 2 50 6.9	9.539*n*	0.757	+1.76	−14.0†
17	12 22 48	16	30 , 6	−2 17.94	−1 9.8	18 26 13.46	− 5 21 26.3	9.621*n*	0.759	+2.00	−12.0†
May 8	13 59 37	17	30 , 6	−2 3.47	−0 59.0	18 58 12.68	−16 50 34.8	9.360*n*	0.846	+2.68	− 4.9†
10	12 53 15	18	25 , 5	−2 9.43	−5 14.1	19 2 4.54	−18 42 46.5	9.527*n*	0.831	+2.74	− 3.9†
17	13 22 57	19	30 , 6	+0 53.29	+3 35.2	19 19 7.14	−27 28 1.0	9.462*n*	0.875	+3.12	+ 0.1†
19	13 23 42	20	30 , 6	−2 16.84	+5 7.8	19 25 22.50	−30 40 51.0	9.468*n*	0.883	+3.24	+ 2.1†
Nov. 9	12 22 29	21	25 , 5	−2 3.53	−5 3.4	4 59 39.57	− 6 59 48.1	9.218*n*	0.795	+4.34	+17.9†
10	10 56 4	22	30 , 6	−1 47.03	+4 29.5	4 57 19.98	− 6 40 0.6	9.481*n*	0.782	+4.38	+18.0†
16	10 2 51	23	25 , 5	+2 56.72	−4 17.8	4 42 38.42	− 4 31 28.2	9.510*n*	0.768	+4.58	+18.5†
Dec. 14	10 41 38	24	25 , 5	−1 51.05	+3 28.3	3 46 23.33	+ 5 17 37.6	8.718	0.685	+5.02	+19.7†
22	9 9 48	25	25 , 5	−1 45.86	+0 5.7	3 36 30.96	+ 7 38 24.1	8.692*n*	0.657	+5.06	+20.2†
1916 Jan. 8	8 25 51	26	24 , 5	−1 11.15	−6 49.9	3 24 16.75	+11 53 11.3	8.336	0.599	+1.74	+ 9.1†
					1915 *e* (Taylor)						
1915 Dec. 6	11 33 31	27	25 , 5	−1 26.58	+4 19.7	5 22 42.88	+ 0 1 35.7	8.993*n*	0.740	+4.94	+11.3†
8	11 40 19	28	25 , 5	−4 41.00	+5 52.2	5 21 36.78	+ 0 24 4.8	8.828*n*	0.736	+4.99	+10.8†
14	12 51 55	29	25 , 5	+0 47.43	−0 28.1	5 18 0.49	+ 1 46 33.6	9.108	0.724	+5.10	+11.1†
18	10 24 31	30	25 , 5	−0 53.38	+0 20.9	5 15 37.15	+ 2 51 5.5	9.104*n*	0.713	+5.16	+10.5†
22	10 46 53	31	25 , 5	−0 54.58	+1 1.1	5 13 13.34	+ 4 6 23.2	8.678*n*	0.699	+5.24	+10.4†
1916 Jan. 3	9 58 41	32	25 , 5	+1 17.78	−2 14.2	5 7 43.87	+ 8 34 18.7	8.592*n*	0.645	+2.14	+ 6.0†
7	8 45 7	33	30 , 6	−1 7.92	−5 38.5	5 6 50.70	+10 12 55.9	9.182*n*	0.630	+2.19	+ 6.1†
25	11 23 12	34	20 , 4	−2 5.46	+4 12.6	5 11 35.12	+17 58 31.5	9.462	0.549	+2.21	+ 6.8†
Feb. 29	8 11 29	35	27 , 9	−0 48.23	+2 3.7	6 3 58.06	+29 23 54.9	8.976	0.170	+2.31	+ 7.6†
Mar. 31	11 12 23	36	35 , 6	+1 35.75	+0 26.5	7 22 7.89	+33 2 11.8	9.700	0.496	+2.32	+ 5.4†
					1916 *a* (Neujmin)						
1916 Mar. 4	9 12 12	37	25 , 5	+1 27.91	+3 21.0	8 59 44.87	+11 29 8.7	9.064*n*	0.609	+2.70	− 8.1†
31	9 9 32	38	29 , 6	−3 44.02	−2 24.0	9 26 44.22	− 1 4 7.8	8.584	0.750	+2.40	−12.9†
Apr. 6	9 47 25	39	30 , 6	+0 59.36	+3 42.7	9 36 47.24	− 3 17 5.5	9.144	0.768	+2.32	−13.9†
May 5	10 35 1	40	25 , 5	+3 26.96	−2 42.2	10 36 31.30	−11 21 42.0	9.505	0.804	+2.21	−18.6†
					1916 *b* (Wolf)						
1916 May 6	11 17 30	41	25 , 5	−2 15.20	+5 51.1	12 36 9.49	+ 3 7 53.2	9.287	0.712	+2.90	−17.8†
June 3	10 9 58	42	25 , 5	−5 2.91	−6 33.9	12 28 53.37	+ 4 32 7.5	9.446	0.703	+2.72	−15.4†
26	9 40 46	43	20 , 4	−3 11.51	−1 5.0	12 30 26.27	+ 4 43 47.3	9.561	0.710	+2.50	−13.6†
July 6	10 34 38	42	20 , 4	−0 37.53	−4 55.6	12 33 18.43	+ 4 33 48.1	9.647	0.728	+2.40	−13.1†

Observers: * C. B. Watts. † H. E. Burton.

Mean Places of Comparison Stars for the Beginning of the Year

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	17 11 12.51	− 6 18 2.6	*A.G. Wien-Ottakring* 5991	23	1 39 37.12	− 1 27 28.9	*A.G. Straszburg* 1250
2	22 43 12.80	−26 21 12.9	*Argentine G. C.* 31046	24	3 48 9.36	+ 5 13 49.6	½ { *A.G. Albany* 1129 *A.G. Leipzig II* 1129
3	22 3 31.89	− 9 47 7.2	*A.G. Camb.*, U.S. 7820	25	3 38 11.76	+ 7 37 58.2	*A.G. Leipzig* II 1361
4	21 45 25.18	− 2 22 16.5	*A.G. Straszburg* 7628	26	3 25 26.16	+11 59 52.1	*A.G. Leipzig* I 1022
5	21 51 23.05	+ 0 45 10.2	*A.G. Nicolajew* 5530	27	5 24 1.52	− 0 2 55.3	*A.G. Nicolajew* 1332
6	21 15 11.27	+ 6 55 56.1	*A.G. Leipzig* II 10963	28	5 26 12.79	+ 0 18 1.8	*A.G. Nicolajew* 1353
7	21 51 23.43	+ 6 50 18.9	*A.G. Leipzig* II 11013	29	5 17 7.96	+ 1 46 50.6	*A.G. Albany* 1671
8	21 52 39.17	+ 8 53 16.2	*A.G. Leipzig* II 11033	30	5 16 25.37	+ 2 50 34.1	*A.G. Albany* 1664
9	17 15 20.87	+ 2 38 18.0	*A.G. Albany* 5730	31	5 14 2.68	+ 1 5 11.7	*A.G. Albany* 1649
10	17 18 1.04	+ 2 25 21.0	*A.G. Albany* 5752	32	5 6 23.95	+ 8 36 26.9	*A.G. Leipzig* II 2042
11	17 27 30.58	+ 1 51 20.2	*A.G. Albany* 5811	33	5 7 56.43	+10 18 28.3	*A.G. Leipzig* I 1553
12	17 39 8.78	+ 1 6 29.0	*A.G. Nicolajew* 4391	34	5 13 38.37	+17 54 12.1	*A.G. Berlin* A. 1459
13	17 52 43.01	+ 0 4 38.8	*A.G. Nicolajew* 4455	35	6 4 43.98	+29 21 43.6	*A.G. Camb.*, Eng. 3003
14	18 1 25.69	− 1 27 35.8	*A.G. Nicolajew* 4487	36	7 20 29.82	+33 1 39.9	*A.G. Leiden* 3125
15	18 16 41.19	− 2 48 34.3	*A.G. Straszburg* 6136	37	8 58 14.26	+11 25 55.8	*A.G. Leipzig* I 3627
16	18 28 29.40	− 5 20 4.5	*A.G. Straszburg* 6202	38	9 30 25.84	− 1 1 30.9	*A.G. Nicolajew* 2893
17	19 0 13.47	−16 49 30.9	*A.G. Washington* 7066	39	9 35 45.56	− 3 20 34.3	*A.G. Straszburg* 3763
18	19 1 11.23	−18 37 28.5	*A.G. Algiers* 8112	40	10 33 2.13	−11 18 41.2	*A.G. Camb.*, U.S. 4026
19	19 18 10.73	−27 31 36.6	*Argelander's Zonen-Beobachtungen von Oeltzen* 19456	41	12 38 21.79	+ 3 2 19.9	*A.G. Albany* 4564
20	19 27 36.10	−30 46 0.9	*Catalogo de Zonas Estelares* 1075	42	12 33 53.56	+ 4 38 56.8	*A.G. Albany* 4527
21	5 1 38.76	− 6 55 2.6	*A.G. Wien-Ottakring* 1334	43	12 33 35.28	+ 4 45 5.9	½ { *A.G. Albany* 4521 *A.G. Leipzig II* 6182
22	4 59 2.63	− 6 44 48.1	*A.G. Wien-Ottakring* 1321				

NOTES

1914 *c* (Neujmin)

Date	Seeing	
1914		
July 28.	Poor.	Comet faint and diffused.

1914 *e* (Campbell)

1914		
Oct. 1.	Fair.	Comet visible in 5-inch finder. Nebulous. Fairly bright nucleus. Moonlight.
Oct. 10.	Poor.	Barely visible in 2-inch finder. Faint nucleus. Thin clouds at times.
Oct. 18.	Fair.	Visible in 2-inch finder. Faint nucleus surrounded by nebulosity. Seems to have faint tail following.
Oct. 23.	Fair.	Visible in 5-inch finder.
Nov. 10.		Clouds interfered (first observation).
Nov. 10.	Good.	Visible as faint patch of light in 5-inch finder (second observation).
Nov. 19.	Fair.	Comet very faint; too faint for satisfactory measures. Not a good observation.

1915 *a* (Mellish)

1915		
Feb. 17.	Fair.	Comet barely visible in 5-inch finder. Small faint nucleus.
Feb. 19.	Fair.	Visible in 5-inch finder. Faint nucleus surrounded by nebulosity.

1915 *a* (Mellish) *Continued*

Date	Seeing	
1915		
Feb. 26.	Poor	Visible in 5-inch finder.
Mar. 9.	Good.	Visible in 5-inch finder. Fairly bright nucleus.
Mar. 18.	Poor.	Barely visible in 2-inch finder. Visible in 5-inch finder.
Mar. 31.	Poor.	Could not see comet in 5-inch finder. Moonlight.
Apr. 7.	Fair.	Comet visible in 2-inch finder. Brighter than when last observed. Bright nucleus surrounded by nebulosity.
Apr. 17.	Poor.	Barely visible in 2-inch finder. Fairly bright nucleus. Perhaps a little haze.
May 8.	Poor.	Visible in 2-inch finder. Tail noticeable in 5-inch finder. Bright nucleus. Seems to be double, having faint component preceding a trifle north. Thin clouds and haze.
May 10.	Fair.	Comet fainter than on May 8, perhaps on account of haze or city lights. Fainter component seen.
May 17.	Poor.	Visible to naked eye. Estimated magnitude about 5th. Bright nucleus.
May 19.	Fair.	Visible to naked eye. Estimated magnitude about 5th.
Nov. 9.	Fair.	Visible in 5-inch finder.
Nov. 10.	Fair.	Visible in 5-inch finder.
Nov. 16.	Good.	Visible in 5-inch finder. Moonlight.
Dec. 14.	Fair.	Could not see comet in 5-inch finder. Moonlight.
Dec. 22.	Fair.	Could not see comet in 5-inch finder. Moonlight.
1916		
Jan. 8.	Poor.	

Measures, with Respect to Bright Nucleus, of Companion Found May 8

	Wash. M.T.	p	s	No. of Meas.	Seeing	
1915						
May 17.	$14^h 29^m 7^s$	288°.08	39″.4	4, 4	Poor.	Looked for intermediate object found by Prof. Barnard. Thought I saw it occasionally.
May 19.	$14^h 23^m 29^s$	291°.05	84″.6	4, 4	Fair.	Looked for intermediate object. Could not be sure of seeing it. Clouds interfered.

1915 *e* (Taylor)

Date	Seeing	
1915		
Dec. 6.	Fair.	Comet barely visible in 5-inch finder. Faint nucleus. Seems to have faint tail preceding a trifle north.
Dec. 8.	Poor.	Comet barely visible in 5-inch finder.
Dec. 14.	Fair.	Could not see comet in 5-inch finder.
Dec. 18.	Poor.	Moonlight. Windy.
Dec. 22.	Fair.	Could not see comet in 5-inch finder. Moonlight.
1916		
Jan. 3.	Poor.	Comet visible in 5-inch finder.
Jan. 7.	Poor.	Visible in 5-inch finder.
Jan. 25.	Fair–poor.	Visible in 5-inch finder. Interrupted by clouds. Poor observation.
Feb. 29.	Good.	Barely visible in 5-inch finder.
Mar. 31.	Fair.	Comet very faint. Not satisfactory observation.

1916 *a* (NEUJMIN)

Date 1916	Seeing	
Mar. 4.	Fair.	Barely visible in 5-inch finder. Faint nucleus surrounded with nebulosity.
Mar. 31.	Fair.	Comet faint. Not visible in 5-inch finder.
April 6.	Poor.	Comet faint. Windy.
May 5.	Fair.	Comet faint.

1916 *b* (WOLF)

May 6.	Fair.	Comet very faint. Probably haze.
June 3.	Poor.	Comet faint. Not satisfactory observation.
June 26.	Fair.	Comet faint.
July 6.	Fair.	Comet very faint. Moonlight. Uncertain about this being the comet.

EDITORIAL NOTICE.

The Editors wish to call attention to a few changes in the future conduct of the *Astronomical Journal.*

The size of a number will depend largely upon the amount of material on hand at the date of publication, but the volume, as heretofore, will contain at least two hundred quarto pages. This should facilitate the prompt publication of articles submitted.

Announcements of important new discoveries will be made. Comet ephemerides will be published when available. For purposes of reference the titles of publications issued by observatories will be listed. To make these lists as complete as possible will require the courteous coöperation of observatories in furnishing the Editor with the titles of their publications as they appear. This does not include the titles of papers published in bulletins or periodicals.

CORRIGENDUM.

In the making of the plates from the original drawings, in the article "On Evidences of Systematic Variation in Recent Longer Series of Determinations of Stellar Parallax," by ALBERT S. FLINT, *Astronomical Journal*, No. 696, sensible changes have occurred in the intervals in certain cases. The required corrections to the several authorities represented are as follows:

Y	$-0''.0034$	FL. II	$+0''.0003$	SCHL.	$+0''.0023$	PR.	$-0''.0032$
FL. I	$+0\ .0014$	RUSS.	$+0\ .0034$	JDKO.	$-0\ .0008$		

CONTENTS.

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OBSERVATIONS OF WOLF'S COMET (*1916 b*).

BY E. E. BARNARD.

When we received the announcement of the discovery of this comet on May 1 there was much uncertainty as to its nature and one was in doubt what to look for. It was possibly an asteroid. A photograph of this region with the Bruce 10-inch lens on 1916 April 24, showed a trail near the place. This, however, proved not to be WOLF's object, but the asteroid *Aeternitas* (446); See *A. N.* 4845. When it was located by a photograph taken with the two-foot reflector by MR. E. P. HUBBLE, to whom I am greatly indebted, its cometary nature was at once evident. A closer inspection then showed it on the photograph of April 24, where its position was measured with respect to a faint star close to it, which was subsequently accurately measured in the sky. This position is incorporated in the measures. On the photograph the comet was excessively faint and could only be found by knowing closely where it should be.

Visually, in the large telescope, the object was always cometary, though it was very small. Under good conditions it was in no wise a difficult or faint object, as the notes that follow the observations will show. It is probable that the comet will become a naked eye object in June of 1917.

Positions of the Comet.

1916	C. S. T.	$\Delta\alpha\cos\delta$	$\Delta\alpha$	$\Delta\delta$	Comps.	α Appt.	δ Appt.	Red. to Appt.	*
	h m s	″	m s	′ ″		h m s	° ′ ″	s ″	
Apr. 24	9 16 30	− 40.7	−0 2.71	−0 5.5	. . .	12 41 42.39	+2 10 23.8	+3.00 −18.7	1
May 6	9 44 27	+159.6	+0 10.65	−0 26.1	10, 8	12 36 10.28	+3 7 45.6	+2.89 −17.8	1½
10	9 0 55	−128.2	−0 8.56	+0 25.2	6, 10	12 34 36.10	+3 24 11.2	+2.87 −17.5	2
10	9 22 20	−132.2	−0 8.83	+0 29.0	8, 5	12 34 35.83	+3 24 15.0	+2.87 −17.5	2
22	10 7 47	−164.6	−0 11.00	−1 48.7	7, 10	12 30 51.91	+4 5 18.7	+2.79 −16.4	3
27	10 32 39	+ 34.7	+0 2.32	±0 0.0	6, 9	12 29 48.60	+4 18 17.4	+2.74 −15.9	4
June 3	10 20 19	. .	. . . . . .	+0 47.8	. 9	. . . . .	. . .	. . . . . .	5
	10 41 28	− 60.1	−0 4.02	. . . .	5 . .	. . . .	. . . . . .	. . . .	5
17	9 35 2	−212.1	−0 14.17	+1 33.9	5, 8	12 28 59.18	+4 45 11.6	+2.56 −14.3	6
21	10 6 8	+160.6	+0 10.73	+2 39.8	10, 10	12 29 30.18	+4 45 26.7	+2.53 −14.1	7
24	9 17 4	+286.7	+0 19.18	+2 29.5	5, 8	12 30 1.31	+4 44 43.2	+2.50 −13.8	8
28	9 21 40	− 71.2	−0 4.76	−0 2.2	6, 10	12 30 54.23	+4 42 26.7	+2.46 −13.6	9
July 1	9 10 55	− 5.3	−0 0.36	−0 50.4	7, 7	12 31 42.19	+4 39 49.1	+2.45 −13.5	10
1	9 44 31	+ 2.1	+0 0.14	−0 50.8	6, 4	12 31 42.69	+4 39 48.7	+2.45 −13.5	10
5	9 21 2	+185.6	+0 12.41	+0 26.6	5, 8	12 32 57.28	+4 35 10.8	+2.35 −13.2	11
8	9 37 40	. . .	. . .	+0 5.0	3	.	+4 30 48.4	. . −13.0	12
8	9 45 44	+207.3	+0 13.86	. . . .	5 .	12 34 1.53	. . . . .	+2.39 . . .	12

Mean Places of Comparison Stars.

*	α 1916.0	δ 1916.0	Authority
	h m s	° ′ ″	
1	12 41 42.10	+2 10 48.0	13.5 mag. Compared with Albany *A. G. C.* 4569.
1½	12 35 56.74	+3 8 29.5	Albany *A. G. C.* 4533.
2	12 34 41.79	+3 24 3.5	Albany *A. G. C.* 4531.
3	12 31 0.12	+4 7 23.8	12.8 mag. Measured by Mr. E. P. Hubble on 2-ft. reflector photograph.
4	12 29 43.54	+4 18 33.3	13 mag. Compared with Albany *A. G. C.* 4510.
5			12½ mag.
6	12 29 10.79	+4 13 52.0	13 mag. Compared with Albany *A. G. C.* 6166.
7	12 29 16.92	+4 13 1.0	12 mag. Compared with Albany *A. G. C.* 6166.
8	12 29 39.63	+4 42 27.5	13± mag.(?) Compared with Albany *A. G. C.* 6166.
9	12 30 56.53	+4 42 42.5	12.5 mag. Compared with Star 7.
10	12 31 40.10	+4 40 53.0	15½ mag. Compared with Star 8.
11	12 32 42.52	+4 34 57.4	13 mag. Compared with Albany *A. G. C.* 4527.
12	12 33 45.28	+4 30 56.4	13 mag. Compared with Star 10.

Measures of Comparison Stars.

1916		Δα cos δ	Δα	Δδ	Comps.
		″	m s	′ ″	
May 22	Star 1 — Albany 4569		−1 41.08	+2 6.8	16 tr, 6
May 27	Star 4 — Albany 4510		+1 56.44	−2 50.0	10 tr, 4
June 17, 24	Star 6 — Leipzig 6166	143.3	−0 9.57	+2 49.5	8, 8
June 21, 24	Star 7 — Leipzig 6166	51.0	−0 3.41	+1 58.6	8, 8
June 24, 28	Star 8 — Leipzig 6166	288.1	+0 19.27	+1 25.0	8, 8
June 28, July 1	Star 9 — Star 8		+1 16.89	+0 15.0	24 tr, 6
July 1	Star 10 — Star 9		+0 43.57	−1 49.6	12 tr, 4
July 5, 8	Star 11 — Albany 4527		−1 11.04	−3 59.4	28 tr, 7
July 8, 15	Star 12 — Star 11		+1 2.76	−4 1.0	14 tr, 3

NOTES ON THE APPEARANCE OF THE COMET

May 6. Round, not very diffused. A tiny nucleus of 16th magnitude north preceding the center. Measured diameter of the comet, 3″.7. At 12h 30m it was quite conspicuous and about 12½ magnitude.

May 10. The comet was very faint from moonlight and haze.

May 22. It was 13½ magnitude, with a star-like nucleus of 14 or 14½ magnitude north preceding the center. Measured diameter = 11″.1.

May 27. It was 13½ or 14 magnitude, round, with a faint nucleus in the north preceding part.

June 3. Excessively difficult in clouds. Position of comparison star not determined. A fainter star preceding the comet 1′—2′.

June 17. Faint in bad sky.

June 21. Very faint in clouds.

June 24. Faint on a bright sky. It was 14th magnitude, with a faint star-like nucleus.

June 28. The comet was $10'' \pm$ in diameter and seemed to brush out south. With a magnifying power of 700 diameters it was a little better seen. It was $14\frac{1}{2}$ magnitude, with possibly a faint nucleus. Sky thick.

July 1. Very small and faint. Better seen during the second set of measures.

July 5. Very small; 14 or 15 magnitude.

July 8. Excessively difficult and uncertain from moonlight and bad seeing, especially at the measures for right ascension.

The comet could have been followed longer but for the cloudy and bad western skies.

Yerkes Observatory, Williams Bay, Wisconsin, 1916, Sept. 5.

NOTE ON THE BRIGHTER "A" STARS.

By CHARLES A. MANEY.

This note serves a two-fold purpose both in replacing a part of the work of a previous article(1) and in presenting some figures which the writer hopes are of value in the study of stellar motions. A reply to Prof. H. C. Plummer's excellent criticism(2) of the article referred to is deemed unnecessary, inasmuch as errors found in the values of the galactic latitudes used have considerably altered the results of that article. The figures given in this note have been carefully checked.

The values of the radial velocities of 401 stars of type "*A*" are used in this discussion. Most of the radial velocities are from Lick(3), Mt. Wilson(4), and Yerkes(5) determinations. Where the same star was observed at more than one place, a mean of the various values was used.

The stars were divided into two groups: 200 stars of galactic latitudes from 0 to *±31.7, and 201 stars of galactic latitudes from ±31.8 to ±90. Solutions for the solar motion yielded the following results:

Table I

Gal. Lat.	V_0	K
0° to ±31°.7	−20.1 ±1.2	+1.2 ±0.8
±31°.8 to ±90°	−14.6 ±1.3	+0.5 ±0.7

*Stars in that hemisphere of galactic co-ordinates containing the north celestial pole are considered of positive latitude.

"V_0" and "K" are quantities in the equation

$$V_0 \cos d + K - V = O$$

The above results indicate a relative motion of several kilometers per second between the "*A*" stars of high and low galactic latitudes. The method used simply gives the components of motion in the direction of the solar motion.

From the fact that different values of wave lengths are used by the three observatories, it is to be expected that the radial velocities from each of the places differ from each other by systematic quantities. Accordingly, Mt. Wilson and Yerkes radial velocities were "reduced" to the Lick scale. There were eighteen Mt. Wilson radial velocities in common with the Lick values. From these as data, it was found that the correction to be added to the Mt. Wilson observations is +1.2 ±0.7. Fifty-two Yerkes observations also had Lick values. The correction to be added to Yerkes observations was found to be −0.9 ±0.4.

The least-square solutions were repeated with the Yerkes and Mt. Wilson radial velocities reduced to Lick values. As was expected, the results were only slightly affected by these corrections. Table II gives, in addition to the values of Vo and K, the number of stars of plus and minus galactic latitudes; *i. e.*, their distribution, and the mean value of the residuals for each group.

TABLE II

Gal. Lat	V_0	K	No. Stars of		Totals	Mean Residual
			+ G. L.	− G. L.		
0° to ±31°.7	−20.0 ±1.2	+1.3 ±0.8	91	109	200	12.6
±31°.8 to ±90°	−14.4 ±1.3	+0.7 ±0.7	136	65	201	9.3

The distribution is seen to be quite irregular, particularly in the groups ±31.8 to ±90.

Using the corrected values of the radial velocities of the 401 stars, the solution for the solar motion gives the results as expressed in Table III.

TABLE III

No. of Stars	V_0	K	Mean Resid.
401	−17.9 ±0.9	+1.2 ±0.5	11.1

TABLE IV

Gal. Lat.	No. of Stars of		Totals	Mean Value of Residuals
	+G. L.	−G. L.		
0° to ±32°	91	109	200	12.9
32° to 60°	83	56	139	10.1
60° to 90°	53	9	62	7.5

Table IV gives the distribution, and mean values of residuals as determined from the general solution.

The figures in the last column are quite in harmony with the Galactic Hypothesis.

The distribution in zones ±60 to ±90 is seen to be very irregular and the value 7.5 is therefore more representative of the zone +60 to +90 than of the zones ±60 to ±90. The nine stars of minus latitude give a mean residual of 9.6 as compared with 7.4 for the 53 stars of plus latitude.

August, 1916, Alma, Michigan.

REFERENCES:

(1) *A. J.* Nos. 679-680. Page 53. 1915. C. A. MANEY.

(2) *A. J.* Nos. 686-687. Page 118. 1915. H. C. PLUMMER.

(3). *L. O. B.* VII, 211. 1912. W. W. CAMPBELL.

(4) *Astrophysical Journal.* Vol. XLII. 1915. WALTER S. ADAMS.

(5). Unpublished data.

OBSERVATIONS OF THE SATELLITES OF *URANUS*.

BY E. E. BARNARD.

The following observations of the satellites of *Uranus* are a continuation of those printed in *A.J.* Vol. XXIX, p. 39. They are not corrected for refraction. The conditions of seeing have been too poor in general from the low altitude of the planet to permit observations of *Ariel* and *Umbriel*. As usual the times are $6^h\ 0^m$ slow of Greenwich Mean Time.

	C. S. Time	P. A.	Dist.	Comp	Remarks
		Uranus AND *Ariel*			
	h m s	°	"		
1915 Oct. 5	9 16 25	152.29		5	
	9 22 28		14.98	8	

Uranus AND *Umbriel*

	C. S. Time	P. A.	Dist.	Comp.	Remarks
	h m s	°	"		
1915 Oct. 5	9 29 31	334.70	....	6	Very difficult.
	9 35 44	.	17.01	8	

Uranus AND *Titania*

	C. S. Time	P. A.	Dist.	Comp.	Remarks
1915 Aug. 17	10 26 7	212.89	...	5	*Titania* ½ mag. brighter than *Oberon*.
	10 32 35	.....	22.54	8	
26	11 47 47	230.76		5	Very faint. Very bad seeing.
	11 53 16	..	19.28	9	
31	11 58 56	96.83	....	5	Very low and very bad seeing.
	12 4 38	.....	17.84	9	
Sept. 16	11 7 35	18.13	....	6	Difficult in clouds.
	11 13 48	.....	25.78	2	Single distances. Lost in clouds.
28	9 19 46	168.85	...	6	
	9 25 29	.....	30.81	8	Seeing very bad.
Oct. 5	8 53 13	99.71	....	6	
	8 57 52	.....	17.98	8	
12	7 24 00	9.60	...	6	Very difficult.
	7 28 58	.....	29.45	8	Seeing very bad.
26	6 22 48	222.52	....	6	Seeing excessively bad.
	6 28 10	.	20.38	8	
Nov. 2	6 7 12	169.10	....	7	*Titania* perhaps slightly brighter than *Oberon*.
	6 12 53	.....	29.96	10	

Uranus AND *Oberon*

	C. S. Time	P. A.	Dist.	Comp.	Remarks
1915 Aug. 17	10 38 28	229.42	....	5	
	10 44 55	...	25.61	8	
26	11 37 12	142.03	..	5	
	11 42 56	.	33.03	8	
31	11 46 40	266.32	....	5	
	11 52 39	.	22.80	9	
Sept. 28	9 7 20	299.80	..	5	Seeing very bad.
	9 13 7	....	26.49	8	
Oct. 5	9 2 14	126.96	...	5	
	9 6 54	...	28.40	8	
12	7 12 31	313.21	.	5	Very difficult.
	7 18 17	...	29.96	9	Very bad seeing.
26	6 9 18	325.61	..	5	Seeing excessively bad.
	6 15 8	.	33.56	8	
Nov. 2	5 55 32	150.53		6	Very faint and difficult. Seeing very bad.
	6 1 12	..	35.06	8	

Yerkes Observatory, Williams Bay, Wis., 1916 September 4.

A CONVERGENT POINT FOR FOUR CLUSTERS OF SMALL PROPER-MOTION STARS,

By BENJAMIN BOSS.

In the course of a detailed study of the proper-motions of the *Preliminary General Catalogue* a number of probable star groups have been disclosed, but in many cases the proper-motions are so small or the group is restricted to so small a sky area that it has been impossible to determine the convergent point from each individual group.

The close agreement between the amount and position-angle of proper-motion of the individual stars in each group made it more than probable that we had to deal with physical clusters. It seemed possible that several of these groups might represent motion toward a common convergent point. Consequently a mean was taken of the right ascensions, the declinations, and the position-angles of proper-motion for each group, and the paths were plotted. There resulted a suspiciously sharp convergent for four of the groups at about R. A. = 4^h 45^m, Decl. = $-15°$. While the convergence might be considered fortuitous, an inspection of the details will lend considerable strength to the supposition of a true convergent. In some of the groups a few stars which share in the common direction of motion have been rejected because of disagreement with the mean amount of proper-motion, an indication that they probably do not belong to the group proper.

The four tables present the details for each group. Column one gives the number of the star in the *Preliminary General Catalogue*; column two the magnitude; three the type; four the right ascension for 1875; five the declination for 1875; six the proper-motion; seven the observed position-angle of motion; eight the position-angle required to bring the star's path to convergence at the point R. A. = 4^h 45^m, Decl. = $-15°$; nine the difference between the last two columns; and column ten the probable error of the position-angle of each star.

It must be noted that a more refined convergent point might slightly decrease the size of the differences. However, the differences as they stand are entirely in keeping with the supposition of group motion.

Table V shows the observed deviations of the position-angles drawn up in order of the probable error for the four groups. It can readily be seen that with seven exceptions the observed deviation is less than the probable error, a far better result than might have been expected. Though masked to a certain extent by the grouping of large accidental errors, the increase of observed with probable errors is well indicated.

There is little evidence to be derived from radial velocities. ADAMS has published the radial velocities of a number of the stars in the *Pleiades* group. (*Astrophysical Journal*, Vol. XIX, No. 5). The values obtained were quite at variance, but as the spectra of these stars are not readily measured we must accept ADAMS' caution and not place too great reliance upon the determinations. Aside from the *Pleiades* group there are but two radial velocities given, as far as I am aware, both derived from the "Contributions from the Mount Wilson Solar Observatory, No. 82." One is for *P. G. C.* 2889 in group IV the value given being +22.0 km. The other value of +32.0 km. for *P. G. C.* 2428 in Group III rests upon one determination only. Consequently no further light can be thrown upon these groups at present.

The detection of further tendencies toward motion in the direction of the convergent for the four groups discussed will form the subject of a later paper.

TABLE I — *Pleiades*

R. A. = 3^h 39^m Decl. = +23° 51′

P.G.C.	M	Type	R. A. 1875	Decl. 1875	μ	θ	θ_c	C—O	p. e.
851	5.8	B_5	3^h 37^m 23^s	+23° 54′	6″.0	159°.4	155°.3	−4°.1	±4°.3
852	3.8	B_5	3 37 27	+23 43	5 .5	157 .6	155 .3	−2 .3	2 .6
855	6.0	B_8	3 37 42	+24 27	5 .9	162 .2	155 .6	−6 .6	5 .1
856	4.4	B_5	3 37 46	+24 4	5 .1	165 .1	155 .5	−9 .6	4 .4
860	3.9	B_5	3 38 23	+23 59	5 .5	146 .0	155 .7	+9 .7	3 .8
865	4.2	B_5	3 38 55	+23 33	6 .0	155 .6	155 .7	+0 .1	3 .6
867	7.4		3 39 55	+23 44	5 .9	159 .1	156 .1	−3 .0	5 .6
869	2.8	B_5	3 40 3	+23 43	5 .3	156 .8	156 .1	−0 .7	1 .6
877	3.7	B_8p	3 41 44	+23 40	5 .5	157 .6	156 .6	−1 .0	2 .3
879	5.3	B_8p	3 41 45	+23 45	5 .4	157 .2	156 .7	−0 .5	±3 .6

Table II — *Præsepe*

R. A. = $8^h\ 43^m$ Decl. = $+19°\ 4'$

P.G.C.	M	Type	R. A. 1875	Decl. 1875	μ	θ	θ_c	C—O	p. e.
2282	6.9	*G*	$8^h\ 28^m\ 8^s$	+20° 1′	4″.3	246°.4	241°.8	−4°.6	±4°.9
2305	6.9	. .	8 32 32	+20 13	3 .4	243 .4	242 .3	−1 .1	7 .5
2308	6.7	*A*	8 32 55	+20 27	4 .2	239 .7	242 .2	+2 .5	5 .8
2309	6.9		8 33 0	+20 25	3 .4	241 .9	242 .3	+0 .4	8 .0
2310	6.8	A_5p	8 33 11	+20 7	3 .9	237 .5	242 .4	+4 .9	6 .5
2311	6.5	*A*	8 33 17	+19 59	3 .4	239 .6	242 .5	+2 .9	7 .5
2314	7.1	. .	8 33 46	+20 1	3 .9	247 .4	242 .5	−4 .9	7 .6
2480	5.7	*K*	9 8 20	+15 28	3 .7	244 .1	248 .0	+3 .9	5 .6
2576	6.6	*A*	9 29 3	+14 56	3 .8	251 .6	250 .1	−1 .5	±4 .7

Table III

R. A. = $8^h\ 28^m$ Decl. = $-46°\ 52'$

P.G.C.	M	Type	R. A. 1875	Decl. 1875	μ	θ	θ_c	C—O	p. e.
2114	5.1	*Kp*	$7^h\ 53^m\ 56^s$	−45° 14′	2″.9	294°.8	282°.4	−12°.4	±13°.8
2134	6.0	*A*	7 57 3	−36 56	3 .4	270 .0	284 .2	+14 .2	10 .7
2141	2.0	*Od*	7 59 11	−39 39	3 .2	284 .5	286 .1	+ 1 .6	4 .8
2319	5.7	B_5	8 35 53	−53 0	3 .6	303 .7	287 .4	−16 .3	12 .0
2326	5.8	*A*	8 36 43	−52 34	3 .2	302 .2	287 .0	−15 .2	12 .6
2340	5.8	B_9	8 38 44	−52 39	3 .5	298 .7	286 .6	−12 .1	10 .8
2359	6.5	*A*	8 41 51	−34 10	3 .8	283 .7	274 .4	− 9 .3	14 .3
2428	5.7	B_5	8 56 42	−41 22	4 .2	270 .0	276 .4	+ 6 .4	8 .6
2483	5.4	B_3	9 9 45	−42 43	3 .0	271 .9	274 .8	+ 2 .9	10 .4
2504	5.4	*A*	9 13 56	−50 32	3 .4	282 .0	277 .9	− 4 .1	± 8 .1

Table IV

P.G.C.	M	Type	R. A. 1875	Decl. 1875	μ	θ	θ_c	C—O	p. e.
2606	5.3	*B*	$9^h\ 37^m\ 29^s$	−80° 23′	4″.2	282°.4	284°.3	+ 1°.9	± 6°.6
2715	7.6	. .	10 11 28	−86 18	4 .5	273 .8	277 .5	+ 3 .7	10 .3
2889	4.6	B_3	10 44 35	−79 53	4 .7	267 .6	267 .3	− 0 .3	5 .1
3134	5.1	B_9	11 53 27	−77 32	4 .3	249 .4	250 .2	+ 0 .8	5 .9
3514	6.6	*A*	13 28 34	−75 3	4 .5	221 .4	227 .0	+ 5 .6	10 .6
4387	6.1	M_a	17 8 8	−80 44	4 .2	178 .6	174 .4	− 4 .2	± 7 .8

Table V

p. e.	C—O	p. e.	C—O	p. e.	C—O	p. e.	C—O
±1°.6	−0°.7	±4°.9	−4°.6	± 7°.5	+ 2°.9	±10°.8	−12°.1
2 .3	−1 .0	5 .1	−0 .3	7 .6	− 4 .9	12 .0	−16 .3
2 .6	−2 .3	5 .4	−6 .6	7 .8	− 1 .2	12 .6	−15 .2
3 .6	+0 .1	5 .6	−3 .0	8 .0	+ 0 .4	13 .8	−12 .4
3 .6	−0 .5	5 .6	+3 .9	8 .1	− 4 .1	14 .3	− 9 .3
±3 .8	+9 .7	±5 .8	+2 .5	± 8 .6	+ 6 .4		
4 .3	−4 .1	5 .9	+0 .8	10 .3	+ 3 .7		
4 .4	−9 .6	6 .5	+4 .9	10 .4	+ 2 .9		
4 .7	−1 .5	6 .6	+1 .9	10 .6	+ 5 .6		
4 .8	+1 .6	7 .5	−1 .1	10 .7	+14 .2		

OBSERVATIONS OF THE COMPANION TO *SIRIUS*,

By E. E. BARNARD.

The following measures are a continuation of those printed in *A. J.*, Vol. XXIX, p. 38. The star seems now to be at, or near, apastrion. The companion, under good conditions, is as bright as the $8\frac{1}{2}$ or 9th magnitude.

Date		P. A.	Dist.	Hour Angle	Remarks
		°		h m	
1915.747	Sept. 28	76.15	10.64	E 0 40	Bright and easy, 8-9 mag.
.785	Oct. 14	76.17	10.54	E 0 30	
.796	18	75.82	10.52	E 0 20	Faint from fog. Seeing poor.
.804	21	75.93	10.77	E 0 15	Very bright, $10\frac{1}{2}$ mag. Seeing = 3.
.818	26	75.62	10.52	W 0 45	Well seen: 9-10 mag.
.837	Nov. 2	74.94	10.84	0 00	Difficult. Seeing poor.
.963	Dec. 18	74.34	10.72	W 0 45	Full aperture. Seen only once in a while.
.963	18	74.68	[10.21]	W 0 55	Aperture 15 in. Fairly well seen but seeing bad.
1916.040	Jan. 15	73.42	10.77	W 0 55	Seen with difficulty.
.089	Feb. 2	74.82	10.76	0 00	Seeing very bad.
.108	9	75.47	10.64	E 0 40	Seeing fair but blurring.
.127	16	75.91	10.53	W 0 30	Seeing poor.
.155	26	74.25	10.75	W 0 30	Seeing poor. Wind shaking telescope.
.185	Mar. 8	74.74	10.69	W 0 10	Seeing 2-3.
.204	15	75.24	10.45	W 0 50	Seeing fair. Observation good.
.229	24	73.87	10.70	W 0 40	Seen only once in a while. Seeing excessively bad.
1915.991		75.09	10.66		

The seeing is on a scale of 5.

The following measures were made of a small star near *Sirius*.

1915.804 Oct. 21 P. A. 183°.18 Dist. 31″.59 14 mag.

Yerkes Observatory, Williams Bay, Wisconsin, 1916 September 4.

CONTENTS.

Editor, Benjamin Boss, Albany, N. Y.; Associate Editors, E. E. Barnard, Ernest W. Brown, F. R. Moulton and R. S. Woodward.
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OBSERVATIONS OF COMET *1915 e* (TAYLOR),

BY E. E. BARNARD.

The following observations with the large telescope were made of TAYLOR's comet, which has proved to be of short period.

Attention has already been called (*A. J. 29*, 138) to the fact that in the first part of February the comet was attended by a small companion. The original statement that it had a double nucleus is misleading, for the object was in every respect a double comet. I shall quote quite fully from my notes on these objects, for their subsequent behaviour will make such notes valuable for the proper interpretation of the observations made before and after the full moon in March.

At first the smaller (the northern) component was the fainter of the two by 1½ or 2 magnitudes. Both objects were fairly well defined and were diffused towards the east into a short brush of tail. For a while the north, or smaller, component grew fainter and the south one — the main comet — developed a small bright condensation that was almost a nucleus. The north comet then began to brighten and became quite strongly condensed. At the same time the main comet became diffuse and fainter. It finally entirely disappeared, leaving only what at first had been the companion, which by this time was a strongly condensed comet. The position angle of the companion did not appreciably change; that of March 22 may be in error some 10° — which is not probable — on account of the extreme faintness of the original comet.

The disappearance of the main body seems to have been permanent. I have not before seen a transformation of this kind in which the original comet disappeared while the companion was still visible. The position measures to March 8 refer to the original comet; those from March 22 to the companion. In the meantime, during moonlight, it was not possible to tell which object was under observation, as there was visible only one feeble condensation on the moonlit sky. I have assumed that the position measures after March 22 refer to the companion (*B*). Only one comet was visible after that date. An investigation of the orbit will show if the later measures really refer to the companion.

SOME OTHER MULTIPLE COMETS

BROOKS' comet, 1889 V, it will be remembered, (*A. N.*, Vol. CXXV, p. 177) had several companion comets, two of which remained with it for several months, and one of which at one time was brighter than the main comet. Both, however, finally faded out and apparently ceased entirely to exist, while the main comet was still bright. They have not been seen again, though the principal comet has returned several times.

Several other comets, SWIFT's of 1899, (*A. J. 20*, 60-61) KOPFF's of 1906 (*A. J. 25*, 83-84) and MELLISH's of 1915 (*A. J. 29*, 40) have also had secondary or companion comets, all of which, after a more or less brief existence, disappeared. Of course the best known example of this disruption of cometary matter was BIELA's comet. One of the comets of 1860 was double. Even HALLEY's comet showed something of this kind in May of 1910. The great comet of 1882 for a time also showed this disruptive effect. It is, therefore, not an unusual thing for a comet to develop companions, or a double condition. This breaking up of a comet would doubtless go far to account for the fact that in several cases two or more comets are found to be following nearly the same path around the *Sun*, and also for the entire disappearance of some of the periodic comets.

In examining the ancient records of comets, there are several cases where apparently a double or triple comet was recorded without the telescope. One of these records, which we find in CHAMBERS' *Astronomy*, undoubtedly refers to a comet with two companions,

the three, of course, being visible to the naked eye. This was in the year 896. The reference follows: "In this year there appeared three extraordinary stars, one larger and two smaller ones. * * * They travelled together for three days. The little ones disappeared first and then the large one."

Measured Positions of the Comet.

1916	C. S. T.	Δα cos δ	Δα	Δδ	Comps.	α Appt.	δ Appt.	Red. to Appt.		*
	h m s		m s			h m s	° ′ ″	s	″	
Jan. 5	8 47 2	−109.25	−0 7.38	+2 54.6	5 – 8	5 7 12.88	+ 9 23 27.5	+2.16	+5.9	1
5	9 45 7	+ 57.16	+0 3.86	−2 31.3	4 – 8	5 7 11.12	+ 9 24 27.2	+2.16	+5.9	2
8	8 49 31	.	−2 3.85	+0 15.0	18 tr. 6	5 6 42.90	+10 39 25.8	+2.17	+6.1	3
Feb. 2	6 55 31	+ 93.73	+0 6.70	+0 58.8	5 – 8	5 18 36.95	+21 7 32.2	+2.23	+7.2	4
9	7 46 58	+129.15	+0 9.10	−0 34.9	5 – 9	5 27 27.44	+23 42 5.8	+2.24	+7.6	5
9	8 7 44	. . . . .	. .	−0 17.3	4	. . . .	+23 42 23.4	. . . .	+7.6	5
19	11 0 10	− 58.20	−0 4.35	−1 6.9	8 – 9	5 44 4.99	+26 54 48.0	+2.27	+7.9	6
19	11 17 0	− 11.63	−0 3.11	. . . . . .	6 .	5 44 6.23	. . . .	+2.27	. . . .	6
19	11 23 3	.	. .	−0 49.5	. 5	. . . .	+26 55 5.4	. . . . .	+7.9	6
26	6 53 17	−139.21	−0 10.58	+1 30.5	5 – 8	5 57 31.57	+28 42 17.3	+2.28	+7.7	7
26	8 10 41	− 53.42	−0 4.06	+2 16.6	5 – 6	5 57 38.09	+28 43 3.4	+2.28	+7.7	7
Mar. 4	7 49 42	− 66.64	−0 5.15	+0 7.9	5 – 10	6 12 58.16	+30 13 37.6	+2.30	+7.7	8
8	8 36 15	− 16.19	−0 1.26	−3 11.2	5 – 8	6 22 26.0	+30 56.9	+2.31	+7.4	9
15	8 26 12	− 3.59	−0 0.28	−0 23.9	5 – 8	6 39 43.53	+31 56 35.1	+2.33	+6.9	10
18	7 49 5	− 23.10	−0 1.82	−0 12.7	4 – 6	6 47 19.74	+32 16 0.4	+2.32	+6.7	11
22	8 21 49	− 80.06	−0 6.33	+2 4.4	6 – 10	6 57 49.0	+32 36.9	+2.32	+6.3	12
24	8 11 26	−127.74	−0 10.12	+1 29.3	7 – 8	7 3 6.85	+32 45 4.7	+2.32	+6.0	13
29	10 37 51	− 89.81	−0 7.13	+0 35.3	5 – 11	7 16 45.42	+32 59 11.3	+2.33	+5.5	14
Apr. 1	8 3 45	+ 95.66	+0 7.61	+0 25.4	5 – 8	7 24 34.86	+33 3 27.4	+2.32	+5.0	15
5	8 22 20	+ 74.12	+0 5.90	−2 6.7	6 – 10	7 35 28.78	+33 3 52.3	+2.31	+4.6	16
8	9 1 35	− 33.95	−0 2.70	−1 12.2	5 – 9	7 43 42.73	+33 0 55.9	+2.31	+4.2	17
May 27	9 46 8	+153.91	+0 11.53	+1 37.4	12 – 5	9 49 18.32	+27 5 56.6	+2.12	−2.7	18

Mean Places of Comparison Stars.

*	α 1916.0	δ 1916.0	Authority
	h m s	° ′ ″	
1	5 7 18.10	+ 9 20 27.0	11½ mag. Compared with ½ (*BD* + *9° 758* + *Star 2*).
2	5 7 5.10	+ 9 26 52.6	10 mag. R. H. TUCKER, *L. O. M. C.*
3	5 8 11.58	+10 39 4.7	*Leipzig A. G. C. 1558.*
4	5 18 28.02	+21 6 26.2	11½ mag. Compared with *Berlin A. G. C. 1734.*
5	5 27 15.80	+23 42 33.1	*BD + 23° 934.* Compared with *Berlin A. G. C. 1764.*
6	5 44 7.07	+26 55 47.0	*BD + 26° 966.* Compared with *Cambridge (Eng.) A. G. C. 2721.*
7	5 57 39.87	+28 40 39.1	*Cambridge (Eng.) A. G. C. 2886.*
8	6 13 1.01	+30 13 22.0	10 mag. Compared with *Leiden A. G. C. 2550.*
9	6 22 25.0	+31 0.0	12 mag. Compared with *BD + 31° 1300.*
10	6 39 11.48	+31 56 52.1	*BD + 32° 1391.* Compared with *Leiden A. G. C. 2807.*
11	6 47 19.24	+32 16 6.1	12 mag. Compared with *Leiden A. G. C. 2845.*

Mean Places of Comparison Stars (Continued)

*	α 1916.0	δ 1916.0	Authority
	h m s		
12	6 57 53.0	+32 34.7	12 mag. Compared with *BD* + *32° 1469*.
13	7 3 11.65	+32 13 29.4	*Leiden A. G. C. 2986*.
14	7 16 50.22	+32 58 30.5	12 mag. Compared with *Leiden A. G. C. 3095*.
15	7 24 24.93	+33 2 57.0	11 mag. Compared with *Leiden A. G. C. 3149*.
16	7 35 20.57	+33 5 54.4	12½ mag. Compared with *Leiden A. G. C. 3217*.
17	7 43 43.12	+33 2 3.9	12½ mag. Compared with *Leiden A. G. C. 3272*.
18	9 49 4.67	+27 4 22.0	12 mag. From photograph with 10-inch Bruce.

Measures of Comparison Stars.

	Δα	Cps.	Δδ	Cps.
	m s			
Star 1 — BD + 9° 758	−0 39.61	10 tr.	+2 19.6	4
Star 4 — Berlin A. G. C. 1734	−2 22.79	14 tr.	+2 55.3	5
Star 5 — Berlin A. G. C. 1764	+2 19.51	14 tr.	+0 58.8	5
Star 6 — Cambridge A. G. C. 2721	−0 51.70	14 tr.	−0 36.5	3
Star 8 — Leiden A. G. C. 2250	+0 1.07	5	−3 47.3	5
Star 9 — BD + 31° 1300	−0 13.37	5	−2 16.3	4
Star 10 — Leiden A. G. C. 2807	−0 51.07	16 tr.	+1 16.9	4
Star 11 — Leiden A. G. C. 2845	−1 25.39	14 tr.	+0 37.4	5
Star 12 — BD + 32° 1469	−0 43.01	16 tr.	+0 43.9	4
Star 14 — Leiden A. G. C. 3095	−0 3.13	3	−5 17.7	3
Star 15 — Leiden A. G. C. 3149	+0 44.49	8 tr.	−4 41.5	3
Star 16 — Leiden A. G. C. 3217	+1 9.95	8 tr.	−2 9.1	4
Star 17 — Leiden A. G. C. 3272	+0 59.77	12 tr.	−1 56.0	4

The measured Δα for stars *8*, *9* and *14* were:

Star 8	+ 13″.82
9	−171 .92
14	− 39 .40

Professor Tucker of the Lick Observatory kindly observed the position of *BD* +*9°758* with the meridian circle. Epoch of observation, 1916.12. Magnitude 9¾.

$$1916.0 \quad \alpha \ 5^h\, 7^m\, 58^s.25 \qquad \delta + 9°\, 18'\, 8''.3$$

He found both stars, *Star 2* and +*9° 758*, faint for the meridian circle, especially *Star 2* which was very difficult. I have compared *Star 1* with both these stars. The resulting right ascensions differ by about 1^s, while the declinations differ by 2″. Following are the observations:

Star 1 — BD +9° 758

$\Delta\alpha$ $-0^m\, 39^s.61$ (26 tr. — 10 on Jan. 5, 16 on Aug. 30)
$\Delta\delta$ $+2'\, 19''.7$ (11 meas. — 7 on Jan. 5, 4 on Aug. 30)

The two sets agree to within $0^s.00$ and 0″.3.

Star 1 — Star 2

$\Delta\alpha$ +184″.6 (4) = $+12^s.47$ $\Delta\delta$ −6′ 26″.7 (4) (Aug. 30).

These would give for *No. 2 — BD +9° 758*

$$\Delta\alpha - 0^m\, 52^s.08$$
$$\Delta\delta + 8'\, 46''.4$$

On Aug. 26 *BD +9° 758* and +*9° 760* were compared with each other:

$\Delta\alpha$ − $0^m\, 28^s.09$ (8 tr.) $\Delta\delta$ + 1′ 44″.3 (2 meas.) Lost in dawn.

Two other small stars were used in the observations. Their relative positions follow:

10^m star − *BD* + 9° 758 $\Delta\alpha - 0^m 38^s.42$ $\Delta\delta + 6' 32''.4$

The same star was also compared with *Star 2*.

10^m star	*Star 2*	$\Delta\alpha + 0^m 13^s.66$	$\Delta\delta - 2' 14''.0$
11^m star	*BD* + 9° 758	$\Delta\alpha - 0^m 28^s.09$	$\Delta\delta + 2' 19''.3$

Position Angles and Distances of the Companion Comet

The comet, as has been stated, was found to be double on February 9. The position angle and distance of the two components were measured thereafter at every opportunity. Very little change occurred in the relative position of the two, the position angle remaining about the same, and the distance slowly increasing. The comet was receding from the *Earth* in all the observations. Following are the measures of the two components.

	C. S. T.	P. A.	Cps.	Dist.	Cps.
1916 Feb.	9^d 8^h 28^m	21°.26	13	9″.50	12
	26 7 45	23 .28	11	12 .06	19
	27 7 42	23 .50	7	12 .33	13
Mar.	1 7 34	22 .81	6	13 .95	10
	8 8 1	21 .48	13	13 .96	12
	22 8 10	33 .08	5	15 .99	12

The position angle of March 22 is uncertain from the faintness and lack of condensation of the main comet.

Notes on the Physical Appearance of the Comet

1916 Jan. 5. It was soft and diffused like some of the periodic comets. The central brightness was 10″ in diameter with no definite nucleus.

Jan. 8. Tenth to eleventh magnitude. Strong condensation but no definite nucleus. Its general light was very soft and widely diffused. It was larger than the field (5′).

Feb. 2. Small, brightly condensed to probably a small nucleus. A 14th magnitude star close south preceding, a 15th magnitude star close north following.

Feb. 9. The comet was double, two perfectly distinct comets whose nebulosity mingled. The south one was the brighter and had a small bright nucleus; the north one less definite. We will call the main comet and the companion *A* and *B* respectively.

Feb. 26. Both condensations almost stellar; about two magnitudes different in brightness. The nebulosity nearly filled the field and extended south following the comets. Both very little brighter preceding the middle. *A* = 12^m. *B* = 13½^m or 14^m. The comet was faintly visible (as one) in the 4-inch finder.

Feb. 27. The companion seemed fainter. It was 2½ magnitudes less than the main comet.

March 1. *A* had lost its definite nucleus and was more diffused than *B*, which had almost a faint nucleus. They were both a little brighter in the middle. *B* was 1½ or 2 magnitudes less than *A*. Estimated magnitudes = 12 and 13½. Their nebulosity extended following.

March 8. On this date the north comet (*B*) was slightly the brighter, and had a small speck of nucleus of 14½ magnitude, like a small, ill-defined star. The south component (*A*) had but little condensation and was much diffused. The two were very closely the same size. The nebulosity extended at right angles following the line between the two. There was a vague suggestion in the appearance of *A* of a possible breaking up again. The south component (*A*) observed for position as in previous observations.

March 15. Very faint in strong moonlight. Could not tell if it were double.

March 18. Very faint in full moonlight. Could see only one comet.

March 22. The north component (*B*) observed for position. It was the brighter and was definitely condensed north preceding its center and easy to observe. The main comet (*A*) was too faint to observe accurately for position. It was excessively difficult. The two seemed to have interchanged individualities.

March 24. The south component (*A*) was not visible. What was left was simply the companion, the main comet having entirely disappeared. The greatest brightness was about ½′ or ¾′ in diameter with possibly a faint speck of light slightly north of the center. The sky was not very transparent.

March 29. Quite strongly condensed with perhaps a very small nucleus in the northern part. No trace of the original comet. Seeing 2, but sky poor.

April 1. Only one comet, 13½ magnitude. A little brighter toward the middle, the nebulosity extending south following. No nucleus. It was 1′ in diameter. Seeing good, sky good.

APRIL 3. Seen faintly for a moment before it was lost in clouds. Single.

APRIL 5. It was $13\frac{1}{2}$ magnitude and $\frac{3}{4}'$ in diameter, very gradually brighter north of the middle. Could see no trace of the original comet.

APRIL 8. It was single.

MAY 27. The comet was 16 or $16\frac{1}{2}$ magnitude. Very faint and diffused, but the observations are believed to be good

I am greatly indebted to the kindness of PROFESSOR VAN BIESBROECK, who gave me the opportunity to measure the components A and B with the large telescope on February 27.

PHOTOGRAPHS OF THE COMET

A photograph was made on 1915, December 7 with the Bruce telescope. The comet was difficult to guide on with the 5-inch guiding telescope. It was estimated to be 10th or 11th magnitude. The exposure was $2^h 50^m$ on a good sky and shows a faint tail about $\frac{1}{4}°$ long.

Another photograph was made on 1916, February 21 with an exposure of $1^h 39^m$. The scale was too small to show the comet double. Though difficult to guide on, it could be seen fairly well when the light was cut off from the wires. The sky was not very good. No other photographs were made as the results did not seem to pay for the time spent on it.

Yerkes Observatory, Williams Bay, Wisconsin, 1916 September 2

PHOTOGRAPHIC MEASURES OF DOUBLE STARS.

BY T. P. BHASKARAN

The measures of the rectangular coördinates of the following double stars were made at the Nizamiah Observatory during the usual course of the Hyderabad Astrographic Work. The position angle and distance have been computed by using the approximate formulæ derived by MR. R. J. POCOCK in *A. N.* 4827, viz.:

$$s = 300'' \cdot (1 - A) \cdot (\Delta x^2 + \Delta y^2)^{\frac{1}{2}}$$

and

$$p = \tan^{-1}\frac{\Delta x}{\Delta y} + \tan^{-1}\frac{D}{1-A} + \frac{1}{12}\,1°\;(x-13)\;\tan D' .$$

where A, D are plate constants and D', the declination of the plate centre.

Column 1 in the following list gives the reference number of each star from BURNHAM'S *General Catalogue of Double Stars.* The magnitudes given in Column 7 have generally been taken from the same catalogue; but in cases where both the components have been observed at Washington the magnitudes from *A. G. Wash.* are given. Occasional notes have been inserted from a comparison with the earlier measures in the Notes to BURNHAM'S *General Catalogue*

No.	Double Star	R. A. 1900.0	Decl. 1900.0	Position Angle	Distance	Magnitudes		Epoch 1900+	Notes
		h m	° ′	°	″				
4334	*H. 4013*	7 50	−18 4	197.1	19.02	7.1	10.7	15.115	No change.
4337	*H. 4015*	7 50	−17 32	228.5	20.33	8.9	9.1	15.115	
. .	*BD −17°2472*	8 19	−17 45	91.8	18.95	9.4	9.5	15.101	
4639	*Arg. 20*	8 24	−17 12	170.2	15.49	8.2	8.5	15.104	
4653	*H. 4100*	8 26	−18 1	177.7	22.30	9.5	11.0	15.104	
4854	*S. 585*	8 51	−17 52	146.8*	66.80	7.2	8.0	15.104	No change.
4911	*S. 588*	8 59	−17 16	325.4	31.11	8.7	8.7	15.120	
5057	*β 337*	9 18	−17 28	331.2	7.31	7.0	11.0	15.123	
5156	*S. 604*	9 31	−19 7	87.7	50.50	7.0	11.0	16.161	
5240	*H. 4261*	9 49	−19 1	82.8	8.22	8.0	10.0	16.164	
5303	*β 1072*	9 59	−17 37	270.4	22.00	7.8	7.9	16.161	*A* and *C*.
5320	*S. 607*	10 2	−18 50	325.7	10.96	10	10	16.161	

*The position angle for this pair given in *A. N.* 4827 should be 145°.3.

No.	Double Star	R. A. 1900.0	Decl. 1900.0	Position Angle	Distance	Magnitudes		Epoch 1900+	Notes
		h m							
5477	*S. 610*	10 29	−17 18	12.6	105.69	8.5	8.7	16.184	Decidedly apparent change.
5501	*H. 4337*	10 33	−18 50	250.2	9.77	9	10.0	16.181	
5640	*H. 1181*	10 56	−17 47	267.7	64.83	8.7	9.4	16.167	*R Crateris.*
	Do			267.0	65.90			16.172	
5796	*β 601*	11 24	−16 18	330.8	28.21	8.5	8.8	15.137	*A* and *BC*; apparently fixed.
5852	*H 1192*	11 33	−16 23	357.9	13.58	9.4	9.2	15.123	Fixed.
6029	*H 4496*	12 1	−18 21	27.1	12.32	8	9	16.172	
6032	*H 1209*	12 1	−16 28	249.8	16.07	10-11	11	16.167	
6047	*H 1212*	12 4	−17 1	99.6	21.70	9-10	11	16.167	
6058	*S 634*	12 6	−16 14	279.1	6.56	8	10	16.167	No change.
6183	*Sh 145*	12 25	−15 58	205.6	26.02	2.3	8.9	15.123	*δ Corvi:* these two stars have common proper-motion.
6299	*S 643*	12 49	−17 30	295.9	23.75	8.6	9.0	15.129	Fixed.
	Do.			291.2	22.99			16.186	
6350	*H 2630*	12 57	−16 58	95.5	14.38	11	11+	16.178	
6538	*H 1233*	13 30	−16 20	105.3	18.51	10	13	16.178	
6564	*H 2665*	13 33	−18 56	161.2	23.06	8	12	16.246	
6637	*H 2683*	13 43	−16 15	17.6	15.03	11	11-12	16.183	
6638	*H 2684*	13 43	−16 18	232.8	28.46	11	11+	16.183	
6682	*H. C. Wilson 12*	13 52	−16 48	223.0	31.44	9	11	16.175	*A* and *C*; probably slight change.
6706	*H 2698*	13 58	−17 57	288.4	23.69	9-10	14-15	16.175	
6714	*S 659*	14 0	−17 36	169.7	31.23	8.8	9.0	16.175	Apparently fixed.
	Do.			169.5	31.72			16.183	
	Do.			167.2	31.81			16.318	
6720	*H 1245*	14 1	−16 41	284.5	12.87	12	13	16.183	
6911	*H 2726*	14 28	−18 34	163.5	12.32	10	11	16.329	
6940	*H 2735*	14 33	−16 26	58.6	29.35	9-10	12	16.257	
7007	*Hu 477*	14 43	−16 35	28.0	2.62	9.0	9.0	16.178	
	BD −16°3949	14 46	−16 35	358.8	29.69	9.0	9.2	16.178	
7022	*Ho 388*	14 46	−17 28	123.2	11.98	8.0	11.5	16.178	Prob. slight change.
	Do.			123.4	13.10			16.274	
7107	*S 665*	14 59	−17 31	91.4	25.55	8.3	8.9	16.241	Fixed.
	Do.			91.3	24.63			16.276	
7164	*Sh 195*	15 9	−18 3	139.5	46.72	7.5	9.0	16.257	*Libræ 97.* No change.
	Do.			138.9	48.52			16.318	
	BD −16°4076	15 19	−16 38	5.5	32.33	8.9	8.9	16.271	
7249	*H 1271*	15 19	−18 15	108.8	7.64	10	10	16.331	
7300	*H 1273*	15 27	−17 35	330.5	14.59	9-10	10-11	16.183	
	Do.			328.6	13.66			16.323	
7324	*Howe 35*	15 31	−16 38	329.5	7.30	9.0	9.5	16.186	
7537	*H 1288*	16 7	−16 29	122.2	19.69	10.0	11.0	16.271	
7697	*H 4879*	16 34	−17 33	339.6	19.66	10	10	16.246	
	Do.			343.6	19.38			16.323	
7874	*H 4923*	17 3	−18 8	156.1	9.92	8	9	16.348	Decided change in angle and distance.
7910	*H. C. Wilson 14*	17 10	−18 4	227.7	11.77	9	12	16.323	*A*, *B* and *C*.
	Do.			223.0	10.78			16.328	

No.	Double Star	R. A. 1900.0	Decl. 1900.0	Position Angle	Distance	Magnitudes		Epoch 1900+	Notes
		h m	° ′	°	″				
7931	*O. Stone 34* .	17 11	−17 3	289.3	16.69	9.0	9.5	16.323	Not identified. This is *A.G. Wash. 6186.*
	Do.			288.4	17.22	. .	. .	16.328	
8027	*H 2806*	17 24	−17 44	189.0	8.72	9.0	9.0	16.276	
	Do.			192.9	8.66			16.318	
	Do.			195.7	7.69			16.348	
8039	*H 590* .	17 25	−17 4	310.5	36.33	9	10	16.276	Apparently fixed.
	Do.			312.2	35.33			16.348	
. . . .	*BD −16°4567*	17 31	−16 32	140.3	24.48	9.2	9.2	16.320	
8214	*Hd 147* . . .	17 48	−17 22	206.7	12.24	10	10	16.276	
8341	*H 2818* . .	18 1	−17 13	140.4	17.35	7.8	9.9	15.512	
8381	*S 700* .	18 5	−16 16	352.3	28.30	8.8	8.9	15.512	*A* and *C*. No change.
8386	*Hu 195*	18 5	−17 10	299.8	18.07	8.8	9.2	15.512	*A*, *B* and *C*.
8435	*H 2824*	18 10	−16 50	74.2	22.33	9	10	15.506	
8452	*H 2826*	18 11	−16 52	86.6	8.86	12	12	15.506	
8816	*H 2840*	18 44	−17 57	346.0	14.55	10-11	10-11	16.328	
8823	*Howe 44*	18 44	−17 1	305.4	33.94	8.5	11	16.328	*A* and *C*.
8843	*H 2842*	18 45	−17 54	337.1	29.11	8.9	10	16.328	
8891	*H 2844*	18 50	−17 45	103.3	22.44	8.0	9.7	15.611	

Nizamiah Observatory, Hyderabad, 1916 July 26.

PAIRS OF STARS USED FOR VALUES OF EQUATORIAL MICROMETER SCREWS.

By ASAPH HALL

(Communicated by CAPTAIN J. A. HOOGEWERFF, U.S.N., Superintendent, U. S. Naval Observatory)

Attention is invited to the pairs of stars which have been employed at the Naval Observatory for the determination of the values of the screws of equatorial micrometers. It is suggested that information additional to that given below be printed. The pairs are:

A. Stars *A* and *Z* in the cluster *h Persei*, that is *BD* + 56°.543 and *BD* +56°.498.

B. *Pleiades* stars *BD* +24°.552 and *BD* +24°.550.

C. *Pleiades* stars *BD* +24°.540 and *BD* +23°.495.

The following determinations of the relative positions of these pairs appear to be available. It is to be noted that in Part V *Recherches Astronomiques de l'Observatoire d'Utrecht*, a small relative proper-motion is found for the *Perseus* stars. As it is usual to measure differences of declination by allowing the stars to transit across the field, the $\Delta\delta$ of each pair has been computed, and reduced to 1914.0

A (1) *Der Sternhaufen h Persei*, von A. KRUEGER, Abdruck aus den Abhandlungen der Finnischen Societät der Wissenschaften, heliometer.

(2) and (3) P. 23, Vol. IX, *Observations de Poulkova*, Gyldén and Nyrén vertical circle.

(4) *Supplement III aux Observations de Poulkova*, Romberg, meridian circle.

(5) No. 24, *Contributions from the Observatory of Columbia University*, stars 78 and 39. YOUNG, 12 Rutherford plates.

(6) *Katalog von 858 Sternen, Zweiter Band, Annalen der K. Universitäts-Sternwarte in Strassburg*, meridian circle.

(7) *Sechster Theil, Astronomische Mittheilungen von der K. Sternwarte zu Göttingen*, stars f and e, Schur, heliometer.

(8) No. 2 Band II, *Astronomische Abhandlungen der Hamburger Sternwarte in Bergedorf*, stars 312 and 122, Messow, 2 plates.

(9) Naval Observatory, 9-inch meridian circle, not published.

(10) Naval Observatory, 6-inch meridian circle, not published.

(11) Naval Observatory, 9-inch meridian circle, not published.

(12) Naval Observatory, 6-inch meridian circle, unpublished.

(13) P. 9, Vol. XI, Serie II, *Publications de l'Observatoire Central Nicolas*, mean of 3 heliometers. Reduced to 1914.0 this mean gives $s = 1312''.72$, $p = 33° 49' 28''$.

(14) YALE, observed with heliometer by SMITH, at request of Superintendent of Naval Observatory, unpublished. As communicated, for 1914.0, $s = 1312''.68$, $p = 33° 48' 02''$.

(15) YALE, observed with heliometer by SMITH, at request of Superintendent of Naval Observatory, unpublished. As communicated, for 1915.0, $s = 1312''.80$, $p = 33° 50' 15''$.

	Δδ 1914.0 1114″	Epoch		Δδ 1914.0 1114″	Epoch
(1)	+1″.41	1861.6	(9)	+0″.80	1910.0
(2)	+1 .83	1870.3	(10)	+0 .87	1914.0
(3)	+1 .66	1873.3	(11)	+1 .29	1916.0
(4)	+1 .20	1875.7	(12)	+1 .31	1915.9
(5)	+1 .45	1874.4	(13)	+1 .48	1891.8
(6)	+1 .53	1885.0	(14)	+1 .74	1914.2
(7)	+1 .38	1893.75	(15)	+1 .43	1916.0
(8)	+1 .37	1899.8			

B (1) First Yale Triangulation, p. 351, Vol. I, *Transactions of Yale Observatory*.

(2) Second Yale Triangulation, p. 389, Vol. I, *Transactions of Yale Observatory*.

(3) Naval Observatory, 9-inch meridian circle, unpublished.

(4) YALE, observed with heliometer by SMITH, at request of Superintendent of Naval Observatory, unpublished. As communicated, for 1914.0, $s = 595''.06$, $p = 7° 03' 13''$.

	Δδ 1914.0 590″	Epoch		Δδ 1914.0 590″	Epoch
(1)	+0″.66	1885.0	(3)	+1″.04	1914.0
(2)	+0 .88	1901.5	(4)	+0 .56	1914.2

C (1) and (2) Taken from the two Yale triangulations, as referred to under *B*.

(3) Naval Observatory, 9-inch meridian circle, unpublished.

(4) Naval Observatory, 6-inch meridian circle, unpublished.

(5) YALE, observed with heliometer by SMITH, at request of Superintendent of Naval Observatory, unpublished. As communicated, for 1914.0, $s = 696''.58$, $p = 19° 01' 28''$.

	Δδ 1914.0 658″	Epoch		Δδ 1914.0 658″	Epoch
(1)	+0″.39	1885.0	(4)	+0″.44	1913.9
(2)	+0 .53	1901.5	(5)	+0 .53	1914.2
(3)	+0 .67	1914.0			

In addition to the above, the following three pairs of stars have been observed at the Naval Observatory with the 9-inch meridian circle, each on four nights, for the same purpose, that is, to be used for the determination of values of equatorial micrometer screws. It is requested that unpublished observations of these stars be printed.

Star *BD*	α 1900.0	δ 1900.0	Epoch
− 5°.3737	13h 32m 25s.39	− 6°08′ 33″.25	1905.042
− 5 .3747	13 35 50 .13	− 5 50 50 .55	1905.082
−15 .4266	16 07 09 .36	−15 45 34 .68	1905.148
−15 .4300	16 13 44 .55	−15 18 11 .18	1905.665
−20 .5118	18 17 22 .33	−20 42 10 .42	1905.290
−21 .5025	18 23 52 .98	−21 00 59 .75	1905.145

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OBSERVATIONS OF THE SATELLITES OF *SATURN* AT THE OPPOSITION OF 1915–16,

By E. E. BARNARD.

These observations of the satellites of *Saturn* are a continuation of those printed in *A. J.*, Vol. XXIX, p. 33. A magnifying power of 700 diameters was used in almost every case. The measures are not corrected for refraction. The two faint stars observed on October 21 were seen on a subsequent date. The times are 6^h 0^m slow of Greenwich Mean Time.

1915–16	C. S. Time	P. A.	Dist.	Cps.	P. A. of Wires	Remarks
			Enceladus and *Mimas*			
	h m s	°	″		°	
Oct. 14	16 1 49	285.63	. .	5	.	
	16 6 46	. . . .	7.92	8	15.7	Seeing very bad.
May 3	8 22 39	145.15	. . .	5	. . . .	*Mimas* one magnitude less than *Enceladus*.
	8 27 11	.	16.40	8	54.8	
			Tethys and *Mimas*			
Oct. 21	17 32 7	99.31	.	5	.	
	17 36 2	. .	16.05	8	9.5	
Feb. 2	12 8 38	122.68	. .	5	. . .	Seeing very bad.
	12 15 33	. .	26.46	10	32.1	
Mar. 22	9 18 50	198.53	. . . .	6	.	
	9 25 7	. . . .	16.95	8	107.8	
Apr. 8	7 30 20	220.71	. . .	5	. .	With eyepiece 5.
	7 38 48	. . . .	19.05	11	130.9	*Mimas* very faint and difficult. Seeing very bad.
	8 8 11	213.18	. . . . .	5	. . .	With eyepiece 4 and occulter.
	8 14 39	. . . . .	19.09	10	123.4	
			Dione and *Mimas*			
Sept. 28	16 26 25	343.78	. . .	5	. . .	Seeing very good.
	16 30 54	. .	22.08	8	73.8	
Jan. 5	12 42 57	99.75	. . .	5	.	Seeing very bad.
	12 47 38	. . .	43.25	8	9.3	
Apr. 1	7 9 42	62.12	. . . . .	5	. . .	*Mimas* faint.
	7 14 47	.	56.75	8	152.1	

1915–16	C. S. Time	P. A.	Dist.	Cps	P. A. of Wires	Remarks
			Rhea and *Mimas*			
	h m s	°			°	
Mar. 8	6 49 5	161.43	...	5	...	
	6 53 37	.	34.54	10	71.3	
15	7 25 49	297.66	.	6		Seeing very bad.
	7 31 1	...	38.84	9	27.7	
May 3	7 40 38	48.77		5		Without occulter.
	7 45 54	.....	50.44	8	137.8	
	8 2 16	46.82	...	5	.	With planet occulted.
	8 7 35	...	49.93	10	136.8	
			Tethys and *Enceladus*			
Sept. 28	16 6 34	248.96	.	6	...	
	16 11 22	..	13.87	8	159.3	
Oct. 18	15 43 1	47.03	...	5	..	
	15 47 32	..	9.38	8	136.9	
Dec. 1	13 24 55	202.52	...	5	....	Seeing bad.
	13 29 34	....	18.80	8	112.0	
Feb. 9	13 19 58	128.50	...	5	...	
	13 23 49	...	22.91	8	38.3	
26	9 23 13	269.04	...	5	...	
	9 27 49	.	54.19	8	179.2	
Mar. 22	9 33 51	200.63	...	5	....	Seeing very bad.
	9 38 13	..	25.51	8	110.3	
Apr. 5	7 43 38	47.03	...	9	...	Seeing very bad.
	7 48 47	.	4.43	8	135.7	
			Dione and *Enceladus*			
Nov. 16	17 40 35	355.92		2	..	? if *Enceladus*. Lost in haze.
Dec. 18	15 8 40	84.13	....	5	.	Very difficult. Seeing very bad.
	15 14 30	...	27.59	8	171.0	
Jan. 5	12 30 51	121.10	.	5	....	Seeing bad.
	12 36 22	.	51.27	8	31.3	
Feb. 2	12 45 42	152.30	..	5	....	Seeing very bad.
	12 50 55	..	40.09	8	62.1	
Mar. 4	6 35 35	75.48	.	5	....	
	6 40 17	..	39.10	8	165.7	
18	8 50 9	74.13	...	5	...	
	8 54 20	.	22.00	8	4.6	
Apr. 12	8 24 45	64.29	..	5	..	*Enceladus* very faint.
	8 30 5		33.99	8	154.9	
			Rhea and *Enceladus*			
Sept. 2	16 39 14	102.71		5	.	
	16 42 40		16.3	1	12.6	Lost in haze.
Oct. 14	16 12 8	238.95	.	5	...	Seeing very bad.
	16 16 32	.	41.43	8	148.7	
26	15 51 51	10.26		6		Images breaking.

1915–16	C. S. Time	P. A.	Dist.	Cps.	P.A. of Wires	Remarks
			Rhea and *Enceladus* (*Continued*)			
	h m s	°	″		°	
	15 46 44	. .	19.72	8	99.9	Measures difficult.
Jan. 15	10 48 21	72.69	. .	5	. .	Very difficult. Seeing very bad.
	11 4 30	. .	52.79	11	2.3	
Feb. 19	11 55 23	91.30	. . . .	5		Very faint from bad seeing.
	12 1 10	. .	59.94	9	1.2	
Mar. 8	6 39 59	107.78	. . .	5	. . .	
	6 43 58	. . . . .	40.39	8	17.9	
Apr. 1	7 20 20	69.85	. . .	5	. .	
	7 24 15	. . .	32.60	8	159.4	
8	7 18 5	177.91	. . . .	6	. . .	
	7 23 20	. . .	41.83	8	87.9	
May 3	8 13 24	29.36	. . . . .	5	. . . .	
	8 17 55		54.93	8	119.3	
			Dione and *Tethys*			
Aug. 31	16 37 27	359.09	. .	5	. . . .	Seeing bad but steady for moments. Measures good.
	16 41 45	. . .	24.78	8	88.6	
Sept. 21	15 46 23	227.45	. . .	5	. . . .	Seeing very bad.
	15 52 20	. . . .	41.50	8	137.4	
28	16 16 0	317.64	. . . .	5	. . .	
	16 20 54	. . . .	45.89	8	47.3	
Oct. 14	15 52 42	239.65	. . . . .	5	. . .	Seeing very bad.
	15 56 54	. .	41.84	8	149.7	
18	15 32 58	204.63	. . .	5	. . .	
	15 37 46	. . . . .	40.71	8	114.9	
21	17 48 29	237.06	. . . .	5	. .	
	17 52 28	. .	41.84	8	147.0	
26	16 0 42	245.59	. . .	5	. .	
	16 4 30	. . . . .	41.68	8	155.6	
Nov. 24	13 6 13	16.30	. . . .	5	. .	
	13 10 40	. . .	35.61	10	105.7	
Dec. 1	13 15 28	123.78	. . .	5	. . . .	Seeing bad.
	13 20 15	. . .	23.98	8	34.0	
Jan. 5	12 13 34	90.23	. .	5	. . .	Very difficult. Seeing excessively bad.
	12 21 50	. . .	145.59	9	3.3	
Feb. 2	12 34 21	247.29	. . .	5	. . .	Seeing very bad.
	12 40 0	. . . . .	67.19	8	157.2	
9	13 44 32	5.38	. . .	5		
	13 49 9	. . . . .	45.40	8	95.3	
26	9 13 48	321.88	. . . .	5	. . .	Seeing very bad.
	9 17 48	. . . .	26.21	8	51.9	
Mar. 4	6 44 44	111.40	. . .	5	. . .	
	6 49 17	. . . .	61.75	8	21.2	
8	7 0 39	259.43	. . . .	6	. . .	
	7 4 59	. . . . .	20.07	8	169.3	
15	6 34 28	125.27	. . .	5	. .	
	6 40 51	. .	19.90	8	35.2	
18	8 58 37	55.47	. .	5		

1915–16	C. S. Time	P. A.	Dist.	Cps.	P. A. of Wires	Remarks
			Dione and *Tethys* (*Continued*)			
	h m s	°	″		°	
Mar. 18	9 3 22		70.30	8	145.6	
22	9 56 50	267.34		5		Seeing very bad.
	10 1 20		79.41	9	177.3	
Apr. 5	7 14 12	227.67	. .	5		Seeing very bad.
	7 17 43		62.85	10	137.7	
8	7 8 11	240.58		5		
	7 12 41		15.91	8	150.9	
12	8 2 0	316.36		5	. .	
	8 7 23		41.98	10	46.3	Seeing very bad.
15	7 58 34	40.56		4		Stopped by clouds.
			Rhea and *Tethys*			
Aug. 31	16 27 4	159.23		5	. . . .	
	16 32 14		18.69	8	69.2	
Sept. 2	16 17 12	207.70		5	. .	
	16 22 22		18.04	8	117.6	
28	15 54 11	68.09		5		
	15 59 29		87.44	8	158.0	
Oct. 5	17 7 35	222.51	.	5	. .	Very difficult. Seeing very bad.
	17 12 20	.	32.84	8	132.5	
21	17 56 51	14.42	. . .	5	. .	
	18 0 36		32.51	8	104.5	
26	15 51 28	48.63		5		
	15 56 4		70.19	8	138.4	
Nov. 2	17 17 8	141.42	. .	5		
	17 22 4	.	61.42	8	51.6	
11	13 29 29	230.21		5		
	13 34 4		60.81	8	140.6	
24	12 56 14	258.05		5		
	13 1 0	.	96.16	8	167.7	
Jan. 15	11 11 42	76.22	. .	5	.	
	11 16 30	. .	100.59	8	7.0	
Feb. 2	12 23 18	19.65	. . .	5		Seeing very bad.
	12 28 31	.	38.79	8	109.6	
9	13 9 58	209.16	.	5	.	
	13 14 39	.	21.18	8	119.3	
19	12 6 25	124.07		5		
	12 10 46		26.64	8	33.5	
Mar. 8	6 24 4	121.74		5		
	6 29 3		81.25	8	31.3	
22	9 46 18	77.78	.	5		
	9 51 23		53.25	9	167.8	
Apr. 1	7 36 37	29.22		5		
	7 40 23		21.86	8	118.4	
5	7 23 4	94.35		5		
	7 28 13		67.53	10	1.7	
May 10	7 36 41	136.95	.	5		Seeing excessively bad.
	7 43 20		17.86	8	46.8	

1915–16	C. S. Time	P. A.	Dist.	Cps.	P. A. of Wires	Remarks
			Titan AND *Tethys*			
	h m s	°	″		°	
Sept. 21	16 42 40	118.73	. . . .	5	. . . .	Seeing very bad; in hazy clouds.
	16 48 35	. . . .	98.11	8	28.5	
Nov. 24	13 15 39	105.47		5	. .	
	13 20 18	.	115.63	8	14.7	
Dec. 1	13 34 35	335.97	. . . .	5	. . . .	Seeing bad.
	13 39 56	. . .	94.67	8	66.0	
18	15 29 37	296.09	. . . .	5	. . . .	Seeing very bad.
	15 34 57	. . . .	140.53	8	26.0	
Jan. 5	12 58 36	261.42	. . . . .	5	. . . .	Seeing excessively bad.
	13 4 16	.	146.27	9	171.3	
Feb. 26	9 33 12	130.81	.	5	. . . .	Seeing very bad.
	9 37 55	. .	96.72	8	40.2	
			Rhea AND *Dione*			
Sept. 21	16 16 35	279.35	. . .	5	. . . .	
	16 24 12	. . . .	94.88	10	9.0	
Oct. 18	15 23 8	300.02	.	5	. . . .	
	15 28 6	. . .	48.74	8	29.9	
Nov. 2	17 5 9	194.19	.	5	. . .	
	17 11 14	. . . . .	57.00	8	104.2	
11	13 37 52	134.99	. . . .	5	. .	
	13 43 26	.	67.79	10	45.1	
16	17 28 31	117.49	.	5	. . . .	
	17 33 44	.	86.95	8	27.6	
Dec. 1	13 5 26	44.01	. . . . .	5	. . . .	Seeing very bad.
	13 10 23	. . . .	88.99	8	135.0	
18	15 19 30	80.50	. . . . .	5	. . . .	Seeing very bad.
	15 24 4	.	28.56	8	170.4	
Jan. 15	10 46 38	46.73	. . . .	5	. . . .	Seeing very bad.
	10 51 54	.	16.41	8	154.3	
Feb. 16	8 10 52	21.23	. . .	5	. . . .	Through clouds. Seeing very bad.
	8 15 5	. .	69.78	8	111.2	
19	12 14 30	122.98	. . . . .	5	. . .	
	12 18 3	. . .	16.94	8	32.5	
26	8 59 51	288.76	. . . . .	5	.	
	9 3 49	. . . .	19.65	8	18.2	
Mar. 4	6 54 0	99.68	. . .	5	. . . .	
	6 57 46	. . . . .	21.81	8	9.7	
18	8 40 31	87.88	. . . .	5	. . . .	
	8 45 45	. . . . .	28.34	8	74.6	
24	9 16 34	261.76	. . . .	6	. . .	Seeing excessively bad.
	9 22 57	. . . . .	47.33	7	171.7	
Apr. 1	7 29 3	75.53	. . . .	5	. .	
	7 33 6	. . . . .	29.10	8	165.4	
8	6 59 17	68.36	. . .	5	.	
	7 3 43	. . . .	27.75	8	158.4	
12	7 51 57	238.65	. . . .	5	. . . .	Seeing = 1–2.
	7 56 33		93.95	8	148.6	

Titan and *Dione*

1915-16	C. S. Time	P. A.	Dist.	Cps.	P. A. of Wires	Remarks
	h m s	°	″		°	
Sept. 28	16 58 33	329.34	.	5	. . . .	Seeing good.
	17 2 56		59.48	8	59.3	
Oct. 21	17 40 29	186.90	. . . .	5	. . .	
	17 44 24	.	52.71	8	97.0	
Mar. 8	7 9 22	269.52	. . .	5	. . . .	
	7 13 27	. . .	135.77	8	179.3	
22	10 6 36	323.32	. . . .	5	. . . .	Seeing very bad.
	10 11 0	.	96.24	8	53.3	

Titan and *Rhea*

1915-16	C. S. Time	P. A.	Dist.	Cps.	P. A. of Wires	Remarks
Oct. 5	16 56 30	127.49	.	5	. . . .	
	17 2 16	. .	80.81	8	37.5	
18	15 52 55	246.79	. . . .	5	. . . .	
	15 58 18	.	108.07	8	156.9	
Jan. 15	11 21 53	52.67		5	. . .	Seeing poor.
	11 27 26		82.16	8	163.3	
Feb. 2	13 4 31	297.76	. .	5	. . .	Seeing very bad.
	13 10 0	. .	108.58	8	27.6	
9	13 28 6	190.06	. . . .	5	. . . .	Faint in dense haze.
	13 38 39	. .	49.08	8	100.3	
16	8 20 10	62.50	. .	5	. . .	
	8 24 15	. . .	98.47	8	152.2	
19	12 23 1	293.80	. . . . .	5	. .	
	12 27 24	. . .	198.79	8	23.5	
Mar. 4	7 2 18	339.03	. . . .	5	. .	Very unsteady.
	7 6 37	. .	100.68	8	68.7	
18	9 8 32	61.06	.	5	. . .	
	9 13 35	. . .	89.90	8	151.1	
Apr. 5	7 33 31	327.73	. . . .	5		
	7 37 45	.	82.08	8	57.7	
8	7 47 35	298.83	. .	5		
	7 52 53		144.24	10	28.9	
12	8 12 53	217.53	. . .	5	. .	
	8 17 13	.	60.94	10	127.4	

Tethys and *Iapetus*(?)

1915-16	C. S. Time	P. A.	Dist.	Cps.	P. A. of Wires	Remarks
Apr. 12	9 1 40	243.81	. .	5	. . .	
	9 7 18	.	216.34	8	153.9	

Tethys and *Hyperion*

1915-16	C. S. Time	P. A.	Dist.	Cps.	P. A. of Wires	Remarks
Mar. 18	9 32 58	321.12		5	. .	
	9 38 32		105.67	8	51.2	

1915-16	C. S. Time	P. A.	Dist.	Cps.	P. A. of Wires	Remarks
			Titan and *Hyperion* (?)			
	h m s	°	″		°	
Oct. 26	16 17 14	290.28	.....	5	...	*Hyperion* (?) = 14 mag.
	16 22 52	. .	71.11	9	20.4	
26	16 28 58	269.52	.....	5	..	*Hyperion* (?) = 14.7 mag.
	16 34 27	.	68.95	8	179.9	
Apr. 8	8 30 59	263.83	.....	5	..	*Hyperion* (?) very faint.
	8 34 58	.	134.15	4	173.9	
	8 38 3	.	135.85	4	...	
			Enceladus and a Star (?)			
Feb. 16	8 2 15	215.73	.....	5	....	
	8 6 34	.....	37.56	8	125.2	
Mar. 15	6 56 0	360.96	....	5	..	Difficult. Seeing very bad.
	7 1 59	.	17.09	8	90.7	
			Tethys and a Star (?)			
Feb. 16	7 53 53	130.18	...	5	. .	
	7 57 50	. ..	14.62	8	40.2	
Mar. 4	8 41 44	68.31	....	5	..	
	8 46 27	..	19.50	8	158.7	
			Dione and a Star (?)			
Mar. 15	6 25 29	51.34	.....	5	....	Seeing very bad.
	6 30 17	.....	41.93	8	141.1	
			Rhea and a Star (?)			
Feb. 16	7 44 57	79.26	. ..	5	...	
	7 49 48	.....	48.55	8	69.2	
Mar. 15	6 45 34	297.93	.....	5	....	Seeing bad.
	6 50 17	. .	97.73	8	27.7	
			Titan and a Star			
Mar. 15	7 7 10	271.83	....	5	....	Seeing very bad.
	7 12 49	.	131.73	8	1.7	
			Two Faint Stars near *Dione*			
Oct. 21	17 12 35	294.03	....	3	....	The *pr. star* = 15½ mag.; *fol. star* = 16 mag.
	17 17 4	.....	15.47	6	24.0	
			Titan and the Preceding of the Above Stars			
Oct. 21	17 20 24	194.47	.....	3	....	
	17 23 4	. .	42.23	4	104.5	

Estimated Magnitudes of the Satellites

1915–16	*Mimas*	*Enceladus*	*Tethys*	*Dione*	*Rhea*	Remarks
Aug. 31	. .	. .	13	13.2	11.8	
Sept. 2	. .	14	12		12½	Identification uncertain. *Titan* = 9 mag.
28	. .	14	12.8	12.7	12.2	
Oct. 5	. .	13	12.7	12.7	12	*Titan* = 9 mag.
14	. .	13	12½	12½	11	*Mimas* very faint.
18	. .	13	12½	12½	12	*Iapetus* (?) = 14 mag.
21	14	13	12	12.2	12	
26		14	12.1	12	12	
Nov. 2		14	13	12	12	
16		12½*	13	12½	12	* ? if *Enceladus*.
24		. . .	12½	12½	12	
Dec. 1	. .	13.7	12.4	12.3	12	
18	.	13½	12½	12.2	12	
Jan. 5	14	12½	12½	12	. .	
15	.	14	12½	13	12	
Feb. 2	14	13	12.6	12.7	12.2	
9	15	13	12½	12½	12	
16		14	13	13.5	12.2	
19		14	12.5	12.8	12	
26	.	14	13	13	12	
Mar. 4		13½	13	12¾	12	
8	14		12.3	12.2	12	
15	.	13.7	12½	12½	11.9	
18	.	13	12.2	12.3	12	
22	14	13	12½	12	12	
24		.	13	13	12	
Apr. 1	16	14	12.8	12.9	12	
3	15	14	12.6	12½	12	
5	.	14	12.4	12.2	12	
8	.	.	13	13	12	*Hyperion* = 15½ mag.
12	.	15	12.8	12.8	12	
15			12	12½	12	
May 3	14	14	12½	12½	12	

Yerkes Observatory, Williams Bay, Wis., 1916 September 4.

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THE INTERRELATIONS OF THE ASTEROID ELEMENTS,

By SAMUEL G. BARTON.

The asteroid elements given in the *Berliner Jahrbuch* have been rearranged so that each element in turn increases. When so arranged they have been subdivided into groups of 10° each for the elements having a range of 360°, 1° for inclination and eccentricity and 25″ for mean daily motion. The average values of each of the other elements corresponding to each group were computed. The results are tabulated below and also plotted.

The results are based upon the *Berliner Jahrbuch* for 1914 with the addition of the new ones given in the 1916 and 1918 issues. Only the numbered asteroids 806 in all were included. The elements of 330 being circular elements, this asteroid was omitted. There are some differences of equinox, but this has very little effect upon the results. The notation is that of the *Jahrbuch*. In addition to the elements given the longitude of perihelion π was computed by the relation $\pi = \omega + \Omega$. The magnitude g represents the magnitude the asteroid would have if placed at opposition at a distance of $\frac{1+\sqrt{5}}{2}$ astronomical units from the *Sun*. Assuming the albedo of all the asteroids to be the same, in the average it gives some idea as to the size of the asteroids. In forming the averages the whole degrees only have been used and a half degree added to the average as the average of the neglected parts. Similarly with the mean daily motion only the whole seconds were used. g was used to one decimal, that is used as given. The groups are designated by the lowest value in the group. Thus inclination 1° means all asteroids having inclinations between 1° and 2°. By adding the various sums needed to form the averages, the sum of the element for the 806 asteroids is obtained. The agreement of this sum for all the different arrangements made a very reliable check upon the whole computation.

In plotting, groups containing less than 10 asteroids were in general omitted. The tables however include all. Where a considerable number are scattered in small groups they are sometimes combined into one group and the averages indicated by crosses. Thus the 38 asteroids with inclinations larger than 20° are shown on the plots as crosses. The straight lines represent the general average value of the element. The first curve in each case shows the number of asteroids in each group.

Since the faint asteroids near the *Sun* would be too faint for discovery if they were located in the outer part of the asteroid zone, we would expect that the nearer asteroids would average smaller, that is their average g would be larger, even for a uniform distribution. Thus g is expected to increase with μ and the proper μ curve shows that it does. This fact is to be held in mind in the discussion of the other curves.

The π Curves

The number curve shows the marked effect of *Jupiter* in causing the perihelia of the asteroids to cluster about that of *Jupiter* at 13°. This was noted long ago by Newcomb* and explained upon the basis of the secular perturbations due to *Jupiter*. The ϕ curve shows a similar form and for a similar reason as was shown by Von Brunn†. Brown‡ pointed out a singular thing regarding these curves, namely, that if separate curves are drawn to include the early asteroids only and the late asteroids only, the curves for the later asteroids will have a wider range than the curves for the earlier ones. He suggests that this may be an effect in which the mass plays a part. To get definite information on this point I have formed the average g and μ for each successive page of 40 asteroids each with the following results.

* *A.N.* No. 1382 band 58.

† *A.N.* No. 4122

‡ *Science*, Jan. 20, 1911.

π CURVES

Page	g	μ
1	6.7	854″
2	7.6	810
3	7.9	770
4	8.1	778
5	8.2	778
6	8.7	775
7	9.3	770
8	9.6	815
9	8.9	812
10	8.7	776

Page	g	μ
11	9.0	833″
12	9.2	775
13	8.9	742
14	9.1	755
15	9.3	749
16	9.3	721
17	9.6	750
18	9.7	786
19	9.6	786
20	9.5	757

The average g for the first half or 400 asteroids is 8.38 and for the second half 9.32. The corresponding average μ's are 792.8 and 765.4. We could expect a smaller μ for the later asteroids since the farther asteroids would be discovered in greater numbers with better means of discovery provided that there is a lower limit to the size of these bodies. With a smaller μ however we should find a smaller g. The opposite being the case the early asteroids not only appear to be brighter but also larger. This could be expected since the brighter asteroids would be discovered first

ω CURVES

and an asteroid is brighter either because it is nearer or because it is larger. Both reasons seem to be operating.

The ω curve is low in the middle. The ☊ curve is very irregular where the number of asteroids is small. The various high and low points of the μ curve are due to the presence of exceptional asteroids in these groups. The g and i curves show no marked variations.

The ω Curves

g shows a decrease corresponding however to a decrease in μ. π shows increase in the first two quadrants, but is regular in the last two. The change from the least value to the greatest at 220° is remarkable. Of the 18 asteroids in the group at 220°, 16 have π less than 180° and those two are the first and last of

Ω CURVES

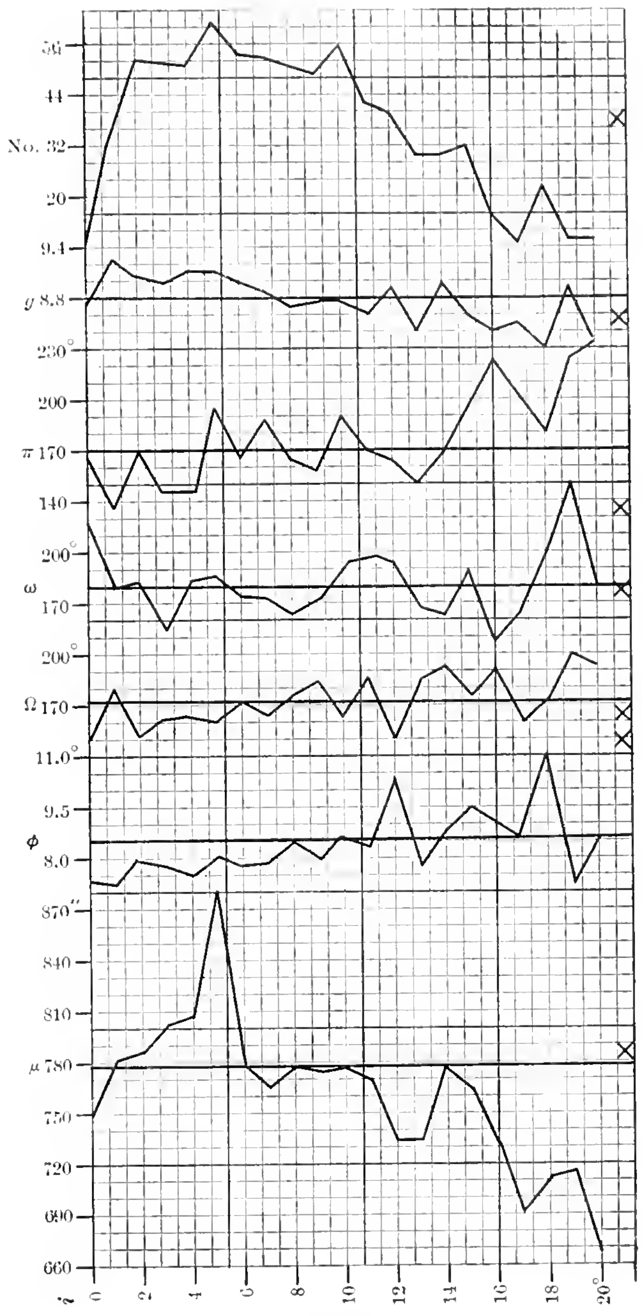
the group. The average π for the remaining 16 is 64°. One of the 16 has π larger than the average of all. The values making the high point at 230 are 20 in number. Six of them are less than 180°. It will be noticed that the Ω curve also jumps at this point as could be expected. The Ω curve has a sine curve form which I believe has its maximum point at 103° and its minimum point at 283°.* This I believe is due to the clustering of *Perihelia* at 13° together with the relation $\omega + \Omega = \pi$. The other curves are uniform.

The Ω Curves

The number curve shows a tendency to cluster in the first two quadrants. There are 450 asteroids in the first two and 356 in the last two quadrants. Newcomb* at first explained this as a clustering about the node of *Jupiter* at 99° but later† decided that in the long run a uniform distribution was to be expected. The ω curve shows the sine curve form already noticed

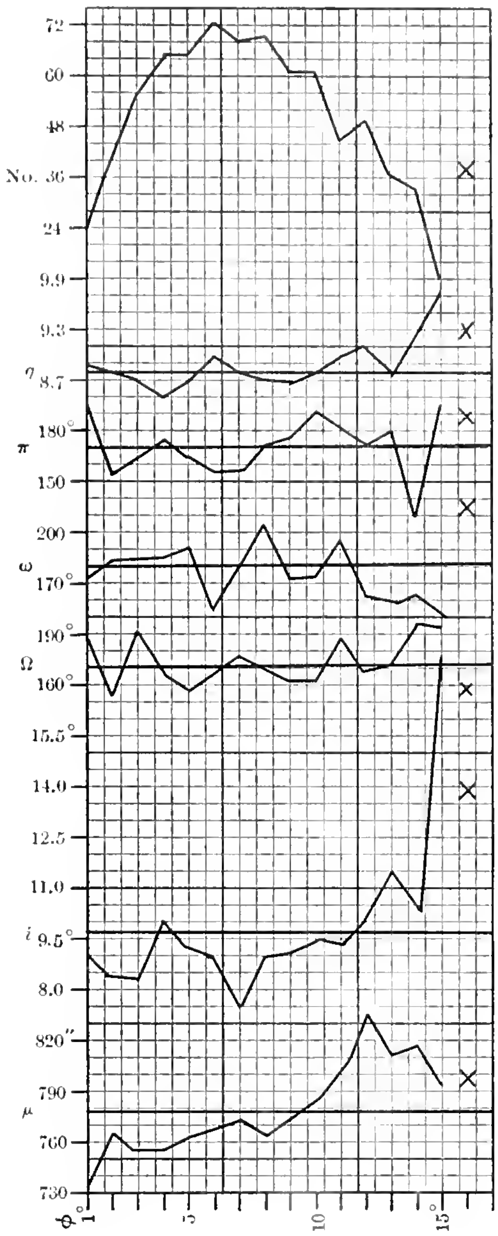
*A.N. No. 1382

†*Enc. Brit.*



i CURVES

in the ω curves and for the same reason. i shows an increase in the first two quadrants. The highest and lowest values succeed each other at 150°, but examination of the separate values shows nothing peculiar. The large μ at 300° is due to the presence of *Eros* in the group.



ϕ CURVES

The ϕ Curves

The number curve shows a pretty regular decrease each way from 5°. g shows increase corresponding to a marked increase in μ. As is usual the inclination increases with the eccentricity. The other curves show no marked variations.

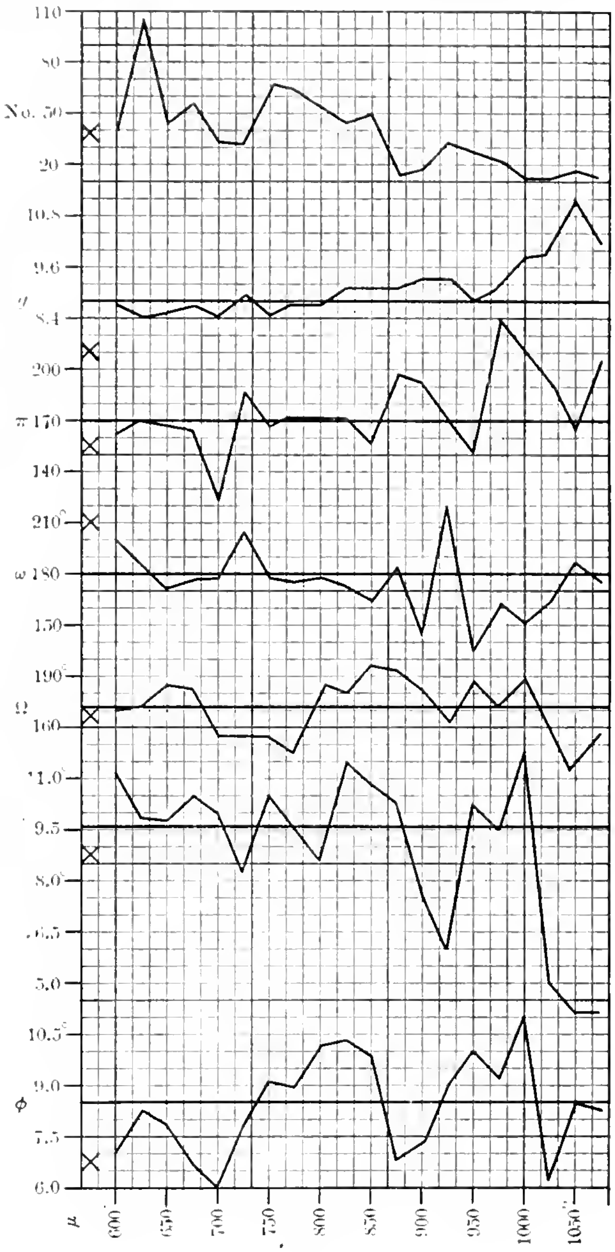
The i Curves

The number is greatest at 6° and decreases rapidly for smaller values and more slowly for larger values. g shows a decrease corresponding to a decrease in μ. π shows an increase. The asteroids beyond 20°, however, have small π. ϕ increases with i as already noticed. The μ curve shows great irregularities. It is surprising that BAUSCHINGER* states that he found no relation between these two quantities. In particular the very high point at 5° is remarkable. Here the number of asteroids 62 is greater than in any other group yet the group departs more from the average than any other except the small group of 10 asteroids at 20°. After 5°, μ in general decreases. The nearer asteroids of large inclination would be thrown far from the ecliptic as seen from the earth in a part of their path and thus their chances of discovery lessened. In this way large inclinations might be eliminated from the nearer asteroids and the trend of the curve explained, but we are confronted by the fact that the 38 asteroids of greatest inclination represented by the cross have μ larger than the average. The explanation is thus rendered unsatisfactory. It is remarkable that there is no irregularity at 5° in the g curve.

The μ Curves

The number curve has been discussed many times. The interval is too large and not well chosen to show the gaps due to *Jupiter's* influence. BODE's law gives $\mu = 756''$ which it will be noted is not the place where the asteroids are most numerous. The average μ, however, is $778''$. The average semimajor axis is 2.81. BODE's law gives 2.8. π shows increase and ω decrease. g increases because of the exclusion of the smaller asteroids in the outer part of the zone. To see if the asteroids are actually larger in those parts I have assumed that we can see asteroids of the 16th magnitude and have excluded those asteroids which if placed in the most distant group by the table would be fainter. That is, asteroids with g greater than 9.2 have been excluded. This corresponds to a magnitude of 14 at $\mu = 525''$ and only 12 asteroids lie beyond this point. The decreased number and corresponding average g called g' are shown in the table. g' is still somewhat small for the farther asteroids possibly because the limit of magnitude is too great and prejudices that part. Not much smaller values could be used without excluding too many asteroids.

* Veröffentlichungen des Königlichen Astronomischen Rechen-Instituts zu Berlin No. 16.



μ CURVES

Where the limit is surely small enough g varies very little so that there is no evidence of a progressive change in the size of the asteroid. For large μ the i curve is very irregular. In particular the last three groups plotted are of interest. Between $\mu = 1025''$ and $\mu = 1125''$ there are 50 asteroids. The average inclination of these is 4°.7 or less than half the general average 9°.6. Moreover only one of the 50 has an inclination greater than 9°.6 and that one is the last in the group. The next to the last one has $i = 8° 40'$ and the first has $i = 9° 33'$. Excluding these three

which are at the extremes we have 47 consecutive asteroids no one of which has an inclination of greater than 6°.7 and of average inclination 4°.2 compared with the general average 9°.6. This condition is surely not accidental. The three asteroids beyond $\mu = 1125''$ have large inclinations, (330) being included as one. It is to be noted that the greatest value of all just precedes these small values. There seem to be no peculiarities among the asteroids at $\mu = 925''$. Fifteen of these 50 asteroids have inclinations of 5°, 11 of 4°, 6 of 3°. An average distribution would lead us to expect 4 of 5° and 3 of 4°. Allowing for the excess which these asteroids would give to μ in the i plot we find that the μ at 4° is below the average but that at 5° is still 53″ too large. Thus this while giving a reason for a high point at 5° only explains it in part.

The average values of the elements are:

$$g = 8.8 \quad \pi = 170° \quad \omega = 180° \quad \Omega = 171° \quad i = 9°.6$$
$$\phi = 8°.5 \quad \mu = 778''.$$

Summary for π

No.	g	π	ω	Ω	i	ϕ	μ
34	9.0	0	182	183	8.4	9.3	771
40	8.4	10	167	198	9.5	9.0	785
39	8.9	20	208	169	10.2	8.6	765
29	8.8	30	167	203	10.2	9.8	783
34	8.9	40	165	166	9.1	10.2	774
32	8.7	50	166	159	7.5	8.2	772
32	9.2	60	186	136	8.8	9.1	803
23	8.6	70	161	165	8.4	7.1	755
20	8.9	80	169	186	8.3	8.9	741
25	8.7	90	173	167	10.7	7.8	711
23	8.8	100	182	188	8.1	7.7	749
11	8.3	110	162	148	10.5	7.0	753
22	8.6	120	182	155	11.0	7.2	829
17	9.2	130	190	177	7.9	7.7	820
20	9.1	140	122	132	11.2	7.1	806
12	8.5	150	159	176	9.3	6.3	738
15	9.1	160	163	171	8.7	6.2	766
10	8.4	170	186	241	12.7	6.2	792
10	8.4	180	152	175	8.3	5.2	721
15	9.2	190	203	207	8.6	5.4	777
13	8.8	200	156	104	11.6	8.3	736
15	9.5	210	140	148	10.0	7.8	823
15	8.7	220	200	217	11.0	8.5	807
15	9.0	230	161	170	9.2	6.0	772
12	8.6	240	142	163	9.4	7.6	779
20	8.5	250	198	166	10.6	6.9	784
20	8.9	260	152	130	10.3	6.7	778
15	8.7	270	147	199	8.2	8.8	813
21	8.7	280	172	164	11.1	8.7	729
17	9.6	290	201	242	10.3	8.2	834
20	8.4	300	160	181	9.4	10.4	798
29	8.9	310	213	165	8.6	8.3	777
35	9.0	320	222	171	9.9	9.2	807
39	9.4	330	200	153	8.8	11.0	799
22	8.5	340	237	141	11.2	10.1	753
35	8.8	350	186	169	10.6	10.4	778

Summary for ω

No.	g	π	ω	Ω	i	ϕ	μ
11	8.5	163	0	159	10.4	7.0	766
22	9.7	165	10	183	8.5	9.3	839
20	9.1	132	20	161	7.9	8.2	773
23	9.2	166	30	177	8.5	8.6	748
33	8.8	152	40	173	11.3	8.8	770
29	8.8	146	50	165	9.4	8.7	801
19	9.0	133	60	162	8.3	6.9	779
21	8.9	153	70	215	9.8	9.5	832
24	9.3	178	80	184	9.9	9.7	843
25	8.7	151	90	185	10.3	7.8	748
26	8.5	176	100	196	8.3	8.2	761
16	9.3	174	110	148	9.9	8.4	794
26	9.5	187	120	201	9.2	8.0	790
14	8.9	161	130	180	6.8	7.4	851
25	8.5	194	140	193	10.8	8.4	782
23	9.4	201	150	203	9.8	10.2	791
22	8.9	170	160	168	8.8	7.7	795
37	8.9	160	170	210	8.2	8.9	803
22	9.0	199	180	194	10.8	7.6	751
29	8.8	181	190	160	8.5	8.4	781
19	8.9	126	200	167	9.6	8.4	769
16	8.9	173	210	184	11.4	7.7	746
18	8.5	86	220	201	10.7	6.3	775
20	8.2	233	230	123	11.4	8.7	734
24	8.8	174	240	154	8.3	7.4	761
19	8.9	198	250	153	10.2	11.0	815
20	8.3	149	260	172	12.5	9.8	770
18	8.7	147	270	152	11.2	7.8	733
21	8.2	167	280	139	8.2	9.2	725
25	9.0	210	290	117	9.9	8.7	797
28	8.4	184	300	136	12.2	9.5	706
19	8.8	151	310	140	7.8	8.8	806
25	8.9	156	320	162	8.9	7.6	738
28	8.8	224	330	134	8.4	8.5	797
22	8.6	210	340	209	10.0	7.5	758
17	8.7	146	350	150	8.5	7.7	803

Summary for Ω

No.	g	π	ω	Ω	i	ϕ	μ
35	8.9	153	158	0	7.6	8.7	776
17	9.3	170	177	10	9.9	8.9	814
22	9.2	219	210	20	7.3	8.0	810
32	9.0	168	177	30	9.6	8.6	756
33	8.8	208	185	40	10.0	8.5	751
8	8.2	191	226	50	8.5	6.8	758
25	8.7	162	183	60	9.7	8.1	761
20	8.4	197	230	70	10.5	8.7	736
31	8.4	166	175	80	9.0	7.7	786
26	9.4	127	213	90	8.8	8.8	821
27	8.9	184	211	100	9.1	9.0	802
11	8.1	144	227	110	9.8	7.3	746
25	8.5	181	186	120	8.3	9.1	762
31	8.7	167	207	130	10.0	8.6	746
30	8.6	173	197	140	11.4	8.0	775
22	8.6	167	191	150	12.6	9.1	808
22	9.2	189	188	160	7.3	7.6	781
33	8.5	189	166	170	11.3	7.3	718
30	9.0	168	188	180	9.6	8.6	780
11	9.2	157	191	190	12.0	10.7	811
27	8.8	169	173	200	9.4	8.5	822
24	9.0	152	162	210	9.8	9.6	811
13	8.9	116	169	220	8.8	7.0	735
22	9.0	175	153	230	9.3	8.0	823
11	9.4	161	174	240	9.2	8.2	821
18	9.0	159	125	250	9.1	9.4	729
16	8.5	161	178	260	8.3	9.3	769
10	8.5	177	119	270	7.8	8.6	774
17	8.6	140	174	280	8.4	7.4	783
24	9.2	121	157	290	8.9	7.8	784
17	8.9	160	151	300	8.9	8.9	878
16	8.8	194	149	310	11.0	10.4	719
19	9.1	180	139	320	11.2	9.8	786
24	8.9	188	199	330	11.2	9.9	762
25	8.4	162	162	340	10.5	7.1	720
29	9.1	172	177	350	8.9	8.0	773

Summary for i

No.	g	π	ω	Ω	i	ϕ	μ
9	8.7	165	217	148	0	7.3	744
35	9.3	135	180	181	1	7.2	781
53	9.1	171	181	153	2	8.0	785
52	9.0	143	156	161	3	7.8	802
54	9.1	142	184	163	4	7.5	807
62	9.1	195	186	160	5	8.2	881
54	9.0	165	173	172	6	7.8	779
53	8.9	188	172	166	7	7.9	766
51	8.7	165	164	177	8	8.6	777
49	8.8	159	172	185	9	7.8	776

Summary for i (*Continued*)

No.	g	π	ω	Ω	i	ϕ	μ
56	8.8	192	196	163	10	8.6	778
42	8.6	171	197	189	11	8.4	770
41	8.9	166	191	150	12	10.4	734
30	8.5	149	168	185	13	7.7	735
30	9.0	165	163	194	14	8.8	779
32	8.6	197	190	176	15	9.5	763
16	8.4	226	146	192	16	9.1	733
9	8.5	201	163	158	17	8.7	692
23	8.2	180	198	170	18	11.0	711
10	8.9	226	242	200	19	7.1	715
10	8.3	234	180	197	20	8.6	667
6	8.4	136	168	208	21	7.7	773
5	8.4	160	189	115	22	10.3	776
6	8.0	183	206	217	23	13.7	851
4	8.4	140	295	115	24	12.5	753
8	9.4	105	150	180	25	10.8	830
8	3.2	83	123	80	26	14.2	656
1	9.5	30	57	334	27	14.8	690
1	8.2	204	140	64	28	5.6	683
1	9.5	104	309	156	29	13.0	839
1	11.8	231	76	155	32	20.5	833
2	7.5	186	182	186	34	12.4	762

Summary for ϕ

No.	g	π	ω	Ω	i	ϕ	μ
7	8.7	91	182	116	6.9	0	682
24	8.9	194	173	186	9.1	1	739
40	8.8	155	183	151	8.4	2	764
55	8.7	165	184	191	8.3	3	756
65	8.5	177	185	169	10.1	4	756
65	8.7	169	194	158	9.3	5	763
73	9.0	156	155	164	9.0	6	769
69	8.8	157	182	179	7.5	7	772
70	8.7	173	203	170	9.0	8	763
61	8.7	176	172	163	9.1	9	778
61	8.8	194	173	163	9.6	10	788
45	9.0	181	193	188	9.4	11	805
50	9.1	172	161	169	10.1	12	834
37	8.8	177	160	173	11.5	13	811
34	9.2	129	164	198	10.4	14	819
12	9.8	196	152	194	18.0	15	795
9	9.3	152	201	231	11.4	16	790
9	9.3	164	206	118	9.4	17	768
5	9.0	202	255	90	20.3	18	864
7	8.5	223	198	179	12.9	19	809
4	9.0	170	223	127	25.5	20	777
1	9.3	17	281	126	3.3	21	581
1	10.2	337	301	36	18.6	22	848
1	11.4	332	89	244	15.2	24	840
1	14.5	337	152	186	10.8	32	854

Summary for μ

No.	q	π	ω	Ω	i	ϕ	μ	No.	q'
2	7.0	119	150	328	14.2	5.3	275	2	7.0
2	6.8	331	315	197	13.3	7.3	300	2	6.8
1	8.1	310	234	76	2.4	4.6	400	1	8.1
1	7.3	282	54	228	7.9	9.3	425	1	7.3
5	7.5	81	198	171	5.7	8.5	450	5	7.5
1	7.7	2	243	119	4.4	4.6	500	1	7.7
5	7.7	177	300	93	11.5	5.3	525	5	7.7
18	7.5	148	186	178	11.1	7.1	550	17	7.4
3	9.3	125	225	140	7.7	9.7	575	1	8.2
40	8.7	163	200	170	11.2	7.0	600	29	8.3
105	8.4	170	185	173	9.9	8.3	625	83	8.1
44	8.5	167	171	184	9.8	7.9	650	35	8.1
56	8.7	165	175	182	10.5	6.6	675	44	8.4
34	8.5	123	178	157	10.0	6.1	700	22	7.8
31	9.0	188	207	155	8.3	7.8	725	19	8.4
69	8.5	169	177	154	10.5	9.1	750	19	8.0

Summary for μ (Continued)

No.	q	π	ω	Ω	i	ϕ	μ	No.	q'
67	8.7	173	175	113	9.7	9.0	775	45	8.0
55	8.7	173	179	184	8.6	10.1	800	37	8.2
14	9.1	173	172	181	11.6	10.1	825	24	8.1
50	9.1	156	163	195	10.7	9.8	850	29	8.1
12	9.1	199	186	192	9.8	6.9	875	5	7.8
17	9.3	195	142	181	7.6	7.1	900	8	8.3
31	9.3	175	220	161	6.0	9.0	925	14	7.7
29	8.9	151	138	187	10.3	10.1	950	20	8.0
21	9.1	232	163	171	9.6	9.2	975	12	7.8
11	9.8	211	153	189	11.8	11.1	1000	5	8.3
11	9.9	191	164	158	5.0	6.1	1025	2	7.9
16	11.2	166	187	137	1.3	8.6	1050	0	
14	10.2	207	176	159	4.2	8.3	1075	3	7.9
9	11.1	115	199	155	5.8	7.9	1100		
1	10.4	297	123	175	22.5	1.2	1300		
1	10.6	121	178	304	10.8	12.9	2000		

Flower Observatory, University of Pennsylvania.

OBSERVATIONS OF ASTEROIDS,

MADE AT THE U.S. NAVAL OBSERVATORY WITH THE PHOTOGRAPHIC TELESCOPE,

By GEORGE H. PETERS.

[Communicated by Captain J. A. Hoogewerff, U. S. Navy Superintendent.]

Name	Mag.	Date 1915	G. M. T.	α 1915.0	δ 1915.0
			h m	h m s	° ′ ″
(*377*) *Campania*	11.1	Oct. 27	13 20.5	0 18 16.3	+ 4 42 55
(*119*) *Althæa*	10.1	28	15 40.3	1 08 31.9	+ 7 00 57
(*119*) *Althæa*	10.1	29	14 26.0	1 07 52.5	+ 6 54 05
(*556*) *Phyllis*	12.3	30	14 27.5	1 07 29.7	+16 35 50
(*225*) *Henrietta*	12.3	30	15 42.1	1 27 37.9	+ 5 41 04
(*377*) *Campania*	11.1	Nov. 1	14 27.0	0 16 05.5	+ 4 10 30
(*239*) *Adrastea*	13.0	1	15 29.0	0 31 59.9	− 0 08 55
(*167*) *Urda*	13.0	1	15 29.0	0 26 44.7	+ 1 06 20
(*556*) *Phyllis*	12.3	3	14 16.0	1 04 20.4	+16 08 43
(*225*) *Henrietta*	12.3	3	14 58.0	1 25 16.4	+ 5 06 03
(*5*) *Astræa*	10.1	3	15 37.0	1 43 00.8	+ 2 11 13
(*279*) *Thule*	13.9	3	16 52.0	2 21 01.6	+12 06 40
(*377*) *Campania*	11.1	5	13 14.0	0 14 48.5	+ 3 48 00
(*5*) *Astræa*	10.1	5	15 59.0	1 41 19.4	+ 2 02 01
(*279*) *Thule*	13.9	5	16 58.0	2 19 51.3	+12 01 11
(*147*) *Protogeneia*	12.4	7	15 18.8	2 03 03.2	+11 08 16
(*331*) *Etheridgea*	12.0	7	16 32.8	2 17 40.1	+16 45 37
(*340*) *Eduarda*	12.3	7	16 32.8	2 18 23.2	+15 43 01

OBSERVATIONS OF ASTEROIDS (*Continued*)

Name	Mag.	Date 1915	G. M. T.	α 1915.0	δ 1915.0
			h m	h m s	° ′ ″
(341) *California*	12.7	Nov. 7	16 32.8	2 30 44.8	+17 35 35
(331) *Etheridgea*	12.0	9	16 40.7	2 15 56.6	+16 40 52
(340) *Eduarda*	12.3	9	16 40.7	2 16 32.7	+15 38 05
(341) *California*	12.7	9	16 40.7	2 28 24.9	+17 31 35
(425) *Cornelia*	13.3	9	17 54.1	3 16 27.4	+16 52 40
(503) *Evelyn*	11.7	9	17 54.1	3 20 06.6	+15 25 46
(65) *Cybele*	11.5	10	15 52.0	3 33 23.5	+14 30 15
(425) *Cornelia*	13.3	10	17 06.0	3 15 35.6	+16 50 24
(503) *Evelyn*	11.7	10	17 06.0	3 19 10.6	+15 24 19
(147) *Protogeneia*	12.4	24	12 41.6	1 52 26.5	+12 59 27
(65) *Cybele*	11.5	26	14 04.0	3 21 46.0	+13 46 09
(140) *Siwa*	11.8	27	14 56.0	4 07 23.9	+17 34 17
(233) *Asterope*	11.2	27	14 56.0	4 07 59.0	+16 19 50
(451) *Patientia*	10.3	27	15 42.0	4 11 24.8	+10 29 28
(498) *Tokio*	11.1	27	15 42.0	4 21 15.7	+12 53 52
(451) *Patientia*	10.3	30	15 19.0	4 08 37.5	+10 36 30
(498) *Tokio*	11.1	30	15 19.0	4 18 07.3	+12 57 36
(140) *Siwa*	11.8	Dec. 6	15 26.5	3 58 52.7	+17 18 33
(233) *Asterope*	11.2	6	15 26.5	3 59 43.3	+15 33 50
(199) *Byblis*	13.2	10	14 56.1	4 41 14.7	+15 57 46
(67) *Asia*	11.9	10	15 56.5	5 55 19.4	+15 11 15
(387) *Aquitania*	11.0	31	17 12.1	7 03 13.5	+11 46 27
		Date 1916	G. M. T.	α 1916.0	δ 1916.0
(67) *Asia*	11.9	Jan. 2	13 19.8	5 32 22.2	+14 51 32
(201) *Penelope*	12.2	2	14 20.0	6 15 20.5	+15 47 23
(12) *Victoria*	10.7	2	15 09.0	6 41 13.3	+13 55 33
(387) *Aquitania*	11.0	2	16 00.0	7 01 31.2	+11 54 51
(12) *Victoria*	10.7	7	14 16.6	6 35 51.9	+13 54 06
(727) *Nipponia*	12.4	7	16 26.1	7 29 01.9	+11 37 08
(34) *Circe*	11.0	7	16 26.1	7 36 15.1	+12 46 41
(542) *Susanna*	12.9	7	17 27.1	8 03 45.4	+ 9 20 24
(727) *Nipponia*	12.4	8	16 01.0	7 28 04.6	+11 45 20
(34) *Circe*	11.0	8	16 01.0	7 35 20.6	+12 49 00
(542) *Susanna*	12.9	8	17 08.3	8 02 56.9	+ 9 24 41
(201) *Penelope*	12.2	24	12 54.0	5 57 57.6	+16 36 25

Corrections kindly furnished by Professor FRITZ COHN have been incorporated in the above list.

CONTENTS.

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MEASURES OF 234 DOUBLE STARS,

By FREDERICK L. BROWN

The principal program for the 18½-inch refractor of the Dearborn Observatory at present is the photographic determination of stellar parallax. This takes the early evening and morning hours, so that the middle of the night can be used conveniently for micrometric work. Using these hours the following measures of double stars have been made during the two years, September 1914 to September 1916.

The micrometer is the one used by Professors HOUGH and FOX; and a description of it can be found in Volume I of the *Annals of the Dearborn Observatory*. The value of the micrometer screw is, $1R = 14''.840$.

In making the measures of position angle it has been my custom to set the wires a few seconds apart and, with the two stars midway between them, rotate the micrometer until the wires were parallel to the line through the two stars. However in the case of some of the wide pairs a single wire has been used, making it bisect both images. From three to five independent settings were made in all cases. The method of double distances has been used throughout in measuring the separation of the pairs, here from three to five settings were made with the movable wire in each position. In most cases the stars have been measured on at least three nights.

For the most part the stars in this list are fixed or have motion of well-known character, a large per cent. of them being STRUVE stars. In extending his observing list the author is adding pairs from the recent AITKEN and ESPIN lists.

The position for 1880 is given for those stars measured which are not found in *β G. C.*

β G.C. 19 Σ 13 App. II

	°	″
1914.750	276.1	76.07
14.898	275.6	75.87
15.714	276.2	76.59
1915.121	276.0	76.18

71 OΣ App. I

1914.698	102.6	76.14
14.756	102.9	75.61
14.764	102.9	76.00
14.898	102.2	76.14
1914.779	102.6	75.97

85 KR 3

1914.706	42.3	3.27
14.720	44.2	3.54
14.756	42.8	3.28
1914.727	43.1	3.36

111 WEISSE 1

	°	″
1915.810	106.4	5.22
15.818	105.6	5.19
15.848	107.1	5.32
1915.825	106.4	5.25

125 ESPIN 41

1915.799	220.7	6.08
15.815	223.3	6.29
15.829	221.3	5.94
1915.814	221.8	6.10

May be some motion.

137 KR 1

1915.799	189.7	2.25
15.829	190.0	2.29
15.848	189.7	2.19
1915.825	189.8	2.24

151 A. CLARK 1

	°	″
1914.709	286.1	1.16
14.750	285.0	1.31
14.906	285.7	1.25
14.797	284.9	1.16
1914.791	285.4	1.22

Slow change.

216 ESPIN 2 = **269** ESPIN

1915.780	112.4	5.79
15.815	113.2	5.66
15.848	115.5	5.82
1915.814	113.7	5.76

250 ESPIN 116

1915.815	256.4	7.25
15.829	257.0	7.33
15.848	257.8	7.57
1915.831	257.1	7.38

271 β 780

	°	″
1915.810	139.6	2.42
15.818	140.2	2.28
15.848	141.1	2.13
1915.825	140.3	2.28

332 β 230

1915.810	323.2	3.67
15.818	326.1	3.95
15.848	323.5	3.68
1915.825	324.3	3.77

360 Σ 46

1914.712	193.0	6.70
14.717	192.8	6.61
14.734	193.4	6.90
1914.721	193.1	6.74

122 Σ 59

1914.706	148.0	2.25
14.720	147.5	2.13
14.723	147.8	2.11
14.737	149.6	2.41
14.745	148.8	2.10
14.756	148.0	2.15
1914.731	148.3	2.29

563 H 10

A and *B*

1914.712	310.7	5.15
14.715	310.5	5.11
14.717	313.3	5.35
1914.715	311.5	5.20

Distance may be changing.

A and *C*

1914.712	53.9	9.58
14.715	56.8	8.61
14.717	54.5	8.93
1914.715	55.1	9.04

615 Hu 602

1914.797	205.5	4.67
14.873	203.8	4.01
14.906	203.8	3.94
1914.859	204.4	4.21

629 Σ 96

1914.706	297.1	1.04
14.720	285.3	1.21
14.737	283.3	1.00
1914.721	288.6	1.08

669 Σ 103

1914.808	247.2	5.74
14.873	246.1	5.50
14.906	245.4	5.38
1914.862	246.2	5.54

690 H 5453

1914.873	209.0	27.65
14.906	208.6	27.74
15.714	207.9	27.72
1915.164	208.5	27.70

Common proper-motion.

721 OΣ App. 16

1914.734	137.3	65.46
15.714	137.5	65.60
15.722	137.6	66.40
1915.390	137.5	65.82

Distance increasing.

759 Σ 118

1914.720	78.7	12.76
14.745	78.8	12.63
14.756	77.7	12.57
1914.740	78.4	12.65

Change due to proper-motion.

760 Σ 122

1914.709	325.1	6.11
14.715	326.9	6.20
14.717	330.1	6.42
14.734	330.1	5.83
14.745	329.0	6.41
1914.724	328.2	6.19

792 Σ 131

1914.706	141.3	13.80
14.720	141.9	13.78
14.723	142.6	14.10
14.745	143.2	14.09
14.756	144.3	14.16
1914.730	143.3	13.99

813 Σ 136

1914.734	77.0	16.18
14.879	77.9	15.58
14.898	77.9	15.43
1914.837	77.6	15.73

821 β 507

1914.906	152.1	2.06
14.808	157.1	2.23
15.580	154.5	2.12
1915.098	154.6	2.14

843 Σ 139

1914.745	42.9	10.13
14.841	43.2	10.25
15.722	40.9	9.75
1915.103	42.3	10.10

951 Σ 163

1914.720	35.7	34.86
14.745	35.6	34.92
14.841	34.5	35.25
1914.769	35.3	35.01

963 Σ 174

1914.873	166.2	2.72
14.879	167.0	2.60
14.898	167.7	2.60
14.906	166.6	2.80
1914.889	166.9	2.68

1008 Σ 4 App. I

A and *B*

1914.879	300.2	188.24
14.898	299.6	189.02
15.709	300.0	187.96
15.722	299.6	187.14
1915.302	299.9	188.09

Proper-motion computed from measures by Σ, Δ, *Je*, β, quoted from β *G.C.*, and the above, give 0″.166 in 76°.0.

A and *a*

1914.879	81.2	19.60
14.898	78.2	17.97
15.709	79.3	18.66
15.722	78.3	18.83
1915.302	79.2	18.76

1034 β 7

1914.717	14.3	2.75
14.906	15.2	2.95
15.788	14.9	2.87
1915.137	14.8	2.86

1070 OΣ 38

γ *Andromedae*

A and *B*

1914.706	63.7	11.00
14.841	64.0	10.25
15.722	63.6	10.01
1915.090	63.8	10.42

1105 Σ 214

1914.898	189.1	5.56
14.906	189.3	5.08
15.722	187.3	5.16
1915.175	188.6	5.27

1115 Σ 218

1914.841	248.7	4.77
14.873	247.3	5.06
14.898	247.1	5.02
1914.871	247.7	4.95

1116 H VI 69

A and *B*

1914.841	36.3	93.61
15.741	36.6	92.85
15.818	36.0	92.91
1915.467	36.3	93.12

A and *C*

1914.841	278.6	105.75
15.741	278.3	105.80
15.818	278.8	105.97
1915.467	278.6	105.84

1125 Σ 222

	°	″
1914.898	34.8	16.50
15.741	35.4	16.81
15.818	35.1	16.81
1915.486	35.1	16.71

1137 Σ 227

1914.879	71.0	4.12
14.906	73.4	3.84
15.818	74.6	4.07
1915.201	73.0	4.01

1166 Σ 235

1914.909	45.3	1.76
15.815	47.1	1.69
15.867	225.7	1.70
1915.530	46.0	1.72

1171 β 786

1915.829	350.6	5.13
15.867	349.8	5.28
16.555	351.3	5.12
16.716	350.4	5.40
1916.242	350.5	5.23

.. Anon. (10.2 . 10.2)

R.A. $2^h\ 9^m\ 40^s$ Decl. 55° 42′.2

1915.799	131.1	4.58
15.815	131.9	4.72
15.829	133.0	4.79
15.867	133.1	4.70
1915.828	132.3	4.70

$0^s.8$ preceding, 104″ north of *A.G.* Hels. Gotha 2062.

. Λ 1273

R.A. $2^h\ 9^m\ 11^s$ Decl. 55° 40′.5

1915.799	335.2	3.98
15.815	338.9	4.07
15.829	336.6	3.71
15.867	335.2	3.80
1915.828	336.5	3.89

1224 Σ 254

	°	″
1914.906	354.7	12.26
15.741	352.6	12.15
15.824	353.8	12.10
1915.490	353.7	12.17

1262 Σ 262

A and *B*

1914.909	251.2	2.05
15.780	252.5	2.40
15.799	253.0	2.23
1915.496	252.2	2.23

A and *C*

1914.909	107.2	7.45
15.780	111.4	7.77
15.799	110.3	7.55
1915.496	109.6	7.59

1350 *A.G.* 43 = **12901**

1914.906	58.3	2.75
15.741	238.9	2.91
15.824	241.0	2.98
1915.490	239.4	2.88

Measures of distance discordant.

1352 Σ 288

1914.775	216.1	11.78
14.906	216.5	11.95
15.824	218.3	12.10
1915.168	217.0	11.94

1401 Σ 299

1914.775	293.0	3.35
14.873	292.3	3.57
14.879	297.3	3.55
1914.842	294.2	3.49

1418 Σ 311

A and *B*

	°	″
1914.873	117.0	3.08
15.709	120.5	3.29
15.824	122.1	3.16
1915.469	119.9	3.18

A and *C*

1914.873	109.5	24.81
15.709	110.5	25.43
15.824	111.0	25.07
1915.469	110.3	25.10

1505 Σ 329

1914.841	272.9	16.13
15.780	273.2	16.06
15.829	271.9	16.01
1915.483	272.7	16.07

1530 Σ 336

1914.879	9.0	8.84
15.709	8.4	8.58
15.843	8.0	8.53
1915.477	8.5	8.65

1562 Λ 456

1914.906	40.8	4.34
15.824	42.8	4.48
15.843	41.3	4.30
1915.524	41.6	4.37

1610 Σ 362

A and *B*

1914.909	142.9	7.33
15.755	140.9	7.20
15.829	140.7	7.00
1915.498	141.5	7.18

A and *C*

1914.909	43.4	26.05
15.755	44.9	26.28
15.829	44.7	26.47
1915.498	44.3	26.27

A and *D*

	°	″
1914.909	241.3	35.31
15.755	241.7	35.39
15.829	241.8	35.44
1915.498	241.6	35.38

A and *E*

1915.755	285.7	30.69
15.829	285.6	31.08
1915.792	285.6	30.88

E is about 11th magnitude.

1708 Σ 389

1914.884	66.6	2.75
15.755	65.2	2.75
16.062	68.5	2.39
1915.567	66.8	2.63

1710 OΣ 54

1914.909	356.4	24.65
15.780	356.9	24.55
16.062	356.5	24.35
1915.584	356.6	24.52

Distance changing slowly.

1871 Σ 448

1914.906	16.6	3.09
15.706	16.3	3.21
15.832	15.0	3.03
1915.481	16.0	3.11

1939 Σ 470

1914.890	348.4	6.62
15.832	347.8	7.15
15.862	347.8	7.00
1915.528	348.0	6.92

1985 Σ 479

A and *B*

1914.884	126.9	6.95
15.706	127.2	7.22
15.832	126.5	7.43
1915.474	126.9	7.20

A and *C*

1914.884	240.9	58.35
15.706	241.3	58.41
15.832	241.0	58.38
1915.474	241.1	58.38

2043 $O\Sigma$ 72

1914.873	327.7	1.51
14.881	329.9	1.39
15.024	327.6	1.12
1914.927	328.4	1.34

2100 *β* 86

1914.873	50.6	4.26
14.884	50.5	3.91
14.890	18.6	4.08
1914.882	19.9	4.08

2310 *BD* 21°694

1914.873	115.0	
14.884	114.6	5.63
14.906	115.0	5.28
15.024	114.8	4.68
15.040	115.7	5.07
1914.955	115.0	5.16

2482 *A.G.* 85

1914.906	175.8	9.63
15.021	176.7	9.51
15.024	176.8	9.65
1914.984	176.4	9.60

2486 *A.G.* 87

1914.909	280.8	31.03
15.021	281.1	31.10
15.024	280.9	31.75
1914.985	280.9	31.29

2584 Σ 654

1914.884	61.5	6.21
14.906	61.6	7.03
15.021	61.8	7.01
15.040	62.7	6.77
1914.964	61.9	6.76

2637 Σ 681

1914.967	180.8	23.25
15.071	181.2	23.16
16.062	180.6	23.40
1915.367	180.9	23.37

2789 Σ 730

1914.884	141.9	9.76
14.906	141.5	9.68
15.021	140.8	9.87
15.024	140.8	9.65
1914.959	141.2	9.74

2837 Σ 748,

*θ*¹ *Orionis*

A and *B*

1914.706	35.8	8.47
14.890	33.3	8.72
15.079	213.4	8.74
1914.892	34.1	8.64

A and *C*

1914.706	128.5	13.06
14.890	130.7	13.01
15.079	131.7	12.66
1914.892	130.3	12.91

B and *C*

1914.706	164.2	17.04
14.890	164.1	16.85
15.079	160.8	16.40
1914.892	163.0	16.76

D and *C*

1914.706	240.7	13.45
14.890	241.8	13.34
15.079	242.2	12.67
1914.892	241.6	13.15

C and *E*

1914.706	125.5	3.58
14.890	128.2	3.69
1914.798	126.8	3.63

A and *E*

1914.706	352.3	4.33
14.890	352.0	4.12
1914.798	352.2	4.23

D and *B*

1914.706	300.7	19.23
14.890	298.5	19.23
15.079	300.1	19.02
1914.892	299.8	19.16

A and *D*

1915.079	276.5	21.03

2917 Σ 780

A and *B*

1914.898	101.7	3.72
14.967	102.5	3.64
14.975	100.7	3.54
16.062	105.5	3.73
1915.225	102.6	3.66

A and *C*

1914.898	152.2	11.60
14.967	151.6	11.72
14.975	152.7	
16.062	150.5	11.42
1915.225	151.8	11.58

2984 Σ 796

1915.024	62.5	3.61
15.843	242.5	3.76
15.870	61.6	3.57
1915.579	62.2	3.65

3106 Barnard 1

1914.906	193.1	1.94
15.019	194.0	1.85
15.024	192.3	1.74
15.870	193.3	1.93
1915.204	193.2	1.86

3242 Σ 877

1914.906	262.3	5.47
15.019	262.6	5.60
15.024	262.6	5.63
1914.982	262.5	5.57

3322 $O\Sigma$ 136

1914.898	75.7	5.82
14.967	75.2	5.45
14.975	78.7	5.51
1914.947	76.5	5.59

3333 Σ 899

1914.906	20.9	2.26
15.019	18.9	2.49
15.024	19.0	2.28
1914.982	19.6	2.34

3423 Σ 918

1914.898	327.1	4.87
14.967	331.3	4.65
14.975	328.7	4.50
15.062	329.9	4.76
1914.975	329.3	4.69

Angle slowly increasing.

3541 Σ 946

	°	″
1915.062	132.6	4.04
15.897	131.3	3.53
16.007	129.4	4.10
16.062	132.7	3.95
1915.757	131.5	3.91

3559 Σ 948

A and *B*

1914.898	107.9	1.92
14.967	112.6	1.85
14.975	111.1	1.82
16.007	109.0	1.50
1915.212	110.2	1.77

Retrograde motion.

A and *C*

1914.898	305.8	8.31
14.967	310.4	8.41
14.975	307.7	8.60
16.007	307.7	8.60
1915.212	307.9	8.48

No change.

3692 Σ 982

1914.906	157.3	6.50
15.019	157.1	6.58
15.021	157.6	6.78
15.024	157.9	6.74
1914.992	157.5	6.65

Slow retrograde motion with increasing distance.

3841 OΣ 165

1915.021	30.4	5.30
15.024	29.1	5.15
16.086	28.1	5.38
1915.377	29.2	5.28

3935 Σ 1056

1915.019	295.1	3.98
15.021	297.2	3.37
15.024	299.3	3.81
1915.021	297.3	3.72

4202 Σ 1122

1914.898	5.6	15.53
14.967	5.3	15.44
15.062	6.9	15.20
1914.976	5.9	15.39

4215 Σ 1127

A and *B*

1914.898	338.6	5.12
14.967	339.7	5.30
14.975	340.6	5.50
15.062	340.5	5.31
1914.976	339.8	5.31

A and *C*

1914.898	175.4	11.08
14.967	175.7	11.22
14.975	176.4	11.51
15.062	177.7	11.45
1914.976	176.2	11.31

4226 OΣ 179

1915.019	236.4	6.72
15.021	236.6	6.66
15.024	231.9	6.40
1915.021	235.0	6.59

Change uncertain.

4468 Espin 71

1915.062	284.5	3.23
16.062	282.5	2.95
16.215	284.0	2.89
1915.780	283.7	3.02

4481 Σ 1192

A and *B*

1914.906	256.3	2.70
14.967	260.5	2.60
14.975	261.9	2.47
1914.949	259.6	2.59

A and *C*

	°	″
1914.906	223.1	49.05
14.967	223.4	48.93
14.975	223.3	48.48
1914.949	223.3	48.82

4601 Σ 1223

1915.019	34.3	4.91
15.021	37.1	4.95
15.024	35.3	5.09
1915.021	35.6	4.98

Position angle evidently requires a correction of 180°.

4602 Σ 1224

1915.019	12.6	5.86
15.021	14.5	5.79
15.024	13.4	4.68
1915.021	13.5	5.78

4620 Σ 1225

1915.062	192.9	3.81
16.062	10.0	3.79
16.215	192.4	3.65
1915.780	191.7	3.75

No material change.

4723 Σ 1258

1914.967	329.9	9.79
14.975	330.7	9.77
15.062	330.6	9.96
1915.001	330.4	9.84

4798 Σ 1282

1915.019	277.1	3.64
15.021	281.2	3.66
15.024	279.2	3.62
1915.021	279.3	3.64

4815 Σ 1280

1914.975	227.2	4.42
15.062	227.9	4.24
16.062	228.4	4.58
1915.366	227.7	4.41

Position angle evidently requires a correction of 180°.

4841 OΣ 195

1915.019	138.3	9.51
15.021	139.3	9.35
15.024	138.4	9.50
1915.021	138.7	9.45

5062 β 105

1915.040	200.7	3.25
15.071	204.5	3.09
16.059	203.4	2.68
1915.370	202.8	3.01

5091 Σ 1355

1914.967	335.9	2.54
15.019	336.4	2.74
15.021	336.5	2.50
15.071	336.8	2.42
1915.020	336.3	2.55

5187 Σ 1377

1914.967	139.8	3.72
15.040	138.0	4.06
15.071	138.9	3.81
15.076	135.8	3.90
1915.038	138.1	3.87

5232 OΣ 209

1915.062	309.0	4.81
16.111	311.2	4.64
16.215	307.8	4.54
1915.816	309.3	4.67

5256 Σ 1396

1915.040	127.8	3.91
15.071	130.6	3.83
15.076	130.4	3.66
1915.062	129.5	3.80

5377 Σ 1421

1915.024	332.1	1.57
15.071	331.9	1.50
15.076	332.8	1.35
1915.057	332.3	1.47

5388 Σ 1424

1915.024	119.8	3.83
15.071	118.1	4.21
15.076	116.6	4.18
1915.057	118.2	4.07

5400 Σ 1427

1915.035	214.7	9.45
15.062	215.9	9.50
16.245	214.2	9.61
1915.447	214.9	9.52

5471 Σ 1447

1915.019	123.6	4.36
15.024	123.8	4.27
15.040	124.2	4.38
15.071	124.2	4.26
1915.038	123.9	4.32

5539 Σ 1466

1915.019	238.5	6.55
15.024	239.9	6.80
15.040	239.5	6.93
15.071	240.5	6.84
1915.038	239.6	6.78

5624 Σ 1496

1915.071	352.2	19.22
15.076	352.6	19.14
16.059	352.4	19.07
16.106	352.5	19.34
1915.578	352.4	19.27

5679 Σ 1510

1915.062	335.3	1.45
16.245	333.7	1.62
16.346	335.0	1.63
1915.884	334.7	1.57

5720 Σ 1521

1915.019	94.5	3.50
15.024	94.7	3.78
15.040	96.4	3.90
1915.028	95.2	3.73

5731 Σ 1523

ξ Ursae Majoris.

1915.019	113.6	3.06
15.024	115.8	3.07
15.071	117.2	3.06
15.076	117.4	3.22
1915.047	116.0	3.10

5735 Σ 1524

1915.071	144.6	7.29
15.076	147.9	7.42
16.106	146.4	7.63
1915.418	146.3	7.44

5812 Σ 1547

1915.019	325.3	15.46
15.071	326.6	15.58
15.076	325.4	15.43
1915.055	325.8	15.49

6076 Σ 1613

1915.071	14.8	1.54
16.106	14.6	1.32
16.212	14.5	1.17
1915.796	13.6	1.34

6082 Σ 1615

1915.019	87.5	27.02
15.071	87.3	27.13
16.212	88.2	26.96
1915.434	87.7	27.04

6131 Σ 1633

1915.019	245.0	9.01
15.771	244.1	8.93
16.106	245.2	8.93
16.212	65.8	8.96
1915.602	245.0	8.96

6239 Σ 1669

1915.019	295.5	5.58
15.035	305.6	5.74
15.071	305.0	5.85
1915.042	302.0	5.71

6311 Σ 1690

1915.019	145.7	5.99
15.071	148.7	.
16.106	146.9	6.18
1915.399	147.1	6.08

6390 Σ 1719

1915.019	1.6	7.05
16.106	1.8	7.31
16.492	1.6	7.35
16.503	2.2	7.42
1916.030	1.8	7.28

Anon. (11.0, 11.0)

R.A. 13h 1m 38s Decl. 1° 52′.9

1916.192	69.9	7.84
16.503	251.8	7.52
16.511	68.7	7.42
1916.502	70.1	7.58

Star 4′ south, 5s preceding *BD* 2°2627.

BD 2 2629

(9.0, 10.5)

R.A. 13h 1m 47s Decl. 1° 58′.5

1916.106	31.4	1.45
16.192	33.6	1.22
16.503	29.5	1.53
16.511	33.6	1.67
1916.403	32.0	1.47

6397 Σ 1721

1916.106	358.4	6.26
16.503	356.9	5.94
16.511	358.9	6.14
1916.373	358.1	6.11

6398 Σ 1722

1915.019	339.4	2.84
15.503	336.9	2.61
16.511	337.2	2.97
1915.678	337.8	2.81

6459 Σ 1737

1915.019	219.2	15.05
16.412	220.7	15.37
16.503	220.4	15.14
1915.978	220.1	15.19

6527 Σ 1758

1915.062	303.5	3.38
16.163	303.0	3.65
16.407	304.7	3.76
16.409	302.6	3.84
1916.010	303.4	3.66

Change uncertain.

6551 *A.G.* 190

1915.062	11.9	2.45
16.407	11.1	2.40
16.409	10.7	2.51
1915.959	11.2	2.45

6558 Σ 1763

1915.019	40.5	2.53
16.106	221.3	2.72
16.503	41.5	2.58
1915.876	41.1	2.61

6573 Σ 1770

1916.409	122.4	1.90
16.497	122.2	1.89
16.503	122.6	1.94
1916.470	122.4	1.91

6696 Σ 1795

1915.062	3.5	7.84
16.341	3.1	7.77
16.407	0.3	7.61
1915.937	2.3	7.74

6763 OΣ 280

1916.341	21.8	6.91
16.497	20.9	7.32
16.503	19.4	7.12
1916.447	20.7	7.12

6797 Σ 1829

1915.062	150.4	5.36
16.341	150.8	5.50
16.407	151.3	5.28
1915.937	150.8	5.38

6963 OΣ 284

1915.062	100.1	6.65
16.341	103.0	7.08
16.497	105.4	7.20
1915.967	102.8	6.98

6988 Σ 1878

1916.341	324.8	3.82
16.407	322.8	3.70
16.409	322.2	4.01
1916.386	323.3	3.84

Slow retrograde motion.

7004 Σ 1882

1915.062	2.4	11.88
16.341	1.4	11.60
16.409	0.3	11.72
1915.937	1.4	11.73

A and *C* (11.0)

1916.409	82.8	9.08

7117 β 119

1916.264	294.5	1.46
16.346	298.8	1.57
16.492	296.6	1.52
1916.367	296.6	1.52

Slow retrograde motion.

7211 Σ 1932

1916.346	3.3	0.91
16.492	185.3	0.78
16.497	8.0	0.90
16.514	2.4	0.74
16.530	9.6	0.84
1916.475	5.7	0.83

7258 Σ 28 App. I

1916.346	172.4	
16.497	171.7	108.48
16.505	171.4	108.42
16.514	171.6	108.23
1916.465	171.7	108.38

7259 Σ 1938

1916.346	51.7	1.30
16.497	53.5	1.47
16.505	54.0	1.37
16.514	55.2	1.34
1916.465	53.6	1.37

7273 Σ 1944

1916.346	329.3	1.39
16.505	323.6	1.26
16.514	321.5	1.26
1916.454	324.8	1.30

Slow retrograde motion.

7276 OΣ 296

1916.264	301.7	1.63
16.346	303.7	1.77
16.492	303.2	1.68
16.505	303.6	1.74
1916.402	303.1	1.71

7318 Σ 1954

1916.264	184.4	4.09
16.346	183.2	3.55
16.492	185.2	3.73
1916.367	184.2	3.79

7531 Σ 2015

1916.393	158.3	2.79
16.407	158.3	2.67
16.409	158.9	2.95
1916.403	158.5	2.80

7551 Σ 2021

1915.487	157.0	3.78
16.360	336.9	3.97
16.393	336.9	3.89
1916.080	336.9	3.88

7558 Σ 2027

1915.487	256.3	1.88
16.360	76.0	2.04
16.409	75.7	1.95
1916.085	76.0	1.95

7569 A 23

1916.360	72.3	1.71
16.393	71.7	1.79
16.409	72.9	1.78
1916.387	72.3	1.76

7615 Σ 2045

1916.360	184.7	2.17
16.407	185.3	2.15
16.423	186.3	2.25
1916.397	185.4	2.19

7621 Σ 2047

1916.360	330.1	2.04
16.407	329.4	2.07
16.423	328.5	2.34
1916.397	329.3	2.15

7665 Σ 2059

1916.393	200.5	0.98
16.407	22.8	1.06
16.445	201.9	0.93
16.492	202.6	0.95
1916.434	202.7	0.98

7703 Σ 2078

A and *B*

1916.407	110.2	3.55
16.423	112.2	3.81
16.497	112.5	3.47
1916.442	111.6	3.61

A and *C*

1916.407	193.9	90.55

7719 Σ 2095

1915.473	164.2	4.93
15.483	162.3	5.14
15.717	163.2	5.20
1915.558	163.2	5.09

7795 Σ 2109

1915.473	312.8	5.93
15.534	313.6	6.13
15.717	312.0	6.02
1915.574	312.8	6.03

7900 OΣ 325

1916.393	207.4	1.25
16.445	216.4	1.25
16.530	214.6	1.16
1916.456	212.8	1.22

7911 Σ 2140

A and *B*

1915.473	114.9	4.79
15.534	113.0	4.67
15.544	112.7	4.49
15.684	114.9	4.75
15.717	114.4	4.67
1915.590	114.0	4.67

A and *D*

1915.473	38.7	83.27
15.684	38.6	83.07
15.717	38.9	83.61
1915.625	38.7	83.32

7925 Σ 2146

1916.472	226.1	2.66
16.497	224.9	2.83
16.514	225.9	2.60
1916.504	225.6	2.70

7915 β 629

1916.445	342.4	1.21
16.492	340.3	0.88
16.530	343.0	1.13
1916.489	341.9	1.07

7970 Σ 2154

1916.360	252.7	1.74
16.393	253.9	1.93
16.407	250.9	1.65
1916.387	252.5	1.77

8004 Σ 2163

1916.445	91.1	1.53
16.481	87.9	1.45
16.486	91.4	1.29
1916.471	90.1	1.16

Retrograde motion, distance apparently constant.

8005 Σ 2162

1916.445	280.3	1.26
16.492	280.9	1.32
16.530	280.2	1.44
1916.489	280.5	1.34

8017 Σ 2165

1915.473	57.7	8.16
15.544	54.7	8.11
15.717	56.3	8.39
1915.578	56.2	8.22

Distance and angle both increasing.

8023 Σ 2168

1916.360	198.4	2.12
16.407	199.8	2.37
16.423	200.7	2.56
1916.397	199.6	2.45

8056 Hu 179

1916.481	51.9	1.99
16.486	52.8	2.26
16.497	52.9	*
16.530	51.5	2.18
1916.498	52.3	2.14

Seeing very poor.

8075 Σ 2186

1916.445	79.7	2.94
16.481	78.7	2.67
16.486	80.3	2.86
1916.471	79.6	2.82

8086 β 1121

1916.445	233.6	0.68
16.492	234.1	0.70
16.530	233.0	0.63
1916.489	233.6	0.67

Change uncertain.

8153 Σ 2205

1916.360	311.9	1.93
16.407	312.8	1.84
16.423	304.8	1.83
1916.397	309.8	1.87

Angle increasing, distance decreasing.

8160 Σ 2217

A and *B*

1915.473	285.1	6.68
15.544	283.6	6.83
15.717	284.1	6.66
1915.578	284.3	6.72

A and *C* (12.0)

1915.473	304.3	25.40

8168 Σ 2222

1915.473	64.5	2.20
15.544	60.8	2.19
15.717	60.8	1.83
15.752	61.1	2.12
15.755	59.3	1.83
1915.648	61.3	2.03

8234 Σ 2243

1916.360	45.5	1.19
16.407	224.9	1.35
16.423	46.0	1.36
1916.397	45.5	1.30

8235 β 130

1915.531	122.4	1.77
15.542	122.6	1.51
15.544	124.0	1.68
1915.539	123.0	1.65

8241 Σ 2244

1915.542	279.2	0.95
15.544	99.5	1.01
15.717	281.9	0.94
1915.601	280.2	0.97

8252 β 447

1916.407	277.4	1.54
16.423	278.4	1.78
16.445	283.0	1.37
1916.425	279.6	1.56

8320 Σ 2271

1916.472	266.1	2.89
16.530	266.5	2.52
16.538	267.6	2.53
1916.513	266.7	2.65

8561 OΣ 350

1916.492	169.5	1.69
16.503	167.3	1.85
16.511	169.1	1.87
1916.502	168.6	1.80

8568 β 461

1916.481	108.5	1.17
16.492	106.1	1.04
16.511	108.2	1.22
16.524	109.3	1.20
1916.502	108.0	1.16

8782 Σ 37 App. I

1914.676	172.3	207.49
14.698	172.5	207.68
14.731	172.4	206.60
1914.702	172.4	207.26

8783 Σ 2382, ϵ_1 *Lyrae*

1914.676	9.4	2.86
14.698	4.4	2.98
14.731	8.6	3.01
1914.702	7.4	2.95

8785 Σ 2383, ϵ_2 *Lyrae*

1914.676	122.5	2.36
14.698	122.5	2.20
14.731	122.8	2.42
1914.702	122.6	2.33

8788 β 968, ζ *Lyrae*

A and *D*

1914.676	149.5	43.81
14.698	148.6	43.72
14.731	149.7	43.60
1914.702	149.3	43.71

8933 β 648

1916.524	59.6	0.94
16.530	61.2	0.89
16.538	60.4	1.07
1916.531	60.4	0.97

9056 Σ 2466

A and *B*

	°	″
1915.544	105.0	2.33
15.684	106.5	2.47
15.714	106.9	2.56
15.717	106.0	2.24
1915.665	106.1	2.40

A and *C* (9.3)

1915.684	141.7	98.93
15.714	141.4	99.49
15.717	141.6	99.14
1915.705	141.6	99.19

9061 Σ 2469

1915.544	125.0	1.37
15.684	130.0	1.34
15.714	125.8	1.23
15.717	125.4	1.20
1915.665	126.5	1.28

9144 Σ 2487

1915.684	82.4	28.40
15.717	82.0	28.00
15.722	81.9	28.31
15.730	82.3	28.33
1915.713	82.2	28.26

9222 Σ 2502

1915.714	208.0	1.40
15.717	204.9	1.57
15.722	204.1	1.60
15.730	204.3	1.51
1915.721	205.3	1.52

9307 Σ 2520

A and *B*

1915.544	234.7	1.73
15.722	232.2	1.89
15.730	233.2	1.90
15.755	232.1	2.16
15.810	233.4	1.86
1915.712	233.1	1.90

A and *C* (9.2)

1915.544	247.6	89.00
15.722	247.1	89.13
15.730	247.5	89.55
15.755	247.6	89.32
15.810	247.7	89.23
1915.712	247.6	89.25

9605 Σ 2579. *δ Cygni*

1914.676	283.9	2.36
16.516	283.3	1.87
16.524	284.9	1.57
16.530	282.1	2.04
1916.062	283.6	1.96

Retrograde motion.

9890 *β* 832

1915.525	108.5	1.52
15.531	103.1	1.68
15.542	103.3	1.39
1915.533	104.9	1.53

9926 Σ 2628

1914.696	344.5	4.13
15.484	345.5	3.75
15.503	345.9	4.06
15.550	344.7	3.83
1915.308	345.1	3.94

9956 Σ 2639

1914.696	302.0	5.93
14.706	302.9	5.81
14.709	301.5	5.62
14.731	303.8	5.67
1914.711	302.5	5.76

9982 Σ 2644

1914.709	209.5	2.98
15.484	28.7	2.92
15.503	210.3	2.98
1915.232	209.5	2.96

10022 Σ 2655

		″
1914.698	1.8	6.75
14.706	2.9	6.38
14.709	0.5	6.20
1914.704	1.7	6.44

10040 *β* 983

1915.531	156.9	0.88
15.717	157.5	0.82
15.730	161.3	0.88
1915.659	158.6	0.86

10041 OΣ 403

A and *B*

1915.531	355.1	0.79
15.542	173.5	0.76
15.580	352.7	0.79
1915.551	353.8	0.78

A and *C*

1915.531	35.2	11.49
15.542	34.2	11.41
15.580	34.0	11.74
1915.551	34.5	11.55

10044 Σ 2658

A and *B*

1915.484	118.3	5.47
15.503	118.4	5.25
15.525	117.9	5.31
1915.504	118.2	5.34

A and *C*

1915.484	209.7	47.13
15.503	209.4	46.96
15.525	209.8	47.25
1915.504	209.6	47.11

Change of *AC* due to proper-motion of *AB*.

10218 Σ 2694

	°	″
1914.720	346.4	4.08
14.756	343.3	4.05
15.484	346.6	3.96
1914.987	345.4	4.03

10423 OΣ 410

A and *B*

1915.531	17.4	0.61
15.542	13.5	0.64
15.717	13.2	0.74
1915.597	14.7	0.66

A and *C*

1915.542	69.9	68.86
15.717	69.5	68.72
1915.630	69.7	68.79

10566 *β* 67

1915.503	299.4	1.50
15.531	295.7	1.54
15.580	291.8	1.45
1915.538	295.6	1.50

10588 *β* 155

A and *B*

1915.531	31.4	0.85
15.542	30.7	0.72
15.717	26.4	0.82
1915.597	29.5	0.80

Measures are erratic.

AB and *C* (12.5)

1915.542	23.1	16.15

10698 Σ 2745

1915.503	192.8	3.31
15.525	190.9	2.78
15.544	195.0	3.01
15.580	194.0	2.68
1915.538	193.2	2.94

10773 Σ 2762

A and *B*

1915.503	310.9	3.28
15.525	311.4	3.22
15.550	310.9	3.50
15.580	311.1	3.68
1915.539	311.1	3.42

A and *C*

1915.503	226.4	58.31
15.525	226.6	58.51
15.550	226.6	57.91
15.580	226.6	57.27
1915.539	226.6	58.00

A and *D*

1915.525	67.0	74.30
15.550	67.1	73.90
1915.537	67.1	74.10

10829 OΣ 535

AB and *C*

1914.696	16.3	47.41
14.706	16.1	47.38
14.712	15.6	47.42
1914.705	16.0	47.40

10926 Σ 2789

1914.720	115.0	6.47
14.753	116.1	6.50
15.714	116.8	6.28
15.717	296.1	6.53
1915.226	116.0	6.47

10985 Σ 2797

1914.909	216.8	3.51
15.550	218.6	3.71
15.555	223.5	3.57
1915.338	219.6	3.60

10991 *BD* 79°701

1914.720	271.5	1.78
14.753	272.8	1.77
15.555	271.9	1.88
1915.009	272.1	1.81

11095 Σ 2839

1914.690	162.6	31.04
14.696	163.5	31.40
14.698	164.4	31.73
1914.695	163.5	31.39

11100 OΣ 443

1915.810	168.5	8.30
15.832	350.5	8.30
15.848	349.8	8.24
1915.830	349.6	8.28

11103 Σ 56 App. I

1915.810	348.5	39.55
15.832	348.9	39.45
15.848	348.9	39.43
1915.830	348.8	39.48

11211 Σ 2822

1914.696	130.8	2.02
14.706	127.0	1.72
14.709	132.2	1.59
14.717	133.6	2.22
1914.707	130.9	1.89

11323 Σ 2840

1914.720	196.7	19.14
14.753	194.7	18.92
15.555	194.8	19.07
1915.009	195.4	19.04

11361 OΣ 451

A and *B*

1915.810	277.8	7.26
15.832	277.2	7.19
15.848	277.7	7.20
1915.830	277.6	7.22

A and *C* 10.2

1915.810	264.5	43.16
15.848	264.8	43.06
1915.829	264.6	43.26

11116 Σ 2850

	°	″
1915.832	262.6	2.83
15.848	261.4	2.90
15.862	262.6	2.78
1915.847	262.2	2.84

11183 Σ 2863, ξ *Cephei*

1914.720	282.6	6.96
14.753	290.7	7.14
15.714	281.3	7.07
15.747	280.0	6.98
1915.226	283.6	7.04

11490 Σ 2862

1915.580	100.7	2.46
15.714	101.2	2.53
15.722	101.5	2.28
15.840	100.3	2.25
1915.707	100.9	2.38

11512 Σ 2872

A and *BC*

1914.909	316.2	21.66
15.714	316.2	21.73
15.717	315.8	21.82
1915.447	316.1	21.74

B and *C*

1915.788	136.8	0.55
15.799	135.2	0.59
15.867	136.9	0.67
1915.818	136.3	0.60

Slow retrograde motion in close pair.

11578 Σ 2880

1914.909	351.0	4.27
15.714	352.8	3.98
15.717	352.9	4.40
1915.447	352.2	4.22

11584 β 376

	°	″
1914.909	151.6	3.60
15.714	151.6	3.61
15.717	152.3	3.52
1915.447	151.8	3.58

11592 Σ 2877

1914.690	7.2	13.22
14.696	7.2	13.39
14.698	6.6	13.52
14.706	6.8	13.27
14.728	7.5	13.16
1914.703	7.1	13.31

11713 Σ 2909, ζ *Aquarii*

1914.696	307.3	2.88
14.698	305.3	3.04
14.709	306.9	3.06
14.717	311.4	2.89
14.728	308.4	2.81
1914.710	307.9	2.94

11811 Σ 2920

1914.696	141.4	13.79
14.698	140.9	13.49
14.712	142.7	13.58
14.728	142.9	13.68
1914.708	142.0	13.63

11905 ξ *Pegasi*

A and *B*

1914.709	138.5	63.53
14.717	138.5	62.82
14.728	138.9	62.76
14.750	139.0	62.96
1914.726	138.6	63.02

Distance probably decreasing.

11914 Σ 2935

1914.909	305.7	2.56
15.580	311.0	2.47
15.695	305.4	2.42
1915.395	307.4	2.48

11916 Σ 2936

	°	″
1914.696	50.5	4.92
14.712	48.1	4.79
14.715	48.2	4.75
14.717	48.5	4.29
14.758	51.0	4.76
1914.720	49.4	4.70

11939 Σ 2938

1914.696	161.9	19.41
14.698	162.4	20.09
14.712	162.0	19.80
14.717	162.7	19.96
1914.706	162.2	19.82

11997 Σ 2947

A and *B*

1915.755	64.0	3.87
15.761	64.7	3.54
15.788	63.5	3.71
15.867	64.3	3.73
1915.793	64.1	3.71

A and *C* (11.0)

1915.755	205.4	110.08
15.761	205.4	.....*
15.788	206.3	110.22
15.867	206.7	110.39
1915.793	205.9	110.21

*Clouds stopped work. Burnham in "Measures of Proper-Motion Stars" gives for *AC*:

1910.73	206.24	109.29

12061 H 975

1914.745	242.6	50.75
14.750	242.5	51.19
14.758	242.8	51.03
15.695	242.8	51.06
1914.987	242.7	51.01

12065 Σ 2958

1914.909	11.4	3.82
15.580	13.0	3.75
15.689	10.5	3.76
15.695	11.1	3.53
1915.468	11.5	3.72

12181 Σ 2984

1914.706	294.8	4.96
14.737	295.8	4.66
14.764	293.2	4.42
1914.736	294.6	4.68

12202 S 823

1914.720	162.7	167.93
14.764	163.1	167.43
15.714	162.8	167.48
1915.066	162.9	167.61

12213 Σ 2988

1915.810	279.2	3.78
15.832	99.8	3.67
15.837	279.0	3.73
15.848	99.9	3.54
1915.832	279.5	3.68

12228 Σ 2990

1915.580	242.8	1.91
15.689	242.5	2.12
15.695	243.0	1.88
15.703	242.2	1.96
1915.667	242.6	1.97

12304 Σ 3001

1914.720	201.2	3.35
14.745	203.7	2.83
14.756	200.8	2.62
14.764	202.3	2.94
1914.746	202.0	2.93

12332 Σ 3007

A and *B*

1915.695	82.6	6.20
15.703	81.7	6.02
15.709	85.5	6.23
1915.702	81.3	6.15

A and *C*

1915.695	318.7	79.57
15.703	318.6	79.97
15.709	318.1	79.67
1915.702	318.5	79.74

12341 Σ 3009

1914.696	230.1	7.07
14.705	230.5	7.24
14.712	231.2	6.97
14.715	230.9	7.28
14.717	229.4	7.00
14.728	230.3	7.26
14.734	229.2	7.19
1914.715	230.2	7.14

12345 β 854

1915.580	270.2	2.29
15.689	90.1	2.41
15.714	270.2	2.31
1915.661	270.2	2.34

13652 A 1240

1915.810	353.9	2.12
15.832	355.3	1.91
15.848	356.2	1.90
15.843	353.2	2.05
1915.833	354.6	2.00

12492 OΣ 501

1914.717	161.8	15.08
14.731	162.6	....
14.906	162.2	14.70
14.909	163.2	14.97
1914.816	162.5	14.92

12510 β 858

	°	
1915.580	262.8	0.71
15.799	254.9	0.69
15.810	254.2	0.53
1915.730	257.3	0.64

BD + 62°2282

(8.8 8.8)

R.A. 23h31m38s Decl 62 45′42″

A and *B*

1915.755	308.7	8.85
15.780	310.6	8.79
15.791	309.2	8.97
1915.775	309.5	8.87

A and *C* (8.7)

1915.755	42.1	32.69
15.780	43.0	32.93
15.791	42.5	32.87
1915.775	42.5	32.83

Magnitude of *AB* is given as 9.3 in *BD*.

A.G. Hels. Gotha 14308

(9.0, 9.1)

R.A. 23h 38m 1s Decl. 64° 17′.4

1915.755	96.6	35.19
15.761	96.6	36.37
15.780	96.6	36.43
15.791	96.8	36.22
1915.772	96.6	36.05

12536 Espin 150

1915.755	32.8	3.38
15.761	32.1	3.21
15.780	34.9	3.43
15.791	34.5	3.31
1915.772	33.6	3.33

13656 A 1243

1915.832	218.0	4.62
15.843	219.5	4.78
15.848	218.7	4.62
1915.844	218.7	4.67

12575 OΣ 508

1914.737	201.4	1.42
14.750	195.6	1.88
14.756	199.1	1.74
14.767	195.1	1.68
1914.752	197.8	1.68

12675 Σ 3050

1914.747	221.7	2.01
14.797	223.2	2.29
14.906	219.4	2.17
14.909	222.7	2.37
1914.832	221.8	2.21

13657 A 1244

1915.810	272.9	2.74
15.832	272.8	2.95
15.848	273.1	2.79
1915.830	273.0	2.83

12666 Σ 3049

1914.720	326.0	3.20
14.737	326.9	3.18
14.745	330.4	3.39
14.764	325.5	3.17
1914.744	327.2	3.23

12739 Σ 3058

1914.906	50.5	12.66
14.909	51.5	12.69
15.744	50.6	12.64
1915.176	50.9	12.66

The Dearborn Observatory, September 22, 1916.

NEW ASTRONOMICAL WORK

Studies in Stellar Statistics.

III. The Distances and the Distribution of the Stars of the Spectral Type B, by C. V. L. Charlier. Published by the Royal Society of Science of Upsala, Series IV, Vol. IV, No. 7. 1916.

Memorie ed Osservazioni. Published under the direction of E. Millosevich, by the R. Osservatorio Astronomico al Collegio Romano, Serie III, Vol. VI Part II. 1916.

La Latitudine di Pino Torinese nel 1915, dal Giovanni Boccardi. Torino, 1916

Annuario Astronomico pel 1917. Publicato dal R. Osservatorio di Pino Torinese. Torino, 1916.

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MEASURES OF ONE HUNDRED DOUBLE STARS MADE WITH THE 26-INCH REFRACTOR OF THE LEANDER McCORMICK OBSERVATORY.

By CHARLES P. OLIVIER.

The present paper is the first of a new series in which are to be published measures of double stars made by either Dr. S. A. MITCHELL or myself. It is planned to publish when one hundred stars have been satisfactorily measured by either of us.

This paper is a continuation of five former ones* by myself in which I undertook to measure all interesting objects between the equator and −35° declination, contained in "Reference Catalogue of Southern Double Stars" by INNES. There are about 1,130 stars of all kinds in that catalogue, within the limits mentioned, and measures of 377 of them are to be found in the five papers* referred to. As it was never intended to measure the wider and less interesting doubles, with the additional stars in this paper, at least half of this program is now completed.

The present list was intended to include as many as possible of the more recent discoveries. There are among these 31 AITKEN, 4 HUSSEY and 4 INNES doubles. The rest of the list is mostly made up of stars known to be in motion or those which needed remeasuring. One new double is included. In the 100 stars here published there are 15 doubles whose distances is less than 1″, 63 doubles from 1″ to 2″, 14 from 2″ to 3″, and 8 wider than 3″.

No essential changes have been made in the methods of observation from those formerly described. Most of the measures, however, have been made in the fall months, when we have our best seeing and high powers have generally been used.

The right ascensions and declinations are given for 1900. In general, each position angle has been derived from the mean of four settings of the circle, and each distance from four double distances. When the seeing was unfavorable more settings were frequently made.

It may be added that in the future double star work will probably receive a larger portion of the time of the 26-inch refractor than it has been possible to devote to it so far at this observatory, on account of the precedence of the stellar parallax work.

*A.N. Nr. 4166, 4227, 4360; *L.O.B.* No. 175 and 190.

Hu 1 β G.C. [4] Δm 1.6

R.A. 0h 1m 38s Decl. −12° 43′

	°	″	h		
14.726	105.4	1.06	+1.1	2½	850
15.706	107.5	1.09	0.0	2½	850
15.216	106.4	1.08			

No motion in 17 years.

Σ 3063 [10] 9.0 − 10.5

R.A. 0h 2m 29s Decl. −5° 6′

14.726	215.5	1.66	+0.9	4	850
14.761	214.0	1.67	..	4½	850
16.741	214.7	1.79	+0.5	3	600
15.409	214.7	1.71			

Very slow motion.

β 391 [30] Δm 0.0

R.A. 0h 4m 15s Decl. −28° 33′

	°	″	h		
14.742	268.2	..	−0.4	1½	850
15.788	268.8	1.13	+0.9	2	850
16.826	268.0	1.17	+0.6	2	850
15.785	268.3	1.15			

A 1802 9.5 − 12.5

R.A. 0h 8m 58s Decl. +11° 34′

16.741	150.3	1.73	+0.6	2½	600
16.765	149.6	1.88	+1.3	4½	850
16.753	150.0	1.70			

No motion.

Hu 3 [111]

R.A. $0^h\ 11^m\ 53^s$ Decl. $-10°\ 54'$

14.726	110.7	1.71	+1.2	2	850
14.780	114.4	1.66	+0.5	2	850
14.800	109.4	1.55	+0.9	3	850
14.769	111.5	1.64			

Motion of 6° in 15 years.

A 1804 Δm 1.6

R.A. $0^h\ 12^m\ 18^s$ Decl. $+9°\ 29'$

14.805	46.0	1.30	+0.1	4	850
15.788	43.1	1.27	+1.1	3½	850
15.296	44.7	1.28			

No motion.

A 1806 9 – 12

R.A. $0^h\ 35^m.9$ Decl. $+4°\ 44'$

16.741	12.2	2.77	−0.6	3	600
16.765	12.2	2.91	+0.4	3½	850
16.753	12.2	2.84			

No motion.

A 1807 9 – 13

R.A. $0^h\ 38^m\ 38^s$ Decl. $+4°\ 56'$

16.741	156.3	2.25	−0.2	2	600
16.765	159.4	2.06	+0.8	3½	850
16.753	157.8	2.16			

No motion.

A 2305 Δm 0.3

R.A. $0^h\ 39^m\ 2^s$ Decl. $+1°\ 56'$

14.805	1.6	1.29	0.0	1	850
15.788	2.9	1.31	+0.9	3	850
15.296	2.2	1.30			

A 2001 Δm 3

R.A. $0^h\ 58^m\ 36^s$ Decl. $+6°\ 14'$

14.805	245.0	1.31	−0.2	4	850
15.788	243.5	1.40	+0.8	3	850
15.296	244.2	1.36			

No motion.

I 443 8.5 – 11.5

R.A. $1^h\ 0^m.3$ Decl. $-20°\ 23'$

15.859	310.5	2.14	−0.7	3	850
16.683	310.0	2.27	−0.4	2	850
16.271	310.2	2.20			

Σ 113 [707] Δm 0.3

R.A. $1^h\ 14^m\ 41^s$ Decl. $-1°\ 2'$

14.726	359.6	1.45	+0.6	2	850
14.800	356.1	1.32	+0.1	2½	850
14.763	357.8	1.38			

H 2036 [711] Δm 0.2

R.A. $1^h\ 15^m\ 3^s$ Decl. $-16°\ 20'$

14.726	7.3	1.61	+0.3	3	850
14.764	8.8	1.51	−0.1	3	850
14.745	8.0	1.56			

A 2314 9 – 13

R.A. $1^h\ 18^m\ 2^s$ Decl. $+0°\ 58'$

16.683	309.2	2.82	+0.5	4	850
16.722	309.6	3.04	−0.5	2½	850
16.735	310.8	2.79	+0.6	3	850
16.713	309.9	2.88			

No motion.

I 444 Δm 0.2

R.A. $1^h\ 22^m.9$ Decl. $-25°\ 51'$

15.859	303.4	0.5±	−0.5	2½	560
16.683	304.6*	0.56	−0.2	2½	850
16.271	304.1	0.56			

A 2317 Δm 0.2

R.A. $1^h\ 23^m\ 57^s$ Decl. $+3°\ 16'$

14.805	41.3	1.09	−0.3	4	850
15.777	39.4	1.20	+1.4	..	850
15.788	39.7	1.16	+0.4	4	850
16.683	40.3	1.16	+0.1	4	850
15.763	40.2	1.15			

No motion.

I 115 B – C Δm 0.2

R.A. $1^h 25^m.2$ Decl. $-23° 2'$

			h		
15.859	355.1	2.19	−0.9	2½	850
16.683	354.4	2.17	−0.5	3	850
16.271	354.8	2.18			

A 1917

R.A. $1^h 41^m 30^s$ Decl. $-0° 23'$

14.805	26.0	1.96	−0.1	4	850
15.777	27.2	1.93	+1.1	2½	850
15.291	26.6	1.94			

No motion.

A 1918 Δm 0.3

R.A. $1^h 48^m 56^s$ Decl. $-0° 19'$

14.805	324.0	1.18	−0.2	3	850
14.956	325.8	1.45	+0.7	3	850
15.777	321.4	1.41	+1.4	2½	850
15.179	323.7	1.35			

No motion.

A 1919 Δm 0.6

R.A. $1^h 49^m 53^s$ Decl. $-0° 17'$

14.805	289.2	2.24	0.0	4	850
14.956	291.1	2.12	0.9	3	850
15.777	291.6	2.18	+1.5	3	850
15.179	290.6	2.18			

No motion.

A 2408 Δm 0.3

R.A. $1^h 50^m 25^s$ Decl. $+1° 17'$

14.805	36.0	0.99	−0.4	3½	850
15.777	36.7*	1.01	+1.5	3	850
16.683	37.2*	0.87	+0.2	3	850
15.755	36.6	0.96			

No motion.

Hastings 1 [1179]

R.A. $2^h 11^m 4^s$ Decl. $-18° 42'$

15.010	3.1	2.05	+1.1	2	850
15.788	5.2	2.08	+0.6	3	850
15.399	4.2	2.06			

Σ 248 [1212] Δm 0.1

R.A. $2^h 11^m 16$ Decl. $+42° 18'$

			h		
14.742	322.5	1.35	2.2	1	850
14.750	324.7	1.36	3.0	1	850
14.746	323.6	1.36			

β 8 [1226] Δm 1.1

R.A. $2^h 16^m 2^s$ Decl. $+8° 24'$

14.726	26.8	1.22	+0.4	2½	850
14.800	29.3	1.21	−0.6	3	850
14.763	28.0	1.22			

Hough 313 A – B [1217]

R.A. $2^h 18^m 21^s$ Decl. $-8° 16'$

15.010	78.6	1.99	+1.3	3	850
15.777	79.8	1.99	+1.7	3	850
15.788	77.8	1.98	+0.6	3½	850
15.525	78.7	1.99			

A – C

15.777	85.13	17.53	+1.8	2	850
15.788	86.04	17.79	+0.7	3½	850
15.782	85.58	17.66			

Hough 314 [1253] 9.5 – 11.5

R.A. $2^h 19^m 5^s$ Decl. $-8° 19'$

15.777	201.9	4.14	+1.9	3	850
16.741	199.6	4.27	−1.1	2	600
16.259	200.8	4.20			

A 2502 9 – 12.2

R.A. $2^h 20^m 19^s$ Decl. $-10° 56'$

16.735	278.0	2.16	+0.2	3	560
16.765	281.8	2.20	+0.7	3	850
16.842	278.8	2.23	−0.6	3	850
16.781	279.5	2.20			

Hough 315 [**1357**]

R.A. $2^h\ 33^m\ 53^s$ Decl. $-2°\ 4'$

			h		
14.726	357.1		+0.7	1½	850
15.010	358.9	1.65	+1.1	3	850
15.788	357.8	1.55	0.4	1	850
15.175	357.9	1.60			

A 2021 8 – 12

R.A. $2^h\ 40^m\ 38^s$ Decl. $+6°\ 49'$

14.805	239.0	1.32		3	850
15.777	234.6		+1.0	2	850
16.735	235.4	1.26	+0.4	3	560
15.772	236.3	1.29			

β 81 [**1610**] Δm 0.6

R.A. $3^h\ 11^m\ 4^s$ Decl. $-6°\ 17'$

15.010	22.9	0.78	+0.4	3	850
15.788	20.2	0.85	+0.5	3	850
15.399	21.6	0.82			

Hu 1058 [**12931**] Δm 0.3

R.A. $3^h\ 18^m\ 24^s$ Decl. $+39°\ 52'$

14.742	111.5	0.73	2.9	3	850
14.750	110.1	0.97	2.2	3	850
16.765	115.1	0.90		1	850
16.864	108.9	0.87	1.8	2½	850
15.780	111.4	0.87			

No motion.

A 2417 Δm 0.8

R.A. $3^h\ 23^m\ 0^s$ Decl. $+3°\ 49'$

16.735	87.7	0.92	−0.3	3	850
16.765	92.0	0.87	+0.5	1	850
16.750	89.8	0.90			

No motion.

A 1931 Δm 0.7

R.A. $3^h\ 26^m\ 25^s$ Decl. $+7°\ 29'$

14.805	65.2	0.72	0.9	3	850
16.735	63.0	0.82	0.0	3	600
15.770	64.1	0.77			

No motion.

A 1933 Δm 1

R.A. $3^h\ 30^m\ 12^s$ Decl. $+6°\ 5'$

	°		h		
14.805	135.8	1.31	−0.8	3	850
16.683	136.3	1.26	−0.6	2	850
15.744	136.0	1.28			

β 534 [**1807**] 7 – 11

R.A. $3^h\ 33^m\ 59^s$ Decl. $-8°\ 50'$

15.922	194.4	2.31	+0.4	2	850
16.735	191.9	2.21	+0.3	2	600
16.328	191.5	2.26			

Leavenworth 3 [**1851**] Δm 1.4

R.A. $3^h\ 39^m\ 28^s$ Decl. $-13°\ 43'$

15.010	358.4			1	850
16.735	357.2	1.09	+0.3	2	600
16.765	359.8	1.15	+0.3	1	850
16.170	358.5	1.12			

β 569 [**1901**] Δm 3

R.A. $3^h\ 44^m\ 42^s$ Decl. $-4°\ 49'$

15.010	269.4	2.62	+1.2	2	850
15.788	271.2	2.61	+0.4	1	850
15.399	270.3	2.62			

A 992 [**12963**] 8.5 – 11.2

R.A. $3^h\ 46^m\ 39^s$ Decl. $+46°\ 9'$

16.826	193.7	2.92	−1.6	4	850
16.842	196.0	3.06	−2.8	3	850
16.834	194.8	2.99			

A 996 [**12970**] 8.5 – 12

R.A. $3^h\ 55^m\ 33^s$ Decl. $+46°\ 43'$

16.826	282.9	1.32	−1.4	2½	850
16.842	279.8	1.21	−2.5	3	850
16.834	281.4	1.26			

A 1836 Δm 0.6

R.A. 4h 16m 20s Decl. +5° 9′

	°	″	h		
14.802	192.1	1.24	+1.0	3	850
14.868	190.1	1.29	+0.4	1	850
14.835	191.1	1.26			

Change of 6° in 6 years.

Σ 536 [2157]

R.A. 4h 17m 13s Decl. −4° 55′

15.010	166.9	1.72	+0.7	2½	850
15.922	166.1	1.64	...	3	850
15.466	166.5	1.68			

New? 9.5−11

R.A. 4h 20m Decl. −1° 37′

16.820	73.7	7.96	+0.7	3	850
16.831	73.8	7.97	+0.3	4	850
16.826	73.8	7.96			

Very near Σ 547.

A 1842 Δm 0.4

R.A. 4h 32m 3s Decl. +5° 16′

14.802	305.0	1.59	+1.5	3	850
14.868	305.0*	1.52	+0.5	3	850
14.835	305.0	1.56			

Olivier 3 8.7 −9.2

R.A. 4h 42m 14s Decl. −22° 56′

16.735	60.1	1.55	−0.4	2½	600
16.765	60.4	1.45	−0.3	4	850
16.750	60.2	1.50			

Olivier 2 9.2 −11.8

R.A. 4h 42m 34s Decl. −21° 37′

16.735	223.3	3.88	−0.1	3	600
16.765	223.4	4.14	−0.2	4	850
16.820	225.1	3.94	−0.5	2	850
16.773	223.9	3.99			

β 184 [2222] Δm 0.2

R.A. 4h 23m 37s Decl. −21° 44′

	°	″	h		
15.974	257.3	1.41	+0.4	3	850
16.735	256.3	1.38	−0.4	2½	600
16.354	256.8	1.40			

Peters [2706]

R.A. 5h 18m 46s Decl. −0° 58′

14.876	162.3	2.04	−0.3	3	560
15.010	164.5	2.15	+0.6	1½	850
14.943	163.4	2.10			

O. Stone 10 [2707] Δm 0.0

R.A. 5h 18m 51s Decl. −10° 31′

14.879	119.5	1.20	+0.1	2½	560
15.010	119.6	...	+0.5	1½	850
15.777	118.8	1.28	−0.4	1½	850
15.974	118.9	1.26	−0.1	2	850
15.410	119.2	1.25			

Olivier 4 9.2 −9.4

R.A. 5h 35m 52s Decl. −21° 39′

14.879	254.0*	2±	+0.2	1½	560
15.922	251.2	1.85	+0.6	3	560
16.735	252.1	1.90	−0.5	4	600
15.845	252.4	1.88			

No motion in 9 years.

Sirius [3596] −1.4 and 9.3

R.A. 6h 40m 44s Decl. −16° 35′

14.868	77.4	10.68	−0.5	3½	850
14.956	78.5	10.28	+0.1	4	850
14.912	78.0	10.48			
15.911	75.2	10.62	+0.3	4	560
15.922	75.0	10.30	...	2½	850
15.916	75.1	10.46			

[illegible] [3598] [illegible] and 9.3 [illegible]

R.A. [illegible] 18^m 41 Decl. −16° 35′

			h		
16.735	75.1	10.58	−1.4	1½	600
16.765	74.0	10.80	2.	[illegible]	850
16.829	74.1	10.19	−0.5	3	850
16.834	74.1	10.04	−0.3	3½	850
16.788	74.1	10.63			

β 898a Δm 0.2

R.A. 6^h 45^m 49 Decl. −15° 57′

14.956	268.1	1.99	+0.9	2½	850
16.820	266.4	1.93	0.0	2½	850
16.834	266.2	1.96	−0.8	3	850
16.202	266.9	1.96			

New? 9.7 — 9.8

R.A. 6^h 59^m Decl. −18° 10′

14.956	77.6	5.77	+0.4	2	850
16.820	78.4	5.93	−0.1	2½	850
15.888	78.0	5.85			

Σ 1593 [6913] Δm 0.2

R.A. 11^h 58^m 25 Decl. −[illegible] 53′

15.210	16.9	1.54	−0.2	3	560
15.287	17.6	1.58	+0.1	3	560
15.248	17.2	1.56			

[illegible] [6243] Δm 0.0

R.A. 12^h 36^m 36^s Decl. −0° 54′

15.040	323.5	6.17	−0.6	3	850
15.287	322.6	6.18	−0.1	2	560
15.355	324.7		+1.4	1	850
15.227	323.6	6.18			
16.355	323.8	5.88	+0.3	2½	560

β 225 [6857] Δm 0.9

R.A. 14^h 19^m 52 Decl. −19° 31′

15.317	276.0	1.48	−0.1	2	600
16.475	275.4	1.32	+1.3	3	560
15.896	275.7	1.40			

β 346 [7006] Δm 0.3

R.A. 14^h 42^m 57^s Decl. −16° 55′

	°	″	h		
15.317	253.8	1.62	0.0	2½	600
16.475	253.7	1.67	+1.4	4	560
15.896	253.8	1.64			

β 106 [7012]

R.A. 14^h 43^m 50^s Decl. −13° 44′

15.317	346.3	1.70	0.0	2½	600
16.475	347.6	1.64	+1.4	3	560
15.896	347.0	1.67			

Σ 2114 [7837] Δm 0.8

R.A. 16^h 57^m 11 Decl. +8° 34′

12.533	164.5	1.36		3	850
12.549	164.6	1.50	+2.9	2	850
12.691	165.2	1.45	+2.5	4	560
12.591	164.8	1.44			

Σ 2283 [8381] Δm 0.2

R.A. 18^h 4^m 41^s Decl. +6° 8′

14.501	82.6	1.08	+0.5	3	850
14.550	84.6	1.08	+0.8	2½	850
14.526	83.6	1.08			

OΣ 358 [8663] Δm 0.1

R.A. 18^h 21^m 25^s Decl. +16° 54′

14.501	191.2	2.04	+1.1	2½	850
14.550	189.0	1.91	+0.8	3	850
14.526	190.1	1.98			

A 1173 [13147] Δm 2.5

R.A. 19^h 4^m 56^s Decl. +8° 23′

14.550	6.8	1.91	+0.5	3	850
14.805	6.8	1.84	+2.7	3	850
14.678	6.8	1.88			

Motion of 6°.

A 1184 [**13171**]

R.A. $19^h 26^m 14^s$ Decl. $+8° 14'$

	°	″	h		
14.550	101.6	1.55	+0.7	3	850
14.805	104.2	1.49	+2.9	3	850
14.678	102.9	1.52			

No motion.

Σ 2544 [**9450**]

R.A. $19^h 32^m 17^s$ Decl. $+8° 6'$

12.557	204.0	1.19	+0.0	3	850
12.691	201.1	1.39	+0.3	4	560
14.805	202.1	1.16	+2.9	3	850
13.351	202.4	1.25			

A 1189 [**13189**] 8.9–12

R.A. $19^h 35^m 2^s$ Decl. $+8° 13'$

14.550	70.4	1.44	+0.6	2½	850
16.678	68.2	1.42	+1.5	3	850
15.614	69.3	1.43			

Σ 2597 [**9719**] Δm 1.5

R.A. $19^h 49^m 57^s$ Decl. $-7° 0'$

12.557	90.4	1.42	+0.6	3	850
12.691	88.3	1.45	+0.4	3½	850
12.624	89.4	1.44			

A 1193 [**13198**] Δm 3

R.A. $19^h 50^m 18^s$ Decl. $+11° 22'$

14.550	30.8	1.63	+0.6	3	850
16.678	28.2	1.98	+1.0	3	850
16.719	30.6	1.66	+0.1	3	850
15.982	29.9	1.76			

No motion

A 722 [**13513**] Δm 0.1

R.A. $20^h 8^m 4^s$ Decl. $+11° 52'$

14.800	343.7*	2.34	+1.6	4	850
15.689	346.9*	2.63	+1.0	2	850
15.700	344.7*	2.27	+0.6	3½	850
15.396	345.1	2.41			

A 1209 [**13528**]

R.A. $20^h 19^m 42^s$ Decl. $+11° 53'$

	°	″	h		
14.750	323.3	1.81	+0.1	4	850
14.805	323.0	1.81	+2.6	3	850
14.778	323.2	1.81			

No motion.

β 63 [**10266**] Δm 2

R.A. $20^h 25^m 34^s$ Decl. $+10° 34'$

14.750	347.8	0.87	+0.1	3	850
14.805	340.8	0.95	+2.3	3	850
15.689	348.9	1.20	+0.5	2	850
16.678	348.7	0.88	+1.1	3	850
15.480	346.4	0.98			

Σ 2701 [**10362**]

R.A. $20^h 32^m 12^s$ Decl. $+11° 42'$

14.800	219.2	1.95	+2.0	3	850
15.689	219.2	1.95	0.0	3	850
15.244	219.2	1.95			

Σ 2723 [**10183**] Δm 2

R.A. $20^h 40^m 8^s$ Decl. $+11° 57'$

14.750	103.3	1.37	+0.4	3½	850
15.689	106.1	1.37	+0.1	3	850
15.700	102.3	1.42	+0.3		850
15.380	103.9	1.39			

β 65 [**10520**] Δm 3

R.A. $20^h 42^m 52^s$ Decl. $+5° 28'$

14.750	191.6	1.44	+0.7	2	850
14.800	191.9	1.43	+2.1	2½	850
14.775	191.8	1.44			

4 *Aquarii* [**10559**] Δm 1.5

R.A. $20^h 46^m 8^s$ Decl. $-6° 0'$

16.722	329.0	0.48	+0.1	4	850
16.765	330.1	0.55	+0.9	3	850
16.744	329.6	0.52			

β 368 [10731] Δm 0.3

R.A. $21^h\ 2^m\ 5^s$ Decl. $-8°\ 38'$

12.691	87.5	0.61	+0.6	4	850
15.730	89.7	0.61	+1.8	3	850
14.210	88.6	0.61			

Σ 2799 [11001] Δm 0.1

R.A. $21^h\ 21^m\ 0^s$ Decl. $+10°\ 38'$

14.750	112.2	1.58	+0.5	3	850
14.570	112.8	1.80	+0.2	2½	560
14.550	117.0	1.75		2	560
14.623	114.0	1.71			

β 72 [11006] 8.1−9.7

R.A. $21^h\ 24^m\ 47^s$ Decl. $-5°\ 50'$

14.750	39.5	1.50	+1.0	2½	850
15.700	38.5	1.65	0.0	3½	850
15.730	38.5	1.76	+1.5	3	850
15.393	38.9	1.64			

β 684 [11007] 9.0−9.3

R.A. $21^h\ 24^m\ 57^s$ Decl. $-5°\ 55'$

14.805	125.1	1.29	+2.1	3	850
15.700	121.3	1.23		3	850
15.252	121.7	1.26			

β 1212 [11125] Δm 0.3

R.A. $21^h\ 34^m\ 22^s$ Decl. $-0°\ 30'$

15.788	292.1	0.61	−0.4	3½	850
16.722	297.7	0.49	+0.3	3	850
16.765	299.6	0.49	−0.1	3	850
16.125	296.5	0.53			

Leavenworth 10 [11190] Δm 1.0

R.A. $21^h\ 38^m\ 11^s$ Decl. $-11°\ 36'$

16.697	282.3	1.15	−1.1	2	850
16.722	283.1	1.19	−0.1	3½	850
16.710	282.8	1.17			

Howe 57 [11188] Δm 2.2

R.A. $21^h\ 38^m\ 13^s$ Decl. $-26°\ 58'$

15.706	296.0	1.77	+0.2	3	850
15.788	300.3	1.62	+0.2	2	850
16.697	298.5	1.61	−1.0	2	850
16.730	298.3	1.68			

Σ 2825 [11246] Δm 0.2

R.A. $21^h\ 41^m\ 47^s$ Decl. $+0°\ 23'$

14.750	120.5	1.10	+0.5	3	850
15.689	116.7	1.13	−0.2	2½	850
15.700	119.1	1.13	+0.3	3	850
15.380	118.8	1.12			

β 75 [11346] Δm 0.3

R.A. $21^h\ 50^m\ 39^s$ Decl. $+10°\ 24'$

14.750	44.6	1.17	+0.2	3	850
14.570	44.3	1.42	0.0	2	560
14.742	42.8	1.17	−0.4	3	850
14.687	43.9	1.25			

Hu 978 [13613] Δm 0.3

R.A. $22^h\ 7^m\ 53^s$ Decl. $+13°\ 25'$

14.570	222.5*	1.13	0.0	3	560
14.761	216.4	0.87	+0.6	4	850
14.780	219.5	1.02	+0.7	3	850
14.704	219.5	1.01			

Probably in motion.

β 172 [11691] Δm 0.2

R.A. $22^h\ 18^m\ 54^s$ Decl. $-5°\ 21'$

14.761	358.6	0.71	−0.5	4	850
15.730	355.6	0.83	+0.5	3	850
15.744	356.6	0.71	+0.2	3	850
15.412	356.9	0.75			

h 680 Δm 1.5

R.A. 22h 35m.9 Decl. −23° 4′

	°	″	h		
14.742	60.7	1.16	+0.3	2	855
16.722	60.4	1.19	−0.3	.	850
15.732	60.6	1.18			

Σ 2933 *rej* (?) 9 − 11.5

R.A. 22h 36m Decl. +10° 32′

15.700	216.7	3.68	+0.5	3	850
15.730	219.7	3.62	−0.2	3	850
15.715	218.2	3.65			

β 711 [**11913**] Δm 1.5

R.A. 22h 40m 29s Decl. +10° 40′

14.780	28.2	1.32	−0.2	3	850
15.744	29.8	1.45	+0.2	3	850
15.262	29.0	1.38			

52 *Pegasi* [**12091**] Δm 0.7

R.A. 22h 54m 12 Decl. +11° 12′

14.550	227.6	1.12	+0.5	2½	560
14.761	229.0	0.99	−0.3	3½	850
14.805	231.9	0.98	+1.1	4	850
15.730	231.0	1.13	+0.4	3½	850
14.962	229.8	1.06			

Hu 987 [**13630**] Δm 0.3

R.A. 22h 50m 46s Decl. +15° 15′

14.761	235.5	0.56	−0.5	4	850
14.805	233.2	0.62	. .	3	560
14.783	234.4	0.59			

Motion of 11° in 10 years.

A 2296 9.6 − 10.6

R.A. 22h 53m 35s Decl. +3° 28′

16.683	48.5	0.63	+1.1	4	850
16.826	51.4	0.64	+0.4	3	850
16.754	50.0	0.64			

Hu 995 [**13612**] Δm 0.5

R.A. 23h 3m 44s Decl. +15° 0′

	°	″	h		
14.800	188.3	1.42	+0.6	3½	850
15.730	188.6	1.29	+1.2	4½	850
15.265	188.4	1.36			

Olivier 51 D.M. −4°.5829 9.0 − 10.6

R.A. 23h 4m 1s Decl. −4° 36′

14.780	346.6	3.17	+0.4	2½	850
14.800	346.4	3.05	+0.9	3½	850
14.790	346.5	3.11			

A 311 [**12187**] Δm 2

R.A. 23h 4m 28s Decl. −4° 31′

14.780	128.2	1.56	+0.8	3	850
14.800	127.8	1.55	+1.4	3	850
14.790	128.0	1.56			

A 1900 Δm 0.7

R.A. 23h 9m 23s Decl. +7° 7′

16.722	212.2	0.94	+0.1	3	850
16.765	214.8	0.99	+1.0	4	850
16.744	213.5	0.96			

No motion.

β 79 [**12276**] Δm 1.5

R.A. 23h 12m 26s Decl. −2° 3′

14.570	72.7	1.12	0.0	2	850
14.761	70.4	1.07	+0.3	3	850
14.764	65.7	1.14	+0.3	4	850
14.698	69.5	1.11			
16.678	60.3	1.08	−0.6	3	850
16.683	65.3	1.26	.	3	850
16.680	62.8	1.17			

β 80 [**12290**] Δm 0.6

R.A. 23h 13m 46s Decl. +4° 52′

14.761	234.9	0.54	−0.1	3	850
14.764	236.6	0.63	0.0	3	850
14.805	234.8	0.60	+1.0	3½	850
14.777	235.4	0.59			

β 719 **[12346]**

R.A. 23h 19m 22s Decl. −13° 56′

14.761	354.2	1.23	+0.8	3	8.50
14.764	356.1	1.15	+1.5	3	8.50
14.800	357.2	1.32	+1.3	3	8.50
14.775	355.8	1.23			

β 81 **[12443]** Δm 1.9

R.A. 23h 30m 3s Decl. −12° 7′

14.780	10.0	1.52	+0.6	2	8.50
15.730	8.8	1.80	+0.4	4	8.50
16.722	10.2	1.68	+0.3	2½	8.50
16.771	8.5	1.76	+0.6	3	8.50
16.001	9.4	1.69			

Leander McCormick Observatory,
University of Virginia, November 11, 1916.

Muller 2 **[12461]** 9.4 − 10.0

R.A. 23h 32m 11s Decl. −12° 1′

16.722	301.3	2.91	+0.2	3½	8.50
16.765	302.2	2.96	+1.0	4	8.50
16.826	300.4	2.81	+0.7	4	8.50
16.771	301.3	2.89			

A 423 **[12508]** Δm 0.9

R.A. 23h 36m 11s Decl. −9° 10′

14.780	172.0	1.73	+0.6	2	8.50
15.730	172.4	1.72	+0.4	4	8.50
15.255	172.2	1.72			

Former measures are:

1902.65	166.5	1.76	A	3 nights
7.728	170.6	1.90	W	2 nights
7.728	170.7	1.77	OL	2 nights

These seem to indicate slow angular motion.

METCALF COMET.

A comet was discovered by the Rev. JOEL H. METCALF on November 21, having the following approximate photographic positions:

G. M. T.	R. A.	Dec.
Nov. 21.5673	3h 38m 0s.5	+18° 32′.8
Nov. 22.5396	3 37 49 .3	17 58 .4

The comet is faint.

Additional observations follow:

G. M. T.	R. A.	Decl.	Observer
Nov. 26.545	3h 34m 21s.3	+15° 32′ 6″	METCALF
Nov. 28.1172	3 27 40	16 57	Cable from Stromgren.

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PRELIMINARY PARALLAX OF BARNARD'S STAR OF LARGE PROPER MOTION.

By HENRY NORRIS RUSSELL.

Professor Barnard's observations of the star of large proper-motion which he recently discovered were very kindly communicated by him to the writer, in advance of their publication, for the purpose of a preliminary investigation of the parallax. Although the observations extend over only five months, they give such definite evidence of the existence of a very large parallax, and of its approximate amount, that it seems desirable to publish the existing material, although next year's observations will of course yield a determination of the parallax which will quite supersede that presented here.

The stars a and k lie nearly opposite one another, in directions differing only about 20° from that of the major axis of the parallactic ellipse. Upon plotting the measured distances of the star P from these stars against the time, the lines obtained are conspicuously curved, the distance Pa passing through a maximum, and Pk through a minimum, about the middle of the series of observations; and the curvature is in the direction which would arise from a large parallax. A solution was therefore made from the differences of the distances Pa and Pk, in which the position, proper-motion and parallax of P were determined as in the case of heliometer observations, (except that the sum of the distances was *not* used to derive corrections for the "instantaneous scale value" which, though necessary with the heliometer, would probably be illusory in the case of micrometric observations).

The equations of condition are given here, to show how conspicuous the influence of the parallax is. x denotes the correction to the assumed difference of the distances, (72″.61 at the mean epoch, 1916.670); v the correction to the assumed rate of change in this difference in *one-tenth* of a year, (+0″.592), and π the parallax. The dates of observation may be recovered from the coefficients of v.

					O − C
x	$-2.17\,v$	$+0.45\,\pi$	=	+1″.21	−0″.07
	−2.09	+0.35		+1 .29	+ .08
	−1.78	+0.00		+1 .10	+ .14
	−1.51	−0.30		+0 .72	− .03
	−0.94	−0.94		−0 .03	− .30
	−0.74	−1.11		+0 .03	− .12
	−0.55	−1.28		+0 .23	+ .21
	−0.44	−1.38		−0 .16	− .10
	−0.36	−1.44		+0 .05	+ .15
	−0.25	−1.52		−0 .08	+ .08
	−0.17	−1.58		−0 .56	− .35
	−0.06	−1.64		−0 .02	+ .23
	+0.02	−1.68		−0 .28	.00
	+0.13	−1.74		−0 .27	+ .05
	+0.21	−1.76		−0 .32	+ .03
	+0.46	−1.83		−0 .34	+ .06
	+0.57	−1.85		−0 .60	− .17
	+0.82	−1.88		−0 .39	+ .06
	+0.87	−1.87		−0 .41	+ .04
	+1.01	−1.85		−0 .37	+ .08
	+1.06	−1.84		−0 .41	− .03
	+1.20	−1.80		−0 .54	− .12
	+1.36	−1.74		−0 .32	+ .07
	+1.64	−1.60		−0 .38	− .08
	+1.69	−1.58		−0 .26	+ .03

The resulting normal equations are

$$
\begin{array}{rrrcr}
25.00\,x & -\ 0.02\,v & -33.41\,\pi & = & -\ 1''.11 \\
-\ 0.02 & +29.66 & -16.32 & = & -12\ .59 \\
-33.41 & -16.32 & +56.55 & = & +10\ .41
\end{array}
$$

whence

* The distance of Pk on this date is interpolated between the preceding and following observations, neither of which is otherwise used.

[illegible] = +0''.87	±0''.080	weight	1.5
= 0 .05	±0 .037		7.2
[illegible] = +0 .69	±0 .058		2.9

Probable error of one observation ±0''.098

The measures of the distance $P\,c$, and of the various position angles, will not permit of a determination of the parallax and proper-motion, independently of one another, till next year; but the parallax may be determined from them if the proper-motion can be taken from other sources. For this purpose, measures were made with the Princeton measuring machine upon positives of Professor BARNARD's plates of 1894 and 1901 sent by him, and upon the Franklin-Adams chart (sheet 110, taken in 1910), of the star P and the six others called by BARNARD a, b, c, g, k, and n. Taking the positions of these stars derived from BARNARD's measures of 1916 as a standard, the relative positions of P at the different dates were found as follows:

Date	$\Delta\alpha \cos\delta$	O−C	$\Delta\delta$	O−C	Prob. Error
1894.65	+16''.0	−0''.8	−228''.1	−0''.6	±1''.7
1901.45	+ 9 .1	−0 .2	−120 .5	−0 .3	±1 .2
1910.45	+ 6 .0	+1 .3	− 62 .8	+1 .4	±1 .3
1916.64	0 .0	0 .0	0 .0	0 .0	±0 .05

The last column gives the probable error of the deduced positions, as derived from the six reference stars. From these data, satisfying the position of 1916 exactly, and giving that of 1894 half the weight of the others, the proper-motion is found to be $-0^s.051 \pm 0^s.004$ in R. A. and $+10''.35 \pm 0''.06$ in declination, or $10''.38 \pm 0''.06$ toward $355°.8 \pm 0°.35$. The representation of the individual observations is highly satisfactory. It may be noted in passing that on the plate of 1901 the image of star c is completedly concealed by that of P, which is much brighter, and was at the time 2'' north and 4'' east of it.

Upon reducing the various measures of angle and distance to a common epoch with this proper-motion, the displacement due to parallax was conspicuous in every case. Solutions, without the unknown π, were made for the three sets of distance measures separately, but for the position angles in pairs, using the differences of the measured angles of two of the reference stars, to eliminate possible displacements of the zero of position from night to night. The resulting values of the parallax were

From the distances:	$P\,a$	0''.85 ±0''.04	from 27	nights
	$P\,b$	0 .69 ±0 .05	" 25	"
	$P\,c$	0 .53 ±0 .07	" 27	"
From the angles:	$k-a$	0 .66 ±0 .09	from 23	nights
	$c-a$	1 .11 ±0 .08	" 20	"
	$c-k$	0 .54 ±0 .13	" 21	"

The measures of angle on one night, (1916.653) were rejected for large and unexplained discordance. The probable errors attached to the various determinations are those resulting from the solutions, but represent only the effects of errors which are not systematic for the case considered. The considerable discordances between the individual determinations show that other sources of error are operative; but it is of no use to speculate upon these now, when next year's observations will decide the matter.

The results from the distances are much more trustworthy than those from the angles. Giving the former three times the weight of the latter, the general mean is 0''.70 ±0''.05, — the probable error being derived from the differences between the separate determinations and the mean, and including the effect of all discordances.

Combining this result with that previously derived, the parallax of Professor BARNARD's star may be taken provisionally as

0''.70 ±0''.06

which makes it nearer than any other known star except *α Centauri*. SCHLESINGER's preliminary value of half a second (*Harvard Bulletin* No. 616) though smaller than the value derived above, also puts the star second in the list.

With the parallax 0''.70, and JOY's photo-visual magnitude 9.61, the absolute magnitude of this star comes out 13.9 on KAPTEYN's scale, — that is, its brightness is 1/4000 that of the *Sun*, — which is fainter than any star previously known. The faint companions of GROOMBRIDGE 34 and KRUEGER 60 are however not more than two or three times as bright. Since the star is more than two magnitudes fainter than the faintest objects used by ADAMS in drawing the curves defining the relationship between absolute magnitude and spectral peculiarities, it is not surprising that the extrapolation of these curves led to too small a parallax; but this is no reason for distrusting the parallaxes obtained by the spectroscopic method in the region where data for drawing the curves are available.

With this parallax, the velocity of the star at right angles to the line of sight is 70 km./sec. Taking the radial velocity as −110 km. (the mean of the Mt. Wilson and Lick determinations) the velocity of the star in space, relative to the *Sun*, comes out 130 km. toward $6^h\,03^m$, +28°. After correction for the solar

motion, this becomes 118 km. towards $6^h\,05^m$, $+33°$. The motion makes an angle of 33° with the present direction of the *Sun* from the star, and the star will continue to approach us for nine thousand years, at which time it will be in Draco, not far from the pole of the ecliptic, will be of magnitude 8.3, and have a parallax of 1″.3, and a proper-motion of 36″ per year. These numbers may be somewhat changed by further investigation, but suffice to show that the star will be an even more remarkable object in the future than it is now.

At the present time the star is diminishing its distance from us by 1/1280 of the whole amount per century. It follows that its annual proper-motion must be increasing at the rate of 0″.16 per century. This change is more rapid than in the case of any other star, and should be taken into account in carrying the motion back to search for older observations.

Princeton University Observatory, 1916, November 23.

ON THE PROPER MOTION AND THE PARALLAX OF THE RAPIDLY MOVING STAR IN *OPHIUCHUS*,

By FRANK SCHLESINGER.

News of his discovery of this interesting object was kindly communicated by Professor BARNARD early in June, and observations of it were at once begun with the Thaw Refractor. Eleven plates, each with two or three sets of images, have thus far been secured; they are described in Table I. The first nine of them were exposed when the star was close to the meridian, as the third column shows; but the last two were secured, just after sunset, when the object was three hours west. The star was referred to four well distributed comparison stars and the measures in both right ascension and declination were reduced by the dependence method, with the results shown in the sixth and the seventh columns. The scale of these plates is 11″.65 to the millimeter.

In addition to these plates, Professor PICKERING very kindly placed at my disposal a number of plates secured at Harvard Observatory many years ago, all of them showing this star. Of these I selected the four on which it appears to best advantage. These were measured with reference to the same comparison stars, with the results shown in the third and the fourth columns of Table II. The scale of these plates is 163″.8 to the millimeter.

A comparison of the two plates secured at Allegheny in July with the four Harvard plates gives the following

TABLE I. PLATES SECURED WITH THE THAW REFRACTOR

Plate Number	Date	Hour Angle	Parallax Factors in α	Parallax Factors in δ	Solution of Measures in α	Solution of Measures in δ	Wt	Residuals in α	Residuals in δ
	1916	min			mm	mm			
5631	June 12	+ 5	+.11	+.46	+.490	−.063	0.6	−.026	−.060
5652	13	+ 5	+.09	+.46	+.491	−.058	0.7	−.003	−.012
5659	16	+ 32	+.05	+.47	+.491	−.052	0.7	+.025	−.013
6043	July 12	− 4	−.39	+.38	+.472	−.005	0.6	+.025	−.015
6053	14	− 7	−.40	+.37	+.471	+.002	0.6	+.010	+.039
6326	Aug. 9	+ 11	−.77	+.22	+.456	+.049	0.8	+.029	+.050
6369	12	+ 2	−.80	+.19	+.452	+.048	0.8	−.010	−.039
6416	15	+ 7	−.83	+.17	+.448	+.057	0.9	−.041	+.029
6513	21	− 5	−.87	+.13	+.450	+.072	0.4	+.022	+.096
7713	Nov. 5	+177	−.71	−.32	+.442	+.194	0.4	+.006	−.061
7732	6	+192	−.70	−.33	+.441	+.200	0.5	−.012	−.007

Table II. The Harvard Observatory Plates

Plate Number	Date	Solution of Measures in α	Solution of Measures in δ	Residuals in α	Residuals in δ
1720	1890 Aug. 28	$+.158^{mm}$	-1.627^{mm}	$-0''.3$	$-1''.3$
1063	1891 Aug. 20	$+.152$	-1.555	$-0\ .6$	$+0\ .5$
6607	1892 July 18	$+.150$	-1.501	$-0\ .2$	$+0\ .0$
8715	1893 July 11	$+.153$	-1.434	$+1\ .1$	$+0\ .8$

determination of the annual proper-motion:

In Right Ascension, $-$**0''.71**; In declination, $+$**10''.25**

In view of the good accordance of the Harvard plates (as shown in the last columns of Table 2), and in view of the long interval involved, these proper-motions should be correct within a few hundredths of a second of arc.

The observations were next examined for the presence of parallax. There are several ways in which the material at hand can be treated, and these methods naturally lead to somewhat different values of the parallax, but they all agree in indicating that this faint star is, next to α *Centauri*, the *Sun's* nearest known neighbor in space. I first carried out a least-squares solution of the Allegheny plates in right ascension in the usual way, with both the proper-motion (μ) and the parallax (π) as unknowns in addition to a "zero" constant. The normal equations are:

$$\begin{aligned} +7.00\,c \quad +1.22\,\mu \quad -3.29\,\pi &= +1''.52 \\ +1.61 \quad -1.46 &= -0\ .48 \\ +2.57 &= -0\ .03 \end{aligned}$$

Their solution yields

$$\begin{aligned} \text{Parallax} &= +0''.47 \pm 0''.02 \\ \text{Proper-motion in 100 days} &= -0\ .23 \\ \text{Annual proper-motion} &= -0\ .85 \pm 0\ .04 \end{aligned}$$

Owing to the fact that the observations cover a short interval of time, the parallax and the proper-motion are not well separated in this solution. A better value of the parallax can be deduced by assuming the proper-motion to be as we have just found it from the Harvard plates. After eliminating c from the above normal equations the reduced normal for the parallax is equivalent to this:

Parallax $= +0''.68 \ +0.24 \times$ the annual proper-motion in right ascension.

If we assume the latter to be $-0''.74$ we have for the parallax

$+0''.50$

This is the most reliable of the several results that can be deduced from this material. A wholly independent determination can be gotten from the declination measures. In this case the normal equations are indeterminate if we attempt to solve for both the parallax and the proper-motion. Eliminating the constant the reduced normal for the parallax is equivalent to

Parallax $= +0''.43 \ +0.49 \times$ the amount by which the true proper-motion in declination differs from $10''.25$.

Assuming the latter to be zero we have $+0''.43$ for the parallax.

Still other determinations of the parallax can be deduced by using the proper-motions derived by Professor Barnard in *Astronomical Journal* 695. This proper-motion in right ascension is $0''.20$ numerically smaller than mine, and would therefore lead to a somewhat larger parallax, namely $+0''.55$. The two proper-motions in declination agree very well so that the use of Professor Barnard's value would not alter sensibly the parallax $0''.43$ derived from the declinations. Considering all things I believe that $0''.50$ is the best value that can be derived from these observations, with a probable error not in excess of $0''.03$.

I am indebted to Mr. Hudson and to Miss Denton for making the measures upon which these computations are based.

Allegheny Observatory of the University of Pittsburgh,
November 15, 1916.

OBSERVATIONS OF COMETS,

MADE WITH THE $10\frac{1}{2}$-INCH EQUATORIAL AT THE UNIVERSITY OF MINNESOTA

Date Minn. M. T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ α	log pΔ δ	Red. to App. Pl. α	Red. to App. Pl. δ
Comet 1914 ε (Campbell)										
1914 Oct. 20 9 18 59 (h m s)	1	5 7	−1 8.07 (m s)	−9 22.9 (″)	21 49 50.55 (h m s)	− 1 2 41.2	9.173	0.799	+3.18	+18.2*
Comet 1913 f (Delavan)										
Oct. 26 6 48 30	2	8, 8	−0 28.87	+4 35.1	14 12 17.89	+26 25 1.3	9.654	0.792	+1.38	−14.8*
30 6 7 36	3	8, 6	−2 29.90	+4 56.0	14 26 47.55	+23 23 32.3	9.653	0.775	+1.39	−14.6*
Nov. 3 6 3 0	4	5, 6	+0 24.99	−2 2.4	14 40 5.87	+20 26 6.3	9.616	0.772	+1.43	−14.7*
4 6 19 26	5	8, 7	−1 49.51	+0 6.7	14 43 17.41	+19 42 4.2	9.041	0.786	+1.44	−11.6†
5 6 23 47	6	8, 3	−0 55.89	−1 16.4	14 46 23.66	+18 58 56.4	9.638	0.790	+1.35	−14.7†
10 6 22 53	7	6, 3	+2 46.20	+4 19.6	15 1 0.90	+15 29 29.1	9.629	0.796	+1.54	−15.1†
Comet a 1915 (Mellish)										
1915 Mar. 26 15 33 0	8	15, 11	+0 28.50	+4 24.2	17 58 37.88	− 0 50 57.4	9.332n	0.797	+1.49	−15.9†
29 15 40 26	9	11, 11	−0 12.95	−0 35.7	18 2 21.36	− 1 16 44.1	9.281n	0.800	+1.58	−15.6†
Apr. 9 15 8 2	10	10, 11	+1 31.92	−4 41.3	18 16 7.19	− 3 17 6.1	9.309n	0.812	+1.79	−14.0†
18 15 36 34	11	9, 7	+2 2.48	+3 2.8	18 27 16.25	− 5 44 1.6	9.070n	0.830	+2.06	−11.8†
28 15 18 14	12	7, 7	−4 38.59	−3 12.6	18 41 41.71	− 9 55 47.1	9.026n	0.851	+2.32	− 8.7†
29 14 25 11	13	11, 10	+1 23.10	−1 52.3	18 43 8.64	−10 27 11.7	9.287n	0.851	+2.38	− 8.7†
May 8 14 33 0	14	9, 11	+0 20.14	+3 35.5	18 58 20.32	−16 54 13.1	9.177n	0.882	+2.69	− 5.0†
9 14 26 50	15	10, 8	+0 21.06	+5 55.1	19 0 16.80	−17 49 53.7	9.217n	0.884	+2.74	− 4.5†
13 14 5 37	16	10, 10	−0 14.95	+4 1.7	19 8 51.41	−22 8 21.5	9.275n	0.896	+2.90	− 2.2†

*F. P. Leavenworth. †Hugh Wilcox.

Mean Places of Comparison Stars.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	21 50 55.14	− 0 53 36.5	*A.G. Nicolajew* 5529	9	18 2 32.73	− 1 15 52.8	*A.G. Nicolajew* 1493
2	14 12 46.76	+26 20 29.2	*A.G. Camb. (Eng.)* 6774	10	18 14 33.48	− 3 12 10.8	*A.G. Strassburg* 6129
3	14 29 15.94	+23 18 50.0	*A.G. Berlin B* 5098	11	18 25 41.71	− 5 46 52.6	*A.G. Strassburg* 6188
4	14 39 39.45	+20 28 23.4	*A.G. Berlin B* 5142	12	18 46 17.98	− 9 52 25.8	*A.G. Strassburg* 6384
5	14 45 5.48	+19 42 12.1	*A.G. Berlin A* 5352	13	18 41 43.16	−10 25 13.7	*A.G. Cambridge* 6437
6	14 47 18.20	+19 0 27.5	*A.G. Berlin A* 5360	14	18 57 57.49	−16 57 43.6	*A.G. Washington* 7045
7	14 58 13.16	+15 25 24.6	*A.G. Berlin A* 5427	15	18 59 50.00	−17 55 44.3	*A.G. Washington* 7061
8	17 58 7.89	− 0 55 5.7	Algiers photo. pl. 1285 No. 113	16	19 9 3.46	−22 12 21.0	*A.G. Córdoba A* 13379

Photographic Positions.

Comet 1913 *f* (DELAVAN)

Minneapolis M.T. 1914	α 1914.0	δ 1914.0	log *p*Δ α	log *p*Δ δ	No. of Exposures	Computer	Photo. by
d h m s	h m s	° ′ ″					
Feb. 23 7 36 49	2 10 47.51	+2 44 39.0	9.481	0.777	2	WILCOX	BEAL
Feb. 24 7 40 10	2 11 3.21	+2 52 30.2	9.496	0.776	1	WILCOX	BEAL
Apr. 12 11 58 26	18 19 51.18	3 59 32.6	9.317*n*	0.816	3	WILCOX	WILCOX
Apr. 16 15 37 4	18 25 6.61	5 6 9.5	9.400*n*	0.825	1	WILCOX, UNDERWOOD	WILCOX

[illegible]

OBSERVATIONS OF THE PLANET *AMBROSIA* (*193*).

BY E. E. BARNARD.

This planet, which was discovered by COGGIA in 1879, after being unobserved for over a third of a century, was refound (photographically) by Senor J. COMAS SOLA at the Fabra Observatory, Barcelona, Spain on 1915 September 15. See *Comptes Rendus*, Vol. CLXI, p. 412, October 4, 1915. Elements and ephemeris calculated from the observations of Senor J. COMAS SOLA are given in *Comptes Rendus*, Vol. CLXI, p. 451, October 18, 1915 by MM. LOUIS FABRY and HENRI BLONDEL.

Mr. E. P. HUBBLE, to whom I am greatly indebted, kindly located the planet on 1915 November 24 by a photograph with the 2-foot reflector of this observatory. The present measures were made with the 40-inch refractor. A greater number of observations was prevented by bad weather. Photographs of it were made on 1915 December 5, 7 and 27 with the 10-inch Bruce lens. These, if required, will give fair positions.

Observations of Ambrosia (193)

Date	C. S. T.	Δα cos δ	Δα	Δδ	Comps.	α Appt.	δ Appt.	Red. to Appt. α	Red. to Appt. δ	*
1915	h m s		m s			h m s	° ′ ″	s		
Nov. 24	7 18 2	+ 38.82	+0 2.62	+2 26.8	8 , 5	23 36 5.18	+8 30 23.9	+3.85	+28.5	1
24	8 7 45	+ 44.24	+0 2.98		3	23 36 5.56		+3.85		1
24	8 20 3	− 45.39	−0 3.06		3	23 36 5.64		+3.85		1
24	8 32 28			+2 36.1	, 5		+8 30 33.7		+28.5	1
24	8 41 14		−0 55½	+0 8.5	2 tr., 4	23 36 9	+8 30 34.6	+3.85	+28.5	2
Dec. 1	6 10 2	−102.69	−0 6.94	−2 29.4	10 , 6	23 40 3.02	+9 8 8.3	+3.84	+28.5	3
1	6 54 25	63.56	−0 4.29		5 , ..	23 42 10.47		+3.76		4
1	6 59 2			−3 7.9	, 4		+9 28 10.0		+28.2	4
1	7 12 59			−3 5.2	, 3		+9 28 12.7		+28.2	4

Mean Places of Comparison Stars

*	α 1915.0	δ 1915.0	Authority
	h m s	° ′ ″	
1	23 35 58.71	+8 27 28.6	Small star ? mag. Compared with Leipzig *A. G. C.* 11740.
2	23 37 0.99	+8 29 57.6	*BD* +8° 5099. Compared with Leipzig *A. G. C.* 11740.
3	23 40 6.45	+9 10 8.9	12.6 mag. Compared with Leipzig *A. G. C.* 11766.
4	23 42 11.00	+9 30 19.7	13 mag. Compared with Leipzig *A. G. C.* 11777.

Measures of Comparison Stars

		Δα	Comps.	Δδ	Comps.	Date of Measures
		m s		′ ″		
Star 1	Leipzig 11740	1 58.10	20 tr.	−2 55.9	9	1916 Jan. 5 and 8
Star 2	Leipzig 11740	−0 55.82	20 tr.	−0 26.9	10	1916 Jan. 5 and 8
c	Leipzig 11766	−2 4.45	10 tr.	−4 8.8	3	1916 Oct. 1 and 3
Star 3 − *c*		+0 11.74*	1	−4 19.6	1	1916 Sept. 22
Star 4 — Leipzig 11777		−1 15.08	12 tr.	+1 10.7	3	1916 Oct. 3

* Measured Δα = 173″.99

The star *c* was used for a step star. Its position from the above measures for 1915.0 is:

$$\alpha\ 23^h\ 39^m\ 54^s.11 \qquad \delta + 9°\ 14'\ 58''.5$$

1916 Nov. 24. *Ambrosia* was 1 or 1½ magnitude less bright than Star 1. It was estimated to be of 12th magnitude. The observations were made through breaks in clouds.

Dec. 1. *Ambrosia* was 3/10 magnitude less than Star 3. It had the mean brightness of Star 3 and a 13th magnitude Star 4′ preceding Star 3.

Dec. 1. Very faint in clouds, which prevented a second set of Δα.

Yerkes Observatory, Williams Bay, Wisconsin, 1916, October 13.

OBSERVATIONS OF THE SMALL STAR WITH LARGE PROPER MOTION,

(1916.423 $\alpha\ 17^h\ 53^m\ 43^s.60$ $\delta + 4°\ 27'\ 18''.0$)

By E. E. BARNARD.

The following observations of this star are a continuation of those printed in *A. J.* Vol. XXIX, p. 181, for 1916, September 15. In that paper the direction of motion should be 356°.7 instead of 359°.7.

Reproductions of the photographs of 1894 and 1916 are given in a paper printed in *Popular Astronomy*, Vol. XXIV, p. 504, for October, 1916.

		P	and *c*	Mag.	Mag.
		°	″		
1916.576	July 29	177.88	128.02	..	..
.615	Aug. 12	178.07	127.94	..	..
1916.645	Aug. 23	178.05	128.19		
.653	26	177.66	128.31		
.664	30	177.95	128.66		.
.672	Sept. 2	177.89	128.59		.
.683	6	177.90	128.67		..
.691	9	177.92	128.85		
.716	18	177.98	129.14		
.727	22	177.88	128.93		
.752	Oct. 1	177.90	129.34		.
.757	3	177.81	129.24		.
.776	10	177.69	129.58		..

P and *c* (*Continued*)

					Mag.	Mag.
1916.790	Oct.	15	177.87	129.80		
.806		24	177.90	129.67		
.834		31	177.87	130.12		
.839	Nov.	2	177.73	130.18		

P and *k*

1916.576	July	29	75.80	72.75
.596	Aug.	5	76.25	73.08
.615		12	76.15	72.70
.626		16	76.29	72.85
.634		19	76.36	72.56
.645		23	76.34	72.55
.653		26	76.09	72.96
.664		30	76.67	72.59
.672	Sept.	2	76.66	72.67
.683		6	76.68	72.59
.691		9	76.68	72.66
.716		18	76.84	72.58
.727		22	76.84	72.83
.752	Oct.	1	77.13	72.58
.757		3	76.84	72.75
.771		8	76.85	72.77
.776		10	77.32	72.44
.790		15	77.11	72.60
.806		21	77.50	72.42
.834		31	77.82	72.35
.839	Nov.	2	77.50	72.33
.872		14	77.91	72.29

P and *a*

1916.576	July	29	248.50	144.85
.596	Aug.	5	248.62	145.32
.615		12	248.62	145.25
.626		16	248.41	145.07
.634		19	248.18	145.03
.645		23	248.35	144.94
.653		26	248.49	144.91
.664		30	248.28	145.13
.672	Sept.	2	248.31	145.00
.683		6	248.09	144.98
.691		9	248.01	145.04

P and *a* (*Continued*)

					Mag.	Mag.
1916.716	Sept.	18	248.05	145.07		
.727		22	248.44	145.14		
.752	Oct.	1	248.03	145.18		
.757		3	247.86	145.36		
.771		8	247.86	145.49		
.776		10	247.74	145.15		
.790		15	247.80	145.24		
.806		21	247.75	145.36		
.834		31	247.70	145.36		
.839	Nov.	2	247.62	145.19		

P and *n*

1916.576	July	29	180.81	180.34
.653	Aug.	26	180.47	180.68
.664		30	180.87	180.98
.672	Sept.	2	180.89	181.20

P and *b*

1916.538	July	15	8.79	249.89
.576		29	8.84	249.47
.615	Aug.	12	8.90	249.58
.664		30	9.01	248.80
.691	Sept.	9	8.85	248.60

P and *g*

1916.587	Aug.	2	319.04	237.90	..	12
.596		5	319.35	237.70		
.615		12	319.19	237.90		12

k and *s*

1916.587	Aug.	2	203.59	21.09		..

b and *d*

1916.538	July	15	12.38	48.84	12	12 1/4
.576		29	12.22	48.93		

g and *t*

1916.576	July	29	337.63	63.58

Yerkes Observatory, Williams Bay, Wisconsin, November 10, 1916.

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MEASUREMENTS OF POSITIONS OF ASTEROIDS,

MADE AT THE EMERSON MCMILLIN OBSERVATORY OF THE OHIO STATE UNIVERSITY

By E. S. MANSON, Jr.

Date	Gr. M.T.	$\Delta\alpha$	$\Delta\delta$	No. of Comp.	α (app.)	log $p\Delta$	δ (app.)	log $p\Delta$	Red. to App. Pl.		*
					(124) *Alkeste*						
1914	h m s	m s	′ ″		h m s		° ′ ″		s	″	
May 25	18 40 54	−0 50.17	+4 05.4	10 , 10	16 43 10.27	8.193	−17 53 24.4	0.871	+3.05	−13.9	1
30	18 04 42	−3 03.35	−6 04.5	8 , 8	16 38 37.81	8.722	−17 40 03.6	0.869	+3.12	−11.0	2
					(2) *Pallas*						
July 18	16 29 38	+1 12.25	+0 31.5	24 , 8	19 32 03.67	9.014*n*	+19 35 38.5	0.490	+3.17	+ 1.2	3
19	14 56 54	+0 26.89	−4 36.5	24 , 8	19 31 18.32	9.433*n*	+19 30 30.8	0.537	+3.18	+ 1.5	3
20	14 47 13	−2 07.88	−2 02.4	24 , 8	19 30 30.36	9.446*n*	+19 24 52.5	0.542	+3.18	+ 1.8	4
					(12) *Victoria*						
July 21	16 26 48	+0 37.34	+3 25.5	23 , 8	20 29 11.15	9.271*n*	− 1 20 37.3	0.761	+3.49	+ 5.9	5
26	16 08 56	+0 34.86	−6 35.7	24 , 8	20 25 08.06	9.246*n*	− 1 13 36.0	0.760	+3.55	+ 6.3	6
28	15 58 29	−1 03.51	−5 56.7	24 , 8	20 23 29.71	9.251*n*	− 1 12 56.7	0.760	+3.57	+ 6.6	6
29	16 20 31	+2 31.59	+3 50.1	24 , 8	20 22 39.52	9.109*n*	− 1 13 04.8	0.761	+3.58	+ 6.4	7
					(433) *Eros*						
Oct. 25	16 23 20	+1 54.75	−5 29.8	8 , 8	22 45 29.58	9.414	+21 13 32.5	0.511	+3.36	+27.2	8
30	15 41 52	+3 04.12	−5 38.0	8 , 8	22 43 13.64	9.384	+20 23 55.4	0.511	+3.30	+27.2	9
					(4) *Vesta*						
Nov. 21	18 06 06	+1 27.26	+0 45.6	23 , 8	5 15 20.52	8.910*n*	+17 01 50.7	0.535	+4.86	−14.0	10
22	16 07 20	+0 35.14	+1 00.1	23 , 8	5 14 28.43	9.467*n*	+17 02 05.2	0.583	+4.89	+14.0	10
23	15 03 29	−0 20.12	+1 16.2	23 , 8	5 13 33.19	9.577*n*	+17 02 21.2	0.623	+4.91	+13.9	10
27	16 53 49	+1 50.38	−3 00.6	23 , 8	5 09 26.49	9.221*n*	+17 03 50.4	0.547	+4.99	+14.1	11

MEASUREMENTS OF POSITIONS OF ASTEROIDS (*Continued*)

Date	Gr. M. T.	Δα	Δδ	No. of Comp.		α app.	log pΔ	δ app.	log pΔ	Red. to App. Pl.		✱
						(7) Iris						
1913 Dec. 11	h m s 15 26 11	m s +0 16.03	′ ″ −0 50.3	12	4	h m s 6 22 15.20	9.528*n*	° ′ ″ +21 39 42.2	0.536	s +5.16	″ +4.7	12
1914 Jan. 8	16 19 47	+1 16.94	+3 20.5	5	5	5 56 48.71	8.821	+20 07 35.4	0.471	+2.12	+7.2	13

Mean Places of Comparison Stars

✱	α 1914.0	δ 1914.0	Authority		✱	α 1914.0	δ 1914.0	Authority	
1	h m s 16 43 57.39	° ′ ″ −17 57 15.9	*A.G. Washington,*	6028	8	h m s 22 43 31.47	° ′ ″ +21 48 35.1	*A.G. Berlin B.*	8754
2	16 44 38.01	−17 33 45.4	*A.G. Washington,*	6018	9	22 40 05.92	+20 29 06.2	*A.G. Berlin B.*	8731
3	19 30 48.25	−19 35 05.8	*A.G. Berlin A.*	7529	10	5 13 48.40	+17 00 51.4	*A.G. Berlin A.*	1461
4	19 32 35.06	−19 26 53.4	*A.G. Berlin A.*	7555	11	5 07 31.42	+17 06 36.6	*A.G. Berlin A.*	1427
5	20 28 30.32	−1 24 08.7	*A.G. Nicolajew,*	5201	12	6 21 23.74	+21 39 57.8	*A.G. Berlin B.*	2382
6	20 21 29.65	−1 07 06.6	*A.G. Nicolajew,*	5179		(1915.0)	(1915.0)		
7	20 20 04.35	−1 17 01.3	*A.G. Nicolajew,*	5157	13	5 54 59.68	+20 04 07.7	*A.G. Berlin B.*	2103

Comparison of Observed Places with Ephemerides

Alkeste

Ephemeris Computed from Elements in Berliner Jahrbuch

	Δα O − C	Δδ
May 25	−0m 39s.2	+2′ 30″
30	−0 39 .5	+2 30

Pallas

Nautical Almanac

	Δα O − C	Δδ
June 18	0s 2 .12	+0′ 1″.0
19	2 .15	− 3 .3
20	2 .10	+ 5 .7

Victoria

Ephemeris Computed from Elements in Berliner Jahrbuch

	Δα O − C	Δδ
July 21	−1s 11 .8	+0′ 43.″
26	−1 11 .3	+0 12
28	−1 10 .9	0 0
29	−1 10 .8	0 8

Eros

Berliner Jahrbuch

	Δα O − C	Δδ
Oct. 25	−0m 00s.6	−0′ 6″
30	−0 00 .4	−0 6

Vesta

Nautical Almanac

	Δα O − C	Δδ
Nov. 21	−0m 00s.12	−0′ 4″.2
22	− 00 .16	−0 0 .7
23	− 00 .07	−0 1 .0
27	− 00 .05	−0 0 .8

The observations were made with the 12½ inch equatorial and filar micrometer. The differences in right ascension were obtained by transits and the differences in declination by micrometric measurement.

SUNSPOT OBSERVATIONS,

MADE AT BERWYN, PENN., WITH A $1\frac{1}{2}$-INCH REFRACTOR

By A. W. QUIMBY

1916		Time	New Grs	Total Grs.	Total Spots	Fac. Grs	Def.
July	1	6	1	5	13	2	fair
	2	6	1	6	11	2	fair
	3	6		5	18	2	fair
	4	6		2	6	2	fair
	5	6		1	1	2	fair
	6	6	2	3	10	3	fair
	7	6	1	4	12	3	fair
	8	6		4	10	2	fair
	9	6		4	5	2	fair
	10	10	–	2	4		poor
	11	6		2	4		poor
	12	9	–	1	1		poor
	13	6	–	1	1		good
	14	6	1	2	2	1	fair
	15	11		2	2	–	poor
	16	9		2	2	1	poor
	17	5	2	4	6	1	fair
	18	6	2	6	9	2	fair
	19	6	–	5	10	2	fair
	20	7	2	6	24	2	fair
	21	1	1	7	37	1	fair
	22	11		7	49	2	fair
	23	5		7	71	4	v. g.
	24	1	–	7	52	2	fair
	26	8		6	40	1	poor
	27	8	–	6	32	1	poor
	28	9	–	6	14	2	poor
	29	6	–	3	10	1	fair
	30	6	–	3	6	1	fair
	31	6		2	2	1	fair
Aug.	1	6	1	4	6	2	v. g.
	2	6	–	4	6	4	fair
	3	6	1	2	2	3	fair
	4	9	–	1	1	1	fair
	5	4	1	2	6	1	fair
	6	5	1	4	6	1	fair
	7	7	1	3	6	1	good
	8	7		4	6	2	fair
	9	7	–	3	3	2	fair
	10	5	–	2	2	2	fair
	11	7	–	2	2	3	fair
	12	5	2	4	13	1	good
	13	5	1	4	15	1	good
	14	6	1	4	9	2	fair
	15	6	1	5	17	2	good
	16	7	–	5	6	1	fair

1916		Time	New Grs	Total Grs	Total Spots	Fac. Grs	Def.
Aug.	17	5		1	9	2	good
	18	5	1	5	17	1	fair
	19	6		1	33	1	fair
	20	6	1	5	10	2	fair
	21	6		5	10	1	fair
	22	6		3	27	1	fair
	23	7		2	13	1	fair
	24	6		1	1	1	fair
	*25	7				1	fair
	*26	7				1	fair
	*27	7					fair
	*28	7					fair
	29	1					fair
	30	6	1	1	2	1	fair
	31	7		1	2	1	fair
Sept.	1	7	1	1	1	1	fair
	2	12	1	1	5	1	fair
	3	5		1	5		fair
	4	7	1	1	3	1	fair
	5	7		1	6		fair
	6	5	1	2	6	2	fair
	7	12	2	4	7	3	fair
	8	7	1	5	10	3	fair
	9	7		3	7	3	fair
	10	4	2	4	7	3	fair
	11	9		1	10	1	fair
	12	7		4	20	2	fair
	13	5	1	5	26	3	fair
	14	7	1	6	20	2	fair
	15	8		5	22	1	poor
	16	8		5	39	2	fair
	17	5	1	6	50	2	v. g.
	18	9		6	39	2	poor
	19	7	2	8	25	2	poor
	20	5		7	25	2	good
	21	7		6	13	1	fair
	22	4	1	5	10	2	fair
	23	7		3	5	3	fair
	24	7		1	1	3	fair
	25	7		1	2	1	fair
	26	5	2	2	10	2	fair
	27	7		2	6		fair
	28	7		2	8	1	fair
	29	10		2	5	1	fair
	30	7		2	3	1	fair
Oct.	1	7		1	3	1	fair

1916		Time	New Grs	Total Grs	Total Spots	Fac. Grs	Def.
Oct.	2	10		1	1	1	fair
	3	10	1	1	1	1	fair
	4	4	2	3	7	3	fair
	5	7		3	10	3	fair
	6	1	2	5	10	3	fair
	7	5	1	6	21	2	fair
	8	7	1	7	30	2	fair
	9	7		6	35	2	fair
	10	7		6	35	1	fair
	11	5	2	8	32	2	fair
	12	9		7	30	2	good
	13	8		6	27	2	poor
	14	9		6	18	1	fair
	15	7	–	6	8	1	poor
	16	10	–	2	3	1	poor
	17	8	1	3	10	2	poor
	18	8		4	15	2	fair
	20	8	–	2	10	1	fair
	21	8		2	8	1	fair
	22	4	2	2	5	2	fair
	23	1	–	2	3	1	fair
	24	4	2	4	8	2	fair
	25	8		2	8	1	fair
	26	8		1	7	–	fair
	27	8		1	8	1	fair
	28	8	1	2	15		fair
	29	8	1	3	22	2	fair
	30	8		2	15	1	fair
	31	1	3	5	27	2	fair
Nov.	1	8		5	15	3	fair
	2	4	1	6	21	3	fair
	3	8		6	29	2	fair
	4	8	1	6	49	3	fair
	5	8	–	6	40	2	poor
	6	8	1	6	42	2	fair
	7	8		6	49	3	fair
	8	8	1	7	40	2	poor
	9	8		7	52	2	fair
	10	8		5	32	1	poor
	11	8		5	28	3	fair
	12	8	1	6	16	4	poor
	14	4	–	1	12		poor
	15	4		3	6	2	poor
	16	8	1	4	11	2	fair
	17	8		3	12	2	fair
	18	8		3	6	3	fair

*With hand refractor.

SUNSPOT OBSERVATIONS (*Continued.*)

1916	Time	New Grs.	Total Grs.	Total Spots	Fac. Grs.	Def.	1916	Time	New Grs.	Total Grs.	Total Spots	Fac. Grs.	Def.	1916	Time	New Grs.	Total Grs.	Total Spots	Fac. Grs.	Def.
Nov. 19	8		1	1	2	fair	Dec. 1	8		5	22	2	poor	Dec. 19	8	1	3	13	1	fair
20	8		1	1	1	fair	5	10	1	6	30	2	fair	20	8		3	3	1	poor
21	8		1	1	1	fair	6	8		6	20	2	fair	21	8		1	1	1	poor
22	12	3	3	12	2	poor	7	8		5	15	2	poor	22	4	1	2	2	1	poor
24	8		3	25	1	fair	8	10		5	39	3	fair	23	3	1	3	8	1	fair
25	8	1	1	39	2	fair	9	4		4	20	2	poor	24	8	1	4	9	1	fair
26	8		3	12	2	fair	10	8		3	16	2	poor	25	4		1	9	2	fair
27	1		3	20	1	fair	11	8	1	4	20	2	fair	26	4		4	5	2	poor
28	8		3	17	1	poor	12	3		3	20	1	fair	28	4	2	5	8	1	poor
29	8	1	1	22	1	fair	13	8		2	15	1	fair	29	8	1	6	18	1	fair
30	8	1	1	20	2	fair	14	8		2	2	1	poor	30	8	1	7	23	1	fair
Dec. 1	8		1	22	2	fair	16	3	2	2	5	2	fair	31	8	1	7	25	1	poor
2	9	1	5	15	3	fair	17	8		2	8	1	fair							
3	8		5	20	2	poor	18	8		2	10		poor							

TWO NEW VARIABLES.

By BERNHARD H. DAWSON.

The star h 4638 = C. P. D. $-16°$ 6590, $1^m 25^s$ following and 3′.4 north of ξ *Centauri*, was noted by HERSCHEL as "curiously triple." It has in reality three companions, whose positions as measured with the large refractor of this observatory are:

AB,	1913.12	313°.6	8″.01	3*n*.
AC,	1913.22	259 .2	9 .44	4*n*.
AD,	1913.21	110 .2	9 .32	5*n*.

Assuming 8.8 as the magnitude of *A* and 10.0 as that of *B*, the estimated magnitudes of *C* range from 10.5 to 13.0 and those of *D* from 10.7 to 13.5. The variability, first suspected in *C* 1913.430 and in *D* 1913.214, is thoroughly established but though many estimates were secured, they are not sufficiently systematic to determine the periods or types.

La Plata, 1916 December 12.

NEW VARIABLE STAR.

(1916.0 α $22^h 32^m 18^s.71$ δ $+52° 32' 56''.0$)
(1900.0 α 22 32 10 .1 δ $+52$ 27 59)

By E. E. BARNARD.

Professor E. C. PICKERING informs me that this star, which I have found on my photographs with the Bruce telescope of the Yerkes Observatory, is new, so far as the H. C. O. records go. The period seems to be a long one, and the variation of brightness runs through several magnitudes. The star is now at its maximum — between the 11th and 12th magnitudes. The above position was determined from comparisons with the star Cambridge U. S. 7783, on 1916 October 28 and November 11.

$\Delta\alpha$ + 1^m 13^s.36 $\Delta\delta$ − 0′ 23″.9

The Variable Star in 1915.0 α $22^h 19^m 44^s$ δ $+55° 5'$. A. J. 682

This star has a period of about 288 days and a range of about five or six magnitudes. It was at maximum about 1916 October 1 at approximately 10.4 magnitude.

Yerkes Observatory, Williams Bay, Wisconsin, 1916 December 27.

ELEMENTS AND EPHEMERIS OF *EROS* FOR THE OPPOSITION OF MAY, 1917.

By F. E. SEAGRAVE

ELEMENTS

Epoch = 1917 April 26.50 G. M. T.

$M = 74°\ 8'\ 17''.13$

$\pi = 121\ 25\ 31\ .98$

$\Omega = 303\ 35\ 8\ .69$

$i = 10\ 49\ 39\ .50$

$\varphi = 12\ 53\ 0\ .62$

$\text{Log}\ a = 0.1638460$

$\text{Log}\ q = 0.0542861$

$\mu = 2014''.8301$

T = May 4 1917

CONSTANTS

$$x = r(9.99461) \sin (34°\ 3'\ 41'' + u)$$
$$y = r(9.94143) \sin (299\ \ 0\ 14\ \ + u)$$
$$z = r(9.70826) \sin (319\ 31\ 38\ \ + u)$$

EPHEMERIS

1917	α	δ	Log r	Log Δ
Jan. 28	14 1 3	−25 24 43	0.07234	9.79725
Feb. 1	14 10 49	−27 26 21	0.07546	9.79266
5	14 19 17	−29 24 10	0.07876	9.78794
9	14 27 56	−31 18 11	0.08222	9.78312
13	14 36 14	−33 8 17	0.08580	9.77803
Feb. 17	14 44 3	−34 51 20	0.08954	9.77280
21	14 51 21	−36 36 20	0.09336	9.76735
25	14 58 9	−38 14 11	0.09728	9.76162
Mar. 1	15 4 15	−39 47 13	0.10126	9.75592
5	15 9 38	−41 17 12	0.10530	9.74996
9	15 14 6	−42 41 11	0.10942	9.74396
13	15 17 41	−44 2 2	0.11358	9.73783
17	15 20 14	−45 17 7	0.11772	9.73168
21	15 21 34	−46 26 9	0.12192	9.72570
25	15 21 50	−47 28 21	0.12610	9.72014
29	15 20 26	−48 24 25	0.13026	9.71435
Apr. 2	15 17 47	−49 10 58	0.13442	9.70940
6	15 13 52	−49 48 1	0.13854	9.70506
10	15 8 37	−50 13 51	0.14264	9.70155
14	15 2 20	−50 27 41	0.14668	9.69904
18	14 55 2	−50 27 30	0.15070	9.69779
22	14 47 6	−50 12 45	0.15466	9.69797
26	14 38 57	−49 43 46	0.15856	9.69974
30	14 30 50	−49 0 22	0.16240	9.70320
May 4	14 22 55	−48 3 16	0.16618	9.70849
8	14 16 45	−46 57 16	0.16988	9.71564
12	14 10 11	−45 41 31	0.17350	9.72454
16	14 5 13	−44 18 13	0.17708	9.73543
20	14 1 46	−42 53 7	0.18056	9.74757
24	13 58 28	−41 25 11	0.18396	9.76139
28	13 56 48	−39 58 7	0.18730	9.77655
June 1	13 56 9	−38 33 25	0.19054	9.79274
5	13 56 29	−37 12 23	0.19372	9.80984

FAINT ASTEROID WITH PECULIAR MOTION.

The Rev. J. H. METCALF has announced the discovery by him of a faint asteroid (13 to 14 magnitude) having a peculiar motion. He derived the following positions from photographs taken with his 12-inch doublet at Winchester.

G. M. T.	R. A. 1916.0	Decl. 1916.0	Daily Motion	
1916	h m s	° ′ ″	s	″
Dec. 16.5867	6 33 53.52	+31 41 21.0	−27	− 1.2
23.6180	6 30 42.68	+31 32 14.1	−38	− 5.2
26.5097	6 28 53.89	+31 18 17.5	−42	− 8.3
28.4916	6 27 30.70	+30 58 18.4	−37	−16.3
29.6403	6 26 38.45	+30 34 33.3	−20	−21.0
30.7639	6 26 25.64	+30 15 16.8		

Two images of the object appear on each of these, and on two other, plates. Measures of their relative positions confirm the motion, and indicate a parallax that would be explained if the object is near the *Earth*.

OBSERVATIONS OF WOLF'S COMET.

G. M. T.	R. A.	Decl.	Observer	Place
Dec. 23.9731	15 48 51	5 59 3	GALLO	Tacubaya
Dec. 25.9941	15 52 33.3	6 1 52	GALLO	Tacubaya
Dec. 27.9841	15 56 11.9	−6 3 31	GALLO	Tacubaya
Dec. 29.9821	15 59 51.9	−6 4 31	GALLO	Tacubaya
Dec. 31.0028	16 1 45.7	−6 1 35.5	BARNARD	Yerkes Obsy.

NOVA PERSEI.

Professor BARNARD has announced that he found on December 16, 1916, a faint nebulosity twenty seconds westward from *Nova Persei*. Under date of December 18, a further statement reads: "There has been but little change in brightness of *Nova Persei* during recent years and the image has been sharply defined with the 40-inch telescope, but has faded about a magnitude since last year, and its image is pale and ill defined. Its magnitude is now about 13.5. The faint diffused nebulosity extends from the star for a distance about 15″ or 20″ preceding and south of *Nova Persei*."

NEW ASTRONOMICAL WORK.

Catalogo Astrofotografico, 1900, de −14° a −17°. Published by the Observatorio Astronomico de Tacubaya, Mexico. Vol. I, containing the rectangular and equatorial coördinates of the stars to the 11th magnitude, comprised between −14° and −17° of declination, and of 0^h to 6^h of right ascension. 1916.

El Eclipse Total de Sol del 3 de febrero de 1916. Published by the Ministerio de Instruccion Publica, Estados Unidos de Venezuela. 1916.

History and Description of the Flower Astronomical Observatory, with a determination of its longitude. Published by the University of Pennsylvania. Vol. I, Part I. 1916.

CONTENTS.

EDITOR, BENJAMIN BOSS, ALBANY, N. Y. ASSOCIATE EDITORS, E. E. BARNARD, ERNEST W. BROWN, F. R. MOULTON AND R. S. WOODWARD.
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THE PARALLAX PROBLEM IN ITS APPLICATION TO THE REAL MOTIONS OF THE FIXED STARS,

By A. v. FLOTOW

As physical methods applied to the heavenly bodies inform us in great measure of the elements we need for investigation of the motions in the system of the fixed stars, it seems desirable to develop methods of calculating some particulars common to all stars or to individual groups.

We may divide the investigations of stellar motions made hitherto into three general classes; first the statistical founded on the distribution according to magnitude, second the spherical founded on the discussion of preferential motion, and third the theoretical founded on the results of the first two classes, submitting them to certain dynamical principles. But as yet there has been little attempt made to determine the real motions in space for all stars with well known proper-motion, whose parallaxes and radial velocities are measured. H. C. Wilson* in his discussion treats the real motions, but he stops with a graphical representation. I have therefore undertaken the development of formulæ to effect a simple solution. The results are illustrated by the positions of the apices of the real motions of 116 stars, from which an attempt will be made to determine two planes of velocity. The existence of the plane coinciding with the galaxy has already been shown, the existence of the other is only suggested by the apparent distribution. In addition a method is derived to determine whether any given apex belongs to an assumed velocity-plane. Certain well known formulæ have been inserted for the sake of completeness.

*H. C. Wilson, On the Real Motions of 100 Stars of Large Proper-Motion, Whose Radial Velocities Have Been Determined. *L. O. B.*, No. 214

I

The following designations are used.

Linear distances.

r the distance of the star from the *Sun*, the major semi-axis of the *Earth's* orbit being taken as the unit.

a the major semi-axis of the *Earth's* orbit in km.

a_0 the equatorial radius of the *Earth's* spheroid in km.

Angular distances in seconds of arc.

π the parallax of the star.

π_0 the *Sun's* mean equatorial horizontal parallax.

Linear velocities in km. sec.

v the star's velocity in space directed towards the stellar apex.

ρ the star's observed radial velocity.

ρ' the star's absolute radial velocity.

v_s the *Sun's* velocity in space directed towards the solar apex.

Angular velocities in $\dfrac{\textit{second of arc}}{\textit{tropical year}}$

μ the star's observed proper-motion.

μ' the star's absolute proper-motion.

τ the *Sun's* proper-motion from the star.

$\mu_1\ \mu_2$, $\mu'_1\ \mu'_2$ spherical components of μ and μ' respectively in the direction of solar motion and at right angles to solar motion, in the sense of position-angles.

Equatorial coordinates.

α, δ the right-ascension and declination of the star.

A, D the right-ascension and declination of the solar apex.

A', D' the right-ascension and declination of the stellar apex.

Relative coordinates referred to the star.

χ, d the position-angle and the angular distance of the solar apex.

ϕ, Δ the position-angle and the angular distance of the stellar apex.

θ the position-angle of the observed proper-motion.

The relative coördinates of the solar apex are obtained from the known equatorial coordinates by the formulæ

(1)

$$\begin{aligned}\sin d \cos \chi &= -\sin\delta \cos D \cos(A-\alpha) + \cos\delta \sin D\\ \sin d \sin\chi &= +\cos D \sin(A-\alpha)\\ \cos d &= +\cos\delta\cos D\cos(A-\alpha) + \sin\delta\sin D\end{aligned}$$

The connection between the equatorial values $\Delta\alpha$ and $\Delta\delta$ of the proper-motion, given in the star catalogues, and the corresponding values μ and θ are represented by the formulæ

$$\begin{aligned}\mu\sin\theta &= 15\,\Delta\alpha\cos\delta\\ \mu\cos\theta &= \Delta\delta\end{aligned} \tag{2}$$

where $\Delta\alpha$ is given in seconds of time and $\Delta\delta$ in seconds of arc. The spherical components of this proper-motion are

$$\begin{aligned}\mu_1 &= \mu\cos(\theta-\chi)\\ \mu_2 &= \mu\sin(\theta-\chi)\end{aligned} \tag{3}$$

and that of the star's absolute proper-motion

$$\begin{aligned}\mu'_1 &= \mu'\cos(\phi-\chi)\\ \mu'_2 &= \mu'\sin(\phi-\chi)\end{aligned} \tag{4}$$

Resolving the velocity $v_\odot$ into two components, one $v_{\odot 1}$ along the line of sight and the other $v_{\odot 2}$ along a direction at right angles to it in the plane $(\alpha\delta, AD)$ we have

$$\begin{aligned}v_{\odot 1} &= v_\odot \cos d\\ v_{\odot 2} &= v_\odot \sin d\end{aligned}$$

The angular velocity of the component $v_{\odot 2}$, taking seconds of arc and the tropical year as units, is the *Sun's* proper-motion from the star

$$\tau = \frac{e}{ra\sin 1''}\, v_\odot \sin d$$

where e is the number of mean seconds of time in one tropical year.

Resolving the velocity v in the same way into two components, one ρ' along the line of sight and the other R along the direction at right-angles to it in the plane $(\alpha\delta, A'D')$ we get

$$\begin{aligned}\rho' &= v\cos\Delta = \rho + v_\odot \cos d\\ R &= v\sin\Delta = \rho'\tan\Delta\end{aligned} \tag{5}$$

and from the component R we derive the angular velocity

$$\mu' = \frac{e}{ra\sin 1''}\, v\sin\Delta$$

On substituting the parallax π

$$\frac{1}{r} = \tan\pi = \pi\sin 1''$$

and putting for the constant value

$$\frac{a}{e} = C$$

we may write the two elements τ and μ' in the simple form

$$\begin{aligned}C\tau &= \pi v_\odot \sin d\\ C\mu' &= \pi v\sin\Delta = \pi R\end{aligned} \tag{6}$$

In order to get the absolute motion, we have to add the *Sun's* motion to the observed relative motion

$$\begin{aligned}\mu_1' &= \mu_1 + \tau\\ \mu_2' &= \mu_2\end{aligned} \tag{7}$$

These are the fundamental equations of the star's real motion in space. On substituting the expressions for the components given in (3), (4) and (6) there results

$$\begin{aligned}C\mu\cos(\theta-\chi) &= \pi\{R\cos(\phi-\chi) - v_\odot \sin d\}\\ C\mu\sin(\theta-\chi) &= \pi\, R\sin(\phi-\chi)\end{aligned} \tag{I}$$

Multiplying the first equation by $+\cos(\phi-\chi)$ and the second by $+\sin(\phi-\chi)$ and adding them, also

multiplying the first equation by $-\sin(\phi-\chi)$ and the second by $+\cos(\phi-\chi)$ and adding, the fundamental equations (I) become

$$\text{(II)}\quad \begin{aligned} \pi R &= C\mu\cos(\theta-\phi) + \pi r_\odot \sin d \cos(\phi-\chi) \\ 0 &= C\mu\sin(\theta-\phi) - \pi r_\odot \sin d \sin(\phi-\chi) \end{aligned}$$

A third form is derived from multiplying the first equation of (I) by $+\cos(\theta-\chi)$ and $+\sin(\theta-\chi)$ respectively, and the second by $+\sin(\theta-\chi)$ and $-\cos(\theta-\chi)$ and adding. There results

$$\text{(III)}\quad \begin{aligned} C\mu &= \pi\{R\cos(\theta-\phi) - r_\odot \sin d \cos(\theta-\chi)\} \\ 0 &= R\sin(\theta-\phi) - r_\odot \sin d \sin(\theta-\chi) \end{aligned}$$

In all of these forms the second equation represents the law of sines, yielding the following proportion

$$C\mu : \pi r_\odot \sin d : \pi R = \sin(\phi-\chi) : \sin(\theta-\phi) : \sin(\theta-\chi)$$

These six values represent the sides and angles of the plane triangle

(a)	$= C\mu$	(α)	$= \phi - \chi$
(b)	$= \pi r_\odot \sin d$	(β)	$= \theta - \phi$
(c)	$= \pi R$	(γ)	$= 180^\circ - (\theta - \chi)$

This plane triangle affords a geometrical interpretation of the fundamental equations, and its solution forms the connection between the star's relative and real motion.

The constant C, defined by

$$\text{(8)}\qquad C = \frac{a}{c} = \frac{a_0}{c\pi_0 \sin 1''}$$

gives the angular velocity in terms of km. per second of time at the distance a. Making use of the best known values

a_0 (J. F. Hayford 1910) $= 6378.388$ km.

tropical year (Newcomb) $= 365.2421988$ mean solar days

or $c = 31556926$ mean seconds of time

π_0 (international) $= 8''.80$

we obtain

$$\text{(9)}\qquad C = 4.73760 \quad [0.67556]$$

The uncertainty of π_0 alone has a perceptible effect on C. If we assume $\Delta\pi_0$ as the correction, the value of C given above has the corresponding correction

$$\Delta C = -\frac{C}{\pi_0}\Delta\pi_0 = -0.538\,\Delta\pi_0 \qquad (10)$$

II

The investigations of proper-motions show an accumulation of the apices at certain points of the sphere and suggest the idea of a common apex for groups of stars. Assuming for a group an indicated apex, and computing the resulting values of the parallax and radial velocity for each star, a comparison with the observed values will determine the reality of the group. On the other hand, if we compute from the observed values of parallax and radial velocity the individual apex of each star, we may investigate the distribution of these apices. That is the purpose of the present discussion.

Among the 654 stars of Boss's *Preliminary General Catalogue* whose proper-motions exceed $0''.15$, there are 130 stars with measured parallax and radial velocity. Considering only those stars whose parallax is positive, the number is reduced to 116. Table I contains the data for further computation. The magnitude is taken from the Revised Harvard Photometry and Supplement (*Harvard Annals*, Vol. L and LIV), and the type and radial velocity from W. W. Campbell's lists as furnished in the *Lick Bulletins*. From several lists of measured parallaxes* the best determined values are selected. For the solar motion the position given by L. Boss in *A. J.* 614 is used, and for the solar velocity 20 km.

$$A = 270^\circ.52 \qquad D = +34^\circ.28$$
$$r_\odot = 20 \text{ km.}$$

*G. Bigourdan, Catalogue de Parallaxes Stellaires. (*Bull. Astr.* t. 26).

J. C. Kapteyn and H. A. Weersma, List of Parallax Determinations. (*Publ. of the Astron. Labor.*, at Groningen, No. 24.)

H. N. Russell, Determinations of Stellar Parallax. (*A. J.* 618-619.)

H. N. Russell, Determinations of Stellar Parallax. (*Carnegie Institution of Washington*, Publ. No. 147.)

A. S. Flint, Results for Stellar Parallax from Meridian Transits at the Washburn Observatory, 2nd series. (*A. J.* 631.)

Transactions of the Astron. Obs. of Yale University. Vol. II, Parallax Investigations.

A. van Maanen, The Photographic Determination of Stellar Parallaxes with the 60-inch Reflector. (*Contributions from the Mount Wilson Solar Observatory*, No. 111.)

These elements, as also the proper-motions of the stars, are referred to the epoch 1875.0. The difference in the epochs of ρ and π is considered negligible.

A value of $C = 4.736$ has been used, corresponding to a value of $\pi_0 = 8''.803$ for the solar parallax.

TABLE I

P. G. C. No.	Mag.	Type	1875.0 α	δ	μ	θ	μ'	χ	d	π	ρ
			°		″	°	″	°	°	″	km
6	6.2	F	0.0	+28.3	0.442	141.9	0.192	300.7	74.1	0.150	+ 7.7
12	2.4	F	0.6	+58.5	0.561	109.0	0.293	289.7	61.1	0.071	+ 12.8
55	4.3	F, G	3.4	−65.6	2.058	55.5	1.717	283.3	122.0	0.148	+ 9.3
71	2.9	G	1.8	−78.0	2.243	81.7	1.756	274.0	124.3	0.143	+ 22.8
127	5.7	K	7.7	25.5	1.386	90.4	0.708	299.5	109.6	0.360	+ 18.
168	3.6	F	10.4	+57.2	1.242	111.8	0.160	297.5	66.7	0.204	+ 10.0
174	5.8	K	10.5	+ 4.6	4.367	146.8	0.746	305.1	95.6	0.172	− 11.8
244	5.3	G	15.0	+54.3	3.761	144.3	3.328	301.8	70.3	0.112	− 97.4
259	2.4	Ma	15.7	+35.0	0.249	122.3	0.070	306.3	81.7	0.036	+ 2.
261	4.5	A	15.9	+54.5	0.234	95.0	0.708	302.4	70.4	0.230	+ 14.
314	2.8	A	19.4	+59.6	0.308	98.8	0.206	303.4	69.4	0.030	+ 7.
313	3.8	K	19.4	− 8.8	0.228	199.7	0.110	303.4	110.6	0.101	+ 47.9
321	5.0	F	20.0	+44.8	0.360	105.8	0.136	307.4	78.5	0.088	+ 11.3
372	5.1	F	23.5	+42.0	0.849	99.9	0.467	309.8	82.2	0.116	+ 4.9
391	3.6	K	24.6	−16.6	1.920	296.4	3.055	300.4	118.9	0.320	− 15.5
394	5.7	G	24.7	+63.2	0.635	111.8	0.404	306.4	69.6	0.061	+ 1.9
477	2.2	K_2	30.0	+22.9	0.242	127.8	0.043	313.3	99.0	0.049	− 14.0
514	5.4	G	32.4	+33.7	1.179	102.0	0.821	315.3	92.9	0.114	− 4.
530	var.	Md	33.3	− 3.5	0.239	179.5	0.423	307.6	118.7	0.142	+ 62.3
588	5.9	G	37.3	+ 6.3	2.320	50.8	2.311	312.9	115.5	0.143	+ 22.5
710	4.2	G	45.0	+19.4	1.268	93.4	1.033	323.8	86.9	0.112	+ 50.5
764	4.3	G	48.7	−43.5	3.165	76.2	2.801	268.2	146.5	0.162	+ 87.6
814	3.8	K	51.8	− 9.9	0.971	270.7	1.825	310.7	137.0	0.336	+ 16.5
825	1.4	G	52.6	+ 0.0	0.535	205.4	0.494	318.0	130.7	0.069	+ 28.9
848	3.7	K	54.3	−10.2	0.749	352.6	1.140	311.8	139.1	0.168	− 5.4
984	4.5	G	62.1	− 7.8	4.082	212.6	3.980	319.5	143.0	0.174	− 41.6
1077	1.1	K	67.2	+16.3	0.205	159.7	0.049	336.5	124.8	0.073	+ 55.1
1246	0.2	G	76.9	+45.9	0.439	168.4	0.146	348.6	98.9	0.066	+ 30.2
1259	4.8	G	77.6	+40.0	0.842	140.9	0.519	349.0	104.8	0.100	+ 66.5
1565	4.5	K	91.9	+29.5	0.273	192.9	0.073	1.2	116.2	0.058	+ 21.0
1732	−1.6	A	99.9	−16.5	1.315	203.3	0.773	23.9	160.3	0.376	− 7.4
1809	6.0	F_2	103.9	+29.5	0.839	168.9	0.608	12.2	114.9	0.068	+ 20.3
1952	4.2	A	110.3	+32.0	0.238	39.4	0.423	17.4	111.9	0.049	− 4.
1979*	2.8, 2.0	A	111.9	+32.2	0.201	237.2	0.144	18.8	110.6	0.058	+ 2.6
2023	1.3	K	113.9	+29.2	0.246	163.3	0.154	20.8	112.8	0.060	+ 45.8
2031	1.2	K	114.4	+28.3	0.623	264.6	0.558	21.1	113.4	0.064	+ 3.9
2053	5.4	F, G	115.2	+33.9	1.710	351.2	1.712	82.2	159.5	0.069	+100.
2161	7.0		120.9	+32.8	0.807	211.5	0.616	26.0	107.0	0.048	+ 27.
2199	6.0	F	123.1	−12.2	1.029	161.2	0.939	48.0	143.0	0.109	+ 31.
2380	6.1	K	131.3	+28.8	0.537	242.9	0.287	31.2	105.6	0.085	+ 27.6
2404	3.4	A	132.7	+48.5	0.500	210.2	0.292	33.7	89.1	0.061	+ 6.
2413	4.4	F	133.1	+42.3	0.502	238.8	0.464	34.1	94.1	0.010	+ 27.3

* Mean.

TABLE I (*Continued*)

P.G.C. No.	Mag.	Type	1875.0 α	1875.0 δ	μ	θ	μ'	λ	d	π	ρ
			°	°	″	°	″	°	°		km
2469	7.7		136.5	+53.2	1.678	249.0	1.159	36.7	83.8	0.162	+ 11.
2552	3.3	F_8	141.1	+52.2	1.096	239.8	0.744	40.1	82.8	0.092	+ 15.7
2573	5.5	K	142.0	+36.4	0.749	249.3	0.614	40.4	94.6	0.038	+ 12.2
2632	3.9	F	145.5	+59.6	0.329	241.3	0.097	44.3	75.7	0.080	+ 28.
2681	5.6	F_2	148.4	+32.5	0.677	229.5	0.460	44.4	93.8	0.051	+ 55.9
2734	5.9	F_5	152.6	+23.7	0.421	255.1	0.197	47.3	97.3	0.092	+ 38.8
2742	2.6	K	153.3	+20.5	0.340	116.3	0.362	48.1	99.1	0.012	− 35.
2775	6.5	F_8	155.1	+49.5	0.898	174.4	0.761	49.6	78.9	0.078	− 8.
2806	7.6		156.5	+49.8	0.294	65.3	0.479	50.6	77.7	0.045	− 26.
2935	7.6	K	164.1	+36.8	4.779	186.7	3.828	53.3	81.3	0.403	− 87.
2943	7.5	G_0	164.6	+25.9	0.406	258.7	0.268	52.7	87.8	0.039	− 9.
2984*	4.9, 4.4	G	167.9	+32.2	0.730	215.2	0.242	51.6	81.5	0.174	− 16.
3069	6.3	F	173.0	+45.8	0.579	272.1	0.157	60.2	70.8	0.038	− 17.6
3101	2.2	A_2	175.7	+15.3	0.507	256.2	0.215	55.7	85.3	0.084	+ 1.
3105	3.8	F_8	176.0	+ 2.5	0.793	110.6	1.151	55.5	92.3	0.118	+ 1.9
3112	6.5	Gp	176.4	+38.6	7.047	145.3	7.096	60.0	72.2	0.102	− 97.
3145	6.8	G_4	179.0	+43.8	0.626	214.1	0.517	63.0	68.0	0.033	− 13.
3279	4.3	G	186.9	+42.0	0.756	291.8	0.567	66.6	63.6	0.093	+ 6.4
3307	3.7	F	188.8	− 0.8	0.561	270.6	0.375	55.4	83.5	0.061	− 20.0
3383	2.9	K	194.0	+11.6	0.270	274.1	0.180	57.5	72.4	0.034	− 13.2
3412	5.2	F_5	196.0	+18.2	0.447	286.1	0.360	59.6	67.3	0.119	− 16.8
3424	4.3	G	196.5	+28.5	1.184	317.7	1.140	64.1	62.1	0.116	+ 5.8
3448	4.8	K	198.0	−17.6	1.528	224.9	0.874	52.2	86.2	0.157	− 6.6
3558	4.5	F_5	205.3	+18.1	0.483	273.4	0.369	60.3	59.7	0.040	− 15.4
3650	5.5	F	212.0	+13.5	0.264	256.9	0.130	57.6	56.5	0.098	− 38.6
3662	0.2	K	212.5	+19.8	2.287	208.9	2.079	61.5	52.9	0.075	− 3.9
3735*	0.3, 1.7	G_1 K_5	217.8	−60.3	3.682	281.5	3.369	42.7	103.9	0.759	− 22.2
3813	5.8	Kp	222.5	−20.8	2.038	149.5	1.924	40.3	71.5	0.172	+ 20.1
3895	5.2	G	228.2	+ 2.2	0.644	144.9	0.663	44.8	50.3	0.094	+ 54.1
4042	4.6	F	237.1	+42.8	0.761	35.7	0.871	96.8	27.4	0.099	− 55.6
4055	3.9	F_8	237.7	+16.1	1.332	167.0	1.303	52.1	34.7	0.029	+ 7.3
4090	4.1	F_8	239.9	+58.9	0.458	317.9	0.312	127.1	31.9	0.066	− 8.4
4108	4.9	G_2	241.1	+36.8	0.325	350.2	0.352	87.2	24.1	0.105	− 12.3
4246	3.0	G	249.1	+31.8	0.601	310.1	0.578	56.4	18.0	0.144	− 70.
4370	5.3	K	256.9	−26.4	1.236	202.6	0.510	12.7	62.1	0.202	+ 1.2
4371	5.3	K	256.9	−26.4	1.225	203.8	0.507	12.7	62.1	0.202	+ 1.2
4372	6.7		257.1	−26.4	1.224	203.5	0.504	12.6	62.0	0.202	− 11.
4403	5.4	G	259.0	+32.6	1.060	173.3	1.076	91.0	10.1	0.123	− 78.1
4497	3.5	G_5	265.4	+27.8	0.817	203.6	0.750	34.6	7.9	0.116	− 15.6
4571	4.1	K	269.8	+ 2.5	1.131	167.2	0.771	1.2	31.8	0.168	− 7.4
4638	3.4	K	273.7	− 2.9	0.898	219.0	0.812	355.7	37.3	0.049	+ 9.5
4656	3.9	K	274.6	+21.7	0.326	143.2	0.299	345.0	13.1	0.029	− 58.0
4665	2.9	K	275.1	−25.5	0.196	193.3	0.090	355.1	60.0	0.069	− 43.1
4672	3.7	F_8	275.8	+72.7	0.641	125.6	0.821	187.1	39.0	0.110	+ 32.4
4722	0.1	A	278.2	+38.7	0.348	36.9	0.298	236.8	7.6	0.094	− 13.8
4753	4.3	F_5	280.1	+20.4	0.344	183.0	0.280	330.6	16.2	0.068	+ 22.6

* Center of mass of system.

TABLE I (*Continued*)

P. G. C. No.	Mag.	Type	1875.0 α	1875.0 δ	μ	θ	μ'	χ	d	π	ρ
											km
4845	5.2	G	283.4	+32.7	0.239	132.3	0.227	281.1	40.6	0.018	− 43.1
4892	6.8	K	287.2	+49.6	0.629	344.6	0.611	224.9	19.6	0.027	− 39.0
4950	5.2	G	289.8	+11.7	0.961	49.2	0.981	325.2	28.6	0.065	− 97.
4961	6.2	K	290.1	+24.7	0.658	196.2	0.638	304.1	19.5	0.056	− 5.0
5009	4.8	G	293.1	+69.4	1.839	162.8	2.207	211.4	37.3	0.197	+ 25.5
5014	4.6	F	293.3	+49.9	0.247	351.7	0.227	235.1	22.9	0.029	− 27.7
5037	6.4	F	294.6	+50.2	0.245	225.0	0.481	236.9	23.7	0.158	− 26.1
5054	5.0	F	295.4	+33.4	0.453	478.2	0.149	279.3	20.6	0.016	+ 5.5
5062	0.9	A	296.2	+ 8.5	0.659	55.0	0.779	321.3	34.9	0.190	− 33.
5144	5.7	K	299.6	+29.6	0.853	128.1	0.749	288.4	25.1	0.063	− 16.1
5146	5.9	G	299.6	+16.7	0.576	224.2	0.636	309.6	31.4	0.104	+ 3.1
5163	5.7	A	300.7	+52.8	0.335	11.7	0.232	241.2	28.5	0.056	− 41.
5180	5.7	G$_5$	301.9	−27.4	1.258	98.9	1.180	332.1	68.5	0.036	− 18.
5219	6.1	G	304.1	+66.5	0.559	58.8	0.266	228.4	37.7	0.118	− 6.1
5324	6.0	F	309.4	+60.1	0.185	2.2	0.168	242.0	36.0	0.047	− 11.6
5344	4.6	G	310.6	+57.1	0.242	195.6	0.253	247.0	35.6	0.007	− 32.0
5346	3.6	K	310.7	+61.4	0.826	7.0	0.714	242.0	37.2	0.097	− 87.0
5433	5.6	K	315.3	+38.4	5.252	52.0	4.748	277.9	36.1	0.311	− 63.
5455	4.6	F$_5$	317.1	+ 9.5	0.308	171.6	0.214	307.6	49.2	0.067	− 14.
5511	5.5	A	320.2	+46.2	0.208	78.0	0.053	270.4	38.9	0.087	+ 1.
5651	6.8	F$_7$	328.3	+29.2	0.539	225.3	0.654	291.6	48.7	0.067	+ 7.
5654	4.7	K$_0$	328.4	−57.3	4.695	123.7	3.568	313.9	103.7	0.284	− 38.7
5742	4.2	A$_0$	332.6	+56.4	0.455	84.8	0.128	269.1	46.9	0.107	− 0.2
5874	4.3	F$_5$	340.1	+11.5	0.548	155.3	0.319	302.5	66.8	0.089	− 4.6
5894	5.3	F	341.5	+ 9.2	0.519	85.5	0.370	303.3	69.2	0.056	+ 13.4
5916	4.3	A	342.7	−30.3	0.367	117.3	0.230	308.0	93.8	0.138	+ 6.7
5976	5.7	K	346.8	+56.5	2.102	81.9	1.593	280.4	54.8	0.157	− 20.4
6077	4.3	F$_2$	353.4	+ 4.9	0.578	139.7	0.167	304.1	81.3	0.148	+ 6.0

III

All the following computations are carried out to the third decimal for the values of the functions, and to a tenth of a degree for the angles.

First the position angle ϕ and the component R are found according to the formulae

$$\tan \phi = \frac{\pi r \sin d \sin \chi + C\mu \sin \theta}{\pi r \sin d \cos \chi + C\mu \cos \theta}$$

(11)

$$R = \frac{\pi r \sin d \sin (\theta - \chi)}{\sin (\theta - \phi)}$$

The proportional factor for separating $\tan \phi$ into sine and cosine is on the right hand $+\frac{1}{C\mu'}$, a positive quantity. Consequently the numerator gives the sign of $\sin \phi$ and the denominator that of $\cos \phi$. The component R is always positive. For values of $\sin (\theta - \phi) < 0.100$ a higher degree of precision must be used. The distance Δ and the velocity v are then obtained from the equations

$$v \cos \Delta = \rho' \qquad \tan \Delta = \frac{R}{\rho'}$$
$$v \sin \Delta = R \qquad v^2 = \rho'^2 + R^2 \qquad (12)$$

where

$$\rho' = \rho + r \cos d$$

Having determined the data for the star's apex, we can transform the relative coördinates ϕ, Δ to equatorial coördinates by means of

(13)

$$\begin{aligned}
\cos D' \cos(A' - \alpha) &= -\sin\delta \sin\Delta \cos\phi + \cos\delta \cos\Delta \\
\cos D' \sin(A' - \alpha) &= + \sin\Delta \sin\phi \\
\sin D' &= + \cos\delta \sin\Delta \cos\phi + \sin\delta \cos\Delta \\
\cos D' \cos p &= - \cos\Delta \cos\delta \cos\phi + \sin\Delta \sin\delta \\
\cos D' \sin p &= - \cos\delta \sin\phi
\end{aligned}$$

where p is the star's position-angle at the apex. For a check computation we may use the relation

$$-\sin^2\Delta \sin\phi \sin p = \sin^2(A' - \alpha) \cos D' \cos\delta$$

hence

$$\begin{aligned}
\cos D' \sin(A' - \alpha) &= + [\cos D' \sin(A' - \alpha)] \\
\cos\delta \sin(A' - \alpha) &= - \sin\Delta \sin p
\end{aligned}$$

Here I will add the formulæ for two star constants K and k defined by

$$K = \frac{\sin\Delta}{\pi} \frac{\sin(\phi - \chi)}{\sin(\theta - \chi)} = \frac{R}{\pi r} \frac{\sin(\phi - \chi)}{\sin(\theta - \chi)} \tag{14}$$

$$k = \frac{R}{r^2} = \frac{\sin\Delta}{r}$$

Both constants, significant in the further development, are always positive and bear the relationship

$$K\pi \sin(\theta - \chi) = kr \sin(\phi - \chi)$$

For all values $\pi \sin(\theta - \chi) < 0.020$ the constant K must be computed with a higher degree of precision.

All the elements computed on the basis of these formulæ are given in Table II.

TABLE II

$P.G.C.$ No.	ϕ	Δ	ρ'	R	r	A'	D'	p	K	k
			km	km	km					
6	313.5	109.9	− 2.2	6.1	6.5	228.3	+24.0	44.4	13.7	0.145
12	108.4	42.4	+ 22.4	20.5	30.4	49.0	+31.2	324.5	16.9	0.022
55	42.5	91.3	− 1.3	55.9	55.9	49.0	+19.0	197.2	8.0	0.018
74	78.2	78.7	+ 11.5	57.7	58.8	80.9	− 8.6	191.9	8.8	0.017
127	11.4	40.3	+ 11.	9.3	14.4	15.3	+14.1	190.6	3.5	0.045
168	110.2	31.1	+ 17.9	10.8	20.9	18.7	+38.5	319.4	6.9	0.025
171	167.8	123.9	− 13.8	20.6	24.8	170.7	−58.7	204.0	8.9	0.034
244	113.3	122.8	− 90.7	140.8	167.5	108.4	−39.4	316.1	8.5	0.005
259	113.6	61.4	+ 5.	9.2	10.4	69.2	− 0.8	311.3	76.9	0.084
264	311.0	35.0	+ 21.	14.7	25.6	307.2	+62.3	70.5	0.8	0.022
314	84.9	66.7	+ 14.	32.4	35.3	101.3	+22.5	326.9	45.8	0.026
313	270.7	60.5	+ 10.9	19.2	22.1	318.7	− 3.8	98.1	4.8	0.039
321	24.9	25.5	+ 15.3	7.3	17.0	46.4	+66.0	227.1	13.0	0.025
372	68.7	68.3	+ 7.6	19.1	20.5	110.2	+29.9	307.0	14.1	0.045
391	297.9	118.1	− 25.2	46.7	53.0	271.2	+32.0	92.6	1.7	0.017
394	103.1	74.1	+ 8.9	31.3	32.5	95.9	+ 8.4	333.7	24.7	0.030
477	100.9	166.2	− 17.1	1.2	17.6	195.1	−24.8	265.3	27.2	0.014
514	83.2	98.1	− 5.	34.0	34.4	133.1	+ 1.0	304.2	12.5	0.029
530	281.2	15.0	+ 52.7	14.1	54.6	18.6	− 0.6	101.7	1.0	0.005
588	37.3	79.7	+ 13.9	76.8	78.0	118.7	+52.9	266.9	6.9	0.013
710	72.7	40.1	+ 51.6	43.5	67.5	105.0	+44.7	298.5	7.1	0.010
764	74.6	19.2	+ 70.9	82.2	108.6	98.7	−17.7	227.2	5.3	0.007
814	290.7	85.8	+ 1.9	25.6	25.7	330.0	+19.5	102.1	1.6	0.039
825	229.8	64.9	+ 15.9	33.9	37.4	354.2	−35.8	70.2	14.2	0.024
848	337.2	122.5	− 20.5	32.1	38.1	274.1	+59.3	48.4	3.3	0.022

TABLE II (*Continued*)

P.G.C. No.	ε	Δ	ρ'	R	r	A'	D'	p	K	k
			km	km	km					
984	218.7	118.0	− 57.6	108.4	122.8	287.1	−38.2	127.9	4.6	0.007
1077	323.1	4.2	+ 13.7	3.2	13.8	64.5	+19.6	142.3	4.2	0.002
1216	168.1	25.9	+ 27.1	13.2	30.1	82.4	+20.4	351.2	16.6	0.015
1259	119.1	21.8	+ 61.1	21.6	66.1	99.0	+27.3	311.2	6.0	0.006
1565	230.5	26.1	+ 12.2	6.0	13.6	71.6	+11.5	13.3	28.4	0.032
1732	202.9	158.9	− 26.2	10.1	28.1	288.0	− 3.0	158.0	1.5	0.013
1809	159.1	71.3	+ 11.9	12.2	43.9	130.7	−40.5	336.0	19.5	0.022
1952	29.6	105.0	− 11.	40.9	42.1	254.6	+35.1	329.2	11.1	0.023
1979	318.6	110.5	− 4.4	11.8	12.5	334.6	+24.0	37.8	22.6	0.075
2023	96.4	17.8	+ 38.0	12.2	39.9	133.6	+25.8	285.6	8.1	0.008
2031	288.0	95.5	− 1.0	41.3	41.5	10.7	+13.1	59.3	17.4	0.024
2053	354.6	55.5	+ 81.	118.0	143.1	110.4	+21.1	175.2	11.9	0.006
2161	217.2	70.7	+ 21.	60.0	63.6	81.1	−26.9	34.7	25.8	0.015
2199	148.9	69.9	+ 15.	40.9	43.6	194.2	−59.1	279.9	9.4	0.022
2380	278.2	35.8	+ 22.2	16.0	27.4	90.5	+27.6	78.3	12.9	0.021
2404	263.3	75.2	+ 6.	22.8	23.5	57.5	+ 6.7	41.5	27.1	0.041
2413	211.0	83.2	+ 25.9	217.2	218.7	68.4	−16.1	42.3	107.5	0.005
2469	267.3	69.0	+ 13.	33.8	36.2	61.5	+15.1	38.3	8.3	0.026
2552	249.8	64.7	+ 18.2	38.4	42.6	82.1	+ 8.4	35.6	14.5	0.021
2573	256.5	82.2	+ 10.6	77.0	77.8	66.3	− 6.1	51.9	31.8	0.013
2632	321.8	9.9	+ 33.	5.7	33.5	130.1	+66.7	128.0	7.3	0.005
2681	231.9	37.8	+ 54.6	42.4	69.1	119.4	+ 6.1	41.8	17.7	0.009
2734	321.1	15.6	+ 36.3	10.1	37.7	140.7	+35.3	135.2	6.2	0.007
2742	108.9	104.9	− 38.	142.4	147.1	251.5	−22.6	286.3	75.7	0.007
2775	153.9	95.0	− 4.	46.0	46.2	190.2	−40.4	338.0	15.1	0.022
2806	59.6	113.8	− 22.	49.9	54.6	284.4	− 0.6	326.2	12.5	0.017
2935	168.1	151.8	− 84.	45.1	95.3	331.2	−63.9	202.2	1.5	0.005
2943	274.3	103.8	− 8.	32.6	33.5	60.3	− 2.2	63.9	37.7	0.029
2984	135.6	153.2	− 13.	6.6	14.6	319.5	−48.4	243.3	7.8	0.031
3069	282.2	100.9	− 11.0	57.0	58.0	66.7	+ 0.5	12.9	32.7	0.017
3101	291.3	76.1	+ 3.	12.1	12.5	94.4	+23.8	79.2	27.2	0.078
3105	89.9	81.9	+ 4.1	16.3	46.5	261.0	+ 0.3	272.4	5.8	0.021
3112	142.0	105.1	− 91.	329.7	342.0	242.2	−49.4	312.3	9.4	0.003
3145	207.2	91.6	− 6.	71.6	74.9	139.7	−14.1	27.3	36.6	0.013
3279	317.9	62.1	+ 15.3	28.9	32.7	88.2	+53.1	56.2	12.7	0.027
3307	294.9	122.5	− 17.7	27.8	32.9	63.5	+20.4	75.3	19.7	0.026
3383	301.0	105.8	− 7.1	25.1	26.1	80.0	+25.5	68.4	42.5	0.037
3412	355.3	122.5	− 9.1	14.3	17.0	21.1	+39.2	5.8	8.8	0.050
3424	339.1	71.9	+ 15.2	46.5	48.9	82.6	+68.2	57.8	8.5	0.019
3448	219.4	101.3	− 5.3	26.5	27.0	74.2	−41.5	126.1	10.9	0.036
3558	285.9	96.9	5.3	43.7	44.0	103.6	+12.8	69.7	32.5	0.023
3650	15.3	167.2	27.6	6.3	28.3	28.6	− 1.1	345.1	4.6	0.008
3662	205.1	86.4	+ 8.2	129.7	129.9	163.3	−56.0	15.6	14.6	0.008
3735	333.6	142.1	27.0	21.0	31.2	145.6	+73.4	129.8	0.9	0.018
3813	129.7	63.4	+ 26.5	52.8	59.1	295.1	−43.9	266.0	5.5	0.015
3895	117.9	26.5	+ 66.9	33.1	74.8	251.9	−10.0	296.3	4.6	0.006

TABLE II *(Continued)*

P G.C. No.	φ	Δ	ρ'	R	r	A'	D'	p	K	k
	°		km	km	km	°	°			
4042	46.9	132.3	− 37.8	41.5	56.1	24.3	− 5.0	327.5	6.5	0.013
4055	164.2	83.6	+ 23.7	211.5	212.8	273.8	−62.6	325.3	35.0	0.005
4090	323.0	68.9	+ 8.6	22.3	23.9	111.1	+43.8	25.5	20.7	0.039
4108	20.9	69.3	+ 6.0	15.9	17.0	6.8	+65.6	316.0	8.2	0.055
4246	328.3	159.5	− 51.	19.0	54.4	80.0	−14.0	27.4	2.5	0.006
4370	217.3	48.5	+ 10.6	12.0	16.0	202.8	−55.9	75.7	9.0	0.047
4371	220.5	48.3	+ 10.6	11.9	15.9	202.3	−53.5	78.3	8.9	0.047
4372	219.9	99.6	− 2.	11.8	12.0	129.6	−37.1	133.8	11.8	0.082
4403	168.5	144.7	− 58.7	41.6	71.9	62.2	−66.4	205.0	4.6	0.008
4497	202.6	81.9	+ 1.2	30.1	30.4	230.7	−48.0	30.6	9.3	0.033
4571	160.4	66.1	+ 9.6	21.6	23.6	304.4	−57.4	321.6	8.0	0.039
4638	225.1	72.1	+ 25.4	78.4	82.5	205.8	−43.3	76.6	21.5	0.012
4656	141.2	128.6	− 38.5	48.2	61.7	40.5	−52.9	254.3	29.3	0.013
4665	312.4	169.4	− 33.1	6.2	33.7	104.3	+32.3	52.1	5.8	0.005
4672	143.8	36.5	+ 47.9	35.5	59.6	302.5	+38.7	347.0	4.2	0.010
4722	33.4	67.9	+ 6.0	11.8	16.0	28.7	+57.1	307.9	11.5	0.058
4753	191.8	25.1	+ 41.8	19.5	46.1	275.1	− 4.2	11.1	7.7	0.009
4815	134.1	111.3	− 23.7	60.7	65.2	9.2	−47.8	295.8	54.4	0.014
4892	341.5	100.6	− 20.2	107.8	109.7	127.9	+27.6	13.4	37.5	0.009
4950	41.6	137.6	− 79.	72.1	107.0	81.3	+20.1	316.1	10.1	0.006
4961	203.0	75.6	+ 13.8	53.9	55.6	257.8	−45.0	30.1	17.8	0.017
5009	172.7	51.9	+ 41.4	52.8	67.1	299.2	+17.7	357.3	3.3	0.012
5014	344.2	104.0	− 9.3	37.3	38.4	130.2	+24.6	11.1	36.3	0.025
5037	231.6	118.4	− 7.8	14.4	16.4	214.1	−45.6	45.9	2.5	0.054
5051	181.2	79.6	+ 24.2	132.0	134.2	293.6	−46.2	1.5	62.0	0.007
5062	18.9	131.3	− 17.	19.4	25.8	98.3	+37.3	336.2	3.3	0.029
5144	131.0	116.6	− 28.3	56.5	63.2	38.1	−47.0	285.7	16.2	0.014
5146	245.2	55.2	+ 20.2	29.0	35.4	250.5	− 9.5	61.8	7.1	0.023
5163	32.3	139.6	− 23.	19.5	30.2	99.6	−16.0	340.3	16.7	0.021
5180	93.4	104.7	− 41.	156.1	161.4	46.5	+ 3.8	242.7	28.7	0.006
5219	70.8	47.6	+ 9.7	10.6	14.4	30.4	+45.7	327.3	13.2	0.051
5321	349.6	84.4	+ 4.6	46.6	46.8	142.1	+35.0	6.3	64.6	0.021
5344	198.6	95.2	− 15.7	173.8	174.5	287.5	−36.0	12.4	136.1	0.006
5346	350.5	153.9	− 71.1	34.9	79.2	135.8	−35.5	5.7	5.3	0.006
5433	45.3	123.0	− 47.	72.3	86.2	98.4	+ 7.3	325.7	3.0	0.010
5455	215.6	93.8	− 1.	15.1	15.2	233.8	−54.2	79.0	21.4	0.066
5511	328.6	9.5	+ 17.	2.9	17.2	311.7	+54.1	142.1	7.5	0.010
5651	242.6	66.7	+ 20.	46.4	50.5	272.4	−10.1	51.9	11.3	0.018
5654	120.4	126.0	− 43.4	59.8	73.9	101.9	+15.9	209.0	3.7	0.011
5742	73.7	22.8	+ 13.5	5.7	14.6	14.1	+55.9	288.7	12.8	0.026
5874	191.2	79.0	+ 3.3	17.0	17.3	313.3	−64.9	26.7	19.0	0.057
5894	64.0	56.7	+ 20.5	31.2	37.4	38.7	+26.6	263.3	21.0	0.022
5916	325.2	55.7	+ 5.1	7.9	9.6	313.0	+17.6	148.8	9.5	0.086
5976	75.7	100.5	− 8.9	48.0	48.8	94.4	− 1.0	327.7	8.3	0.020
6077	235.6	30.7	+ 9.0	5.3	10.5	327.9	−12.3	57.3	11.9	0.049

The graphical representation of the points A', D' shows a decided distribution along the galaxy, and a large area of avoidance. A secondary distribution, not as strikingly conspicuous follows a great circle very steeply inclined to the galaxy. The two curves define two velocity-planes. In the figure, 1 and 2 represent the respective poles of the two planes as found in Section VI. A mean of vertices of preferential motion derived from average proper-motions is represented by 3, while the mean vertex derived from large proper-motions is represented by 4. They lie close to a node of the two great circles, surrounded by a large accumulation of apices. The assumed solar apex is represented by 5.

IV

The problem now arises to determine whether a given star belongs to an assumed velocity-plane. Let the equatorial coördinates of the pole of the assumed plane be γ and κ. Then we can transform the equatorial coordinates A', D' to the coördinates G, H of the velocity-plane, taking the ascending node of this plane on the equator as origin and reckoning increasing G in the sense of right-ascensions. The formulæ of transformation are

$$\begin{aligned} a_2 = \cos H \sin G &= -\sin\kappa \cos D' \cos(A'-\gamma) + \cos\kappa \sin D' \\ a_1 = \cos H \cos G &= + \cos D' \sin(A'-\gamma) \\ a_3 = \sin H &= +\cos\kappa \cos D' \cos(A'-\gamma) + \sin\kappa \sin D' \\ \cos H \cos q &= -\sin D' \cos\kappa \cos(A'-\gamma) + \cos D' \sin\kappa \\ \cos H \sin q &= + \cos\kappa \sin(A'-\gamma) \end{aligned} \quad (15)$$

where q is the position-angle of the pole γ, κ at the apex, q is to be reckoned in a direct sense. For a check computation we may use the relation

$$\sin G \cos H \cos\kappa = \cos D' \sin(A'-\gamma) \sin q$$

hence

$$\cos H \cos G = [\cos H \cos G]$$
$$\cos\kappa \cos G = \cos D' \sin q$$

As a condition that the apex $A'D'$ shall lie in the velocity-plane we have

$$\sin H = \cos\kappa\cos D'\cos(A'-\gamma) + \sin\kappa\sin D' = 0$$

This condition may be satisfied or not. If $\sin H$ differs from zero, three cases arise:

1. The computed coördinates A', D' are erroneous in consequence of the unavoidable errors in the elements of observation.

2. Instead of the real pole (γ, κ) an approximate pole (G_0, H_0) has been employed.

3. The star in question does not belong to the assumed plane.

The problem only requires a consideration of the first two cases.

In the first case assuming dA' and dD' as corrections of the computed coördinates, the following conditional equation is formed

$$\sin H + d\sin H = 0$$

or

$$\sin H - \cos H\sin q\cos D'\,dA' + \cos H\cos q\,dD' = 0$$

In order to avoid complications, I will consider here only the effect of errors in the measured parallax and radial velocity.

Differentiating the fundamental formulæ with respect to the star's apex and to the elements π and ρ', the following differential formulæ are derived

$$\pi R d\phi = -v_{\odot}\sin d\sin(\phi-\chi)\,d\pi$$
$$dR = +R\cot(\theta-\phi)\,d\phi$$

and

$$r d\Delta = +\cos\Delta\,dR - \sin\Delta\,d\rho'$$
$$dr = +\sin\Delta\,dR + \cos\Delta\,d\rho'$$

Arranged for computation these give

$$\pi R d\phi = -v_{\odot}\sin d\sin(\phi-\chi)\,d\pi \qquad (16)$$
$$r^2 d\Delta = -C\mu\frac{\rho'}{\pi^2}\cos(\theta-\phi)\,d\pi - R d\rho'$$

These formulæ give the effect of errors in π and ρ' upon the relative coördinates ϕ, Δ of the star's apex. As we are assuming true values of the solar motion $d\rho$ may be used for $d\rho'$. On the other hand if we turn to the transformation of the relative coördinates ϕ, Δ into the equatorial coördinates A', D' we have the differential formulæ

$$\cos D'\,dA' = -\cos p\sin\Delta\,d\phi - \sin p\,d\Delta \qquad (17)$$
$$dD' = +\sin p\sin\Delta\,d\phi - \cos p\,d\Delta$$

and inversely

$$\sin\Delta\,d\phi = -\cos p\cos D'dA' + \sin p\,dD' \qquad (18)$$
$$d\Delta = -\sin p\cos D'dA' - \cos p\,dD'$$

The substitution of the expressions for $d\phi$ and $d\Delta$ from equations (16) into equations (17) gives, in terms of the star constants K and k defined by (14)

$$\cos D'\,dA' = K\{\sin p\cos(\theta-\phi)\cos\Delta + \cos p\sin(\theta-\phi)\}\,d\pi + k\sin p\,d\rho'$$

$$dD' = K\{\cos p\cos(\theta-\phi)\cos\Delta - \sin p\sin(\theta-\phi)\}\,d\pi + k\cos p\,d\rho'$$

These relations can be simplified by introducing a spherical triangle with the following elements

$(a) = \Delta$	$(\alpha) = \epsilon$
$(b) = h$	$(\beta) = 90° - (\theta-\phi)$
(c)	$(\gamma) = 90° - p$

Then we have for determining ϵ and h the equations

(19)

$$\sin\epsilon\cos h = +\sin p\cos(\theta-\phi)\cos\Delta + \cos p\sin(\theta-\phi)$$
$$\sin\epsilon\sin h = +\cos(\theta-\phi)\sin\Delta$$
$$\cos\epsilon = +\cos p\cos(\theta-\phi)\cos\Delta - \sin p\sin(\theta-\phi)$$

Hence the differential formulæ may be written in the form

$$\cos D'\,dA' = K\sin\epsilon\cos h\,d\pi + k\sin p\,d\rho' \qquad (20)$$
$$dD' = K\cos\epsilon\,d\pi + k\cos p\,d\rho'$$

They show the effect of errors in π and ρ' upon the equatorial coördinates of the apex.

Returning to the equation of condition for the first case, substitute the expressions of (20), considering $d\pi$ and $d\rho'$ as corrections of the observed values. Putting

$$\cos\zeta = \cos q\cos\epsilon - \sin q\sin\epsilon\cos h$$
$$= \cos(p+q)\cos(\theta-\phi)\cos\Delta - \sin(p+q)\sin(\theta-\phi) \qquad (21)$$

where the double expression is useful for a check computation, we get

$$\tan H + K \cos \zeta \, d\pi + k \cos (p + q) \, d\rho' = 0$$

or the simple form

(22) $$\tan H + \Pi \, d\pi + P \, d\rho' = 0$$

where the coefficients

(23) $$\Pi = K \cos \zeta \qquad P = k \cos (p + q)$$

are known functions of the observed data. From equations (22) we are now able to decide, whether the correction $d\pi$, required to fit the apex to the assumed velocity-plane, is reasonable or not. P is small in comparison with the coefficient Π. We conclude from our given star material an average value of about 0.02 for P, and an average proportion $P : \Pi$ of about 1 : 150. Solving with regard to the correction $d\pi$

$$d\pi = -\frac{\tan H}{\Pi} - \frac{P}{\Pi} d\rho'$$

and putting

(24) $$(\partial\pi)_0 = -\frac{\tan H}{\Pi} \qquad (\partial\pi)_1 = -\frac{P}{\Pi}$$

we have

(25) $$d\pi = (\partial\pi)_0 + (\partial\pi)_1 \, d\rho'$$

$(\partial\pi)_0$ is the value of $d\pi$ for $d\rho' = 0$, and $(\partial\pi)_1$ its variation for $d\rho' = +1$ km. The practical illustration of this criterion will be given in Section VI in connection with the following developments.

In the second case we will assume instead of γ and κ the approximate values G_0 and H_0 which need the corrections $d\gamma$ and $d\kappa$, so that

$$\gamma = G_0 + d\gamma \qquad \kappa = H_0 + d\kappa$$

For all the stars belonging to the plane (γ, κ) we have the equation of condition

$$E = \sin H + \cos H \cos G \cos H_0 \, d\gamma + \cos H \sin G \, d\kappa = 0$$

or in terms of (15)

(26) $$E = a_0 + a_1 \cos H_0 \, d\gamma + a_2 \, d\kappa = 0$$

A least square solution determines $\cos H_0 \, d\gamma$ and $d\kappa$.

V

In the first case it is supposed that the elements γ and κ are known; and in the second case a group of stars belonging to a given plane of velocity. Both conditions are not realized in practice. They are interdependent. However, rough elements of a given plane can be derived for use in formula (25). After having selected a group of stars in this way, the solution of the equation (26) will lead to a better approximation. This process might be repeated with the corrected elements, but it is possible to derive the criterion (25) using the corrections $d\gamma$ and $d\kappa$, when they may be considered as differentials. This method is desirable if the progressive measurement of parallaxes and radial velocities will produce an increase of a group. Then the part of the computation depending on the approximate values G_0 and H_0 is settled once for all and needs only a supplementary computation depending on the corrections $d\gamma$ and $d\kappa$.

Consider in the equation $E = 0$ the corrections $d\gamma$ and $d\kappa$ as given constants, and the coördinates G, H as variable with regard to π and ρ'. The new equation of condition for the corrected values is

$$E + dE = 0$$

and it remains to express dE in convenient form for practical use. We have

$$dE = d \sin H + \cos H_0 \, d\gamma \, d(\cos H \cos G) + d\kappa \, d(\cos H \sin G)$$

The coördinates G, H and their differentials must be transformed into the corresponding equatorial elements. Introducing the three constants L, M and N given by

$$\begin{aligned} L &= \cos H_0 \, d\gamma \\ M &= \cos H_0 \, d\kappa + \sin H_0 \\ N &= \sin H_0 \, d\kappa - \cos H_0 \end{aligned} \qquad (27)$$

with the reciprocal connection

$$\begin{aligned} M \sin H_0 - N \cos H_0 &= 1 \\ M \cos H_0 + N \sin H_0 &= d\kappa \end{aligned}$$

it is found that

$$\begin{aligned} dE = &+ \{L \cos (A' - G_0) + N \sin (A' - G_0)\} \cos D' dA' \\ &- \{L \sin D' \sin (A' - G_0) - M \cos D' \\ &\qquad - N \sin D' \cos (A' - G_0)\} \, dD' \end{aligned}$$

On substituting from (20) the corrections $d\pi$ and $d\rho'$ we find as factor of $K\,d\pi$

$$-L\,\{\sin(A'-G_0)\cos\epsilon\sin D' - \cos(A'-G_0)\sin\epsilon\cos h\}$$
$$+M\cos\epsilon\cos D'$$
$$+N\,\{\cos(A'-G_0)\cos\epsilon\sin D' + \sin(A'-G_0)\sin\epsilon\cos h\}$$

and as factor of $k\,d\rho'$

$$-L\,\{\sin(A'-G_0)\cos p\sin D' - \cos(A'-G_0)\sin p\}$$
$$+M\cos p\cos D'$$
$$+N\,\{\cos(A'-G_0)\cos p\sin D' + \sin(A'-G_0)\sin p\}$$

These expressions may be considered in connection with the following two spherical triangles

$$\begin{array}{llll} (a) & = 90^\circ - D' & (\alpha) & = \lambda_1 \\ (b) & = \mu_1 & (\beta) & = 90^\circ - \epsilon \\ (c) & & (\gamma) & = 90^\circ + (A' - G_0) \end{array}$$

and

$$\begin{array}{llll} (a) & = 90^\circ - D' & (\alpha) & = \lambda_2 \\ (b) & = \mu_2 & (\beta) & = 90^\circ - p \\ (c) & & (\gamma) & = 90^\circ + (A' - G_0) \end{array}$$

From them we derive the relations

$$(28)\quad \begin{aligned} \sin\lambda_1\cos\mu_1 &= -\sin(A'-G_0)\cos\epsilon\sin D' + \cos(A'-G_0)\sin\epsilon \\ \sin\lambda_1\sin\mu_1 &= +\cos\epsilon\cos D' \\ \cos\lambda_1 &= +\cos(A'-G_0)\cos\epsilon\sin D' + \sin(A'-G_0)\sin\epsilon \\[1ex] \sin\lambda_2\cos\mu_2 &= -\sin(A'-G_0)\cos p\sin D' + \cos(A'-G_0)\sin p \\ \sin\lambda_2\sin\mu_2 &= +\cos p\cos D' \\ \cos\lambda_2 &= +\cos(A'-G_0)\cos p\sin D' + \sin(A'-G_0)\sin p \end{aligned}$$

and compare them with the expressions given above, replacing $\cos h$ by $1-2\sin^2 {}^1\!/_2\, h$. These formulæ, of which we only desire the expressions on the left hand, are independent of the given corrections $d\gamma$ and $d\kappa$. Putting

(29)

$$Q_1 = L\sin\lambda_1\cos\mu_1 + M\sin\lambda_1\sin\mu_1 + N\cos\lambda_1$$
$$Q_2 = L\sin\lambda_2\cos\mu_2 + M\sin\lambda_2\sin\mu_2 + N\cos\lambda_2$$
$$S = \cos(\theta-\phi)\sin\Delta\tan{}^1\!/_2\,h\,\{L\cos(A'-G_0) + N\sin(A'-G_0)\}$$

we can express dE in the simple form

$$dE = K\,(Q_1 - S)\,d\pi + k\,Q_2\,d\rho'$$

The equation of condition for the combined case is therefore

$$(30)\qquad E + \Pi' d\pi + P' d\rho' = 0$$

where the coefficients

$$(31)\qquad \Pi' = K\,(Q_1 - S) \qquad P' = kQ_2$$

are known functions of the given data. They are, with the exception of the factor $E/\tan H$, the same as the coefficients Π and P computed with the corrected values $G_0 + d\gamma$ and $H_0 + d\kappa$. The criterion for establishing the membership of a given apex in a given plane is

$$d\pi = (\partial\pi)_0 + (\partial\pi)_1\,d\rho'$$
$$(\partial\pi)_0 = -\frac{E}{\Pi'} \qquad (\partial\pi)_1 = -\frac{P'}{\Pi'} \qquad (32)$$

In the formulæ (19), which determine ϵ and h, h is to be taken less than 180°. Therefore $\tan{}^1\!/_2 h$ is always positive. For values of ${}^1\!/_2 h > 80^\circ$ the expression S must be computed with a higher degree of precision.

VI

Now I shall turn to the investigation of the computed apices. A rough estimate of the poles of two velocity-planes may be drawn from the figure. The fact that the one plane of velocity coincides with the Milky Way suggests the use of NEWCOMB's coördinates of the pole of the galaxy* for the first plane. The uncertainty in locating the pole of the second plane is considerably greater. A position convenient for quick computation is therefore taken. The assumed poles are

*S. NEWCOMB, Contributions to Stellar Statistics. (*Carnegie Institution of Washington*. Pub. No. 10).

Plane I $G_0 = 193$ $H_0 = +27$
Plane II $G_0 = 165°$ $H_0 = -45$

We are now able to obtain a first approximation, free from the graphical representation, by means of the criterion (25). This requires an estimate of the admissible errors in π and ρ'. The value ± 5 km. will be taken as the limiting value of the correction $d\rho'$. This is certainly too high. The error in the absolute parallax is questionable, so I have preferred to determine the limits from the star material itself. Assuming that all the apices in question belong to one or the other of the two planes we may assign every apex to that plane requiring the smaller value of $(\partial\pi)_0$. An average value of $d\pi$ can now be determined (I will call it the material correction of π) assuming the weight of $(\partial\pi)_0$ as in inverse proportion to the influence of $d\rho'$, that is to $(\partial\pi)_1$. In the formula

$$\text{weight} = \frac{1}{10\,(\partial\pi)_1}$$

whole numbers from 1 to 100 were used with the restriction that every $(\partial\pi)_1 = 0''.000$ gives the weight 100, and every $(\partial\pi)_1 > 0''.200$ the weight 1. These weights gave an approximate material correction of

$$d\pi = 0''.070 + 0.009\, d\rho'$$

With an assumed maximum $d\rho'$ of 5 km, we may expect a maximum $d\pi$ of $0''.115$. In order to allow for several stars with a large proper-motion this value was extended to $0''.165$. Being only a first approximation this seemed admissible. In deriving the final result strict regard was maintained not to exceed the calculated $d\pi$.

With the weight as defined above, the first approximation free from graphical adjustment was made.

Plane I $G_0 = 194$ $H_0 = +34°$
Plane II $G_0 = 152$ $H_0 = -50°$

With these values of the poles the coefficients H and P were computed. Table III exhibits the data necessary for criterion (25).

TABLE III

P.G.C. No.	tan H	$(\partial\pi)_0$	$(\partial\pi)_1$	tan H	$(\partial\pi)_0$	$(\partial\pi)_1$	Distributed to Plane
6	+1.634	+ 0.295	−0.008	−0.176	− 0.044	−0.036	II
12	−0.304	+ 0.057	−0.002	−0.610	− 0.050	−0.002	I, II
55	−0.518	− 1.715	−0.056	−0.418	− 0.255	+0.005	
74	−0.443	− 0.276	−0.007	+0.338	− 0.529	−0.016	
127	−0.898	+22.13	+1.091	−0.835	− 0.397	+0.015	
168	−0.188	+ 0.072	−0.003	−0.737	− 0.129	−0.004	I
171	−0.083	+ 0.019	+0.008	+3.989	+ 1.526	−0.002	I
244	−0.321	+ 0.255	+0.004	+1.587	− 0.350	+0.004	
259	−0.549	+ 0.102	+0.003	+0.091	+ 0.002	−0.002	I, II
264	+0.365	− 1.250	−0.148	−3.042	−123.14	−0.577	
314	+0.184	− 0.017	+0.000	+0.083	+ 0.005	−0.004	I, II
343	0.590	+ 0.378	+0.022	−0.703	− 0.155	−0.000	
321	+0.234	− 0.087	+0.009	−1.203	− 0.103	−0.004	I, II
372	+0.384	− 0.980	+0.004	+0.033	+ 0.005	−0.005	I, II
394	+0.507	0.657	+0.019	−0.907	+ 1.614	+0.021	
394	0.034	− 0.007	0.001	+0.251	+ 0.041	−0.005	I, II
477	+0.607	0.041	+0.000	+1.124	+ 0.041	+0.000	I, II
514	+0.452	0.104	+0.002	−0.710	+ 0.560	−0.017	I
530	1.506	1.474	0.000	0.482	− 0.550	+0.003	
588	+0.699	0.749	0.013	0.300	0.157	−0.002	
740	+0.440	+ 0.445	0.006	0.233	− 0.042	−0.001	II
764	0.249	0.073	0.002	+0.750	− 19.26	−0.022	I
814	0.403	− 1.556	+0.446	1.693	− 3.293	+0.012	
825	3.428	1.053	0.007	0.035	0.005	−0.000	II

TABLE III (*Continued*)

P.G.C. No.	tan H	$(d\pi)_0$	$(d\pi)_1$	tan H	$(d\pi)_0$	$(d\pi)_1$	Distributed to Plane
848	+0.665	− 1.973	+0.039	−1.506	+ 1.501	+0.019	
984	−0.411	+ 0.203	+0.003	+0.116	+ 0.131	+0.001	
1077	−0.325	+ 0.174	+0.000	−0.237	− 0.061	+0.001	II
1246	−0.091	+ 0.012	−0.001	−0.057	− 0.004	−0.001	I, II
1259	+0.197	− 0.277	+0.003	−0.008	− 0.001	−0.001	II
1565	−0.343	+ 0.018	−0.002	−0.047	− 0.002	−0.000	I, II
1732	−0.087	+ 0.070	+0.009	−0.466	− 0.508	+0.009	I
1809	−0.080	+ 0.017	−0.001	+3.110	+ 0.977	−0.006	I
1952	+0.865	+ 0.257	+0.005	−0.667	− 0.496	−0.003	
1979	−0.382	+ 0.018	−0.001	−2.011	− 0.146	−0.004	I
2023	+0.774	− 0.314	+0.002	+0.222	+ 0.028	−0.000	II
2031	−0.925	+ 0.170	−0.002	−0.883	− 0.127	+0.000	
2053	+0.303	+ 0.132	−0.001	+0.170	− 0.026	−0.001	II
2161	−0.642	+ 0.096	−0.002	+0.632	+ 0.247	−0.001	I
2199	−0.054	+ 0.018	−0.001	+0.479	− 0.174	−0.001	I
2380	+0.088	− 0.008	−0.002	−0.083	− 0.023	+0.002	I, II
2404	−0.628	+ 0.101	−0.007	−0.140	− 0.012	−0.001	I, II
2413	−0.786	+ 0.072	−0.000	+0.292	+ 0.054	−0.000	I, II
2469	−0.430	+ 0.161	−0.010	−0.210	− 0.068	−0.002	II
2552	−0.230	+ 0.045	−0.004	+0.107	+ 0.026	−0.001	I, II
2573	−0.681	+ 0.161	−0.003	+0.129	+ 0.034	+0.000	II
2632	+0.871	− 0.191	−0.001	−0.528	− 0.161	+0.001	
2681	+0.290	− 0.022	−0.001	+0.514	− 0.078	−0.001	I, II
2734	+1.062	− 0.172	−0.001	+0.072	+ 0.047	+0.004	II
2742	+0.200	+ 0.016	+0.000	+0.200	+ 0.012	+0.000	I, II
2775	+0.278	− 0.246	−0.018	+1.862	+ 0.375	−0.001	
2806	−0.012	− 0.002	+0.003	−0.470	+ 0.493	+0.007	I
2935	−1.207	− 1.874	+0.006	+0.443	+ 0.337	+0.004	
2943	−0.736	− 0.132	+0.005	+0.010	+ 0.001	+0.001	II
2984	−1.094	+ 0.446	−0.011	+0.158	+ 0.022	+0.003	II
3069	−0.573	− 0.085	+0.003	+0.046	+ 0.009	−0.000	I, II
3101	+0.100	− 0.009	−0.006	+0.007	+ 0.001	+0.002	I, II
3105	+0.346	− 0.226	−0.012	−0.220	− 0.159	+0.009	
3112	−0.064	− 0.027	+0.001	+0.712	+ 0.744	+0.000	I
3145	−0.041	− 0.025	+0.008	+5.525	− 1.128	+0.001	I
3279	+0.328	− 0.063	−0.005	−0.494	− 0.099	+0.001	I, II
3307	−0.325	− 0.093	+0.006	−0.259	− 0.022	+0.001	I, II
3383	−0.064	− 0.018	+0.010	−0.151	− 0.007	+0.000	I, II
3412	−0.296	− 0.170	+0.029	−1.376	− 0.213	−0.004	I
3424	+0.444	− 0.179	−0.008	−0.805	− 0.373	+0.003	
3448	−0.925	− 0.420	+0.011	+0.768	+ 0.365	+0.017	
3558	+0.119	+ 0.056	+0.010	+0.255	+ 0.034	+0.001	I, II
3650	−1.396	− 2.164	+0.009	−0.360	− 0.106	−0.002	II
3662	−0.064	+ 0.121	−0.013	+6.398	+ 5.903	−0.001	I
3735	+0.961	+ 2.947	+0.057	−0.660	− 0.832	+0.016	
3813	−0.580	+ 0.202	−0.005	+0.162	+ 0.268	+0.012	

TABLE III (*Continued*)

P.G.C. No.	tan *H*	$(\partial\pi)_0$	$(\partial\pi)_1$	tan *H*	$(\partial\pi)_0$	$(\partial\pi)_1$	Distributed to Plane
		"			"		
3895	+0.357	− 0.087	−0.001	+0.024	+ 0.020	+0.001	I, II
4042	−1.701	− 0.451	+0.002	−0.344	+ 0.080	+0.003	II
4055	−0.474	+ 0.188	−0.002	+0.645	+ 0.231	−0.000	
4090	+0.519	− 0.122	−0.007	−0.184	− 0.030	−0.004	I, II
4108	+0.170	+ 0.280	+0.071	−2.283	− 0.476	−0.004	I
4246	−0.521	− 0.224	+0.002	+0.408	− 5.014	+0.029	
4370	−0.004	+ 0.001	−0.002	+1.693	+ 0.303	−0.008	I
4371	+0.039	− 0.013	−0.001	+1.693	+ 0.293	−0.008	I
4372	−0.051	+ 0.016	−0.002	+2.682	+ 1.331	+0.041	I
4403	−1.081	− 2.994	−0.000	+0.988	+ 2.428	+0.000	
4497	+0.030	− 0.059	−0.031	+0.862	+ 0.692	−0.023	I
4571	−0.805	+ 0.543	−0.018	+0.359	+ 0.140	−0.008	II
4638	+0.210	− 0.079	−0.000	+1.338	+ 0.192	−0.002	I
4656	−1.996	+ 0.183	+0.001	+0.530	− 0.072	+0.001	II
4665	+0.317	− 0.159	−0.003	−0.044	− 0.008	−0.000	II
4672	+0.445	− 0.044	−0.002	−2.283	− 12.72	−0.024	I
4722	+0.034	− 0.011	−0.010	−1.518	− 0.422	−0.015	I
4753	+0.087	− 0.023	−0.001	−0.308	− 0.053	−0.001	I, II
4815	−3.857	+ 0.788	+0.000	+0.230	− 0.014	+0.001	II
4892	−0.671	+ 0.260	+0.002	+0.167	− 0.024	+0.001	II
4950	−0.409	+ 0.058	+0.000	−0.064	+ 0.009	+0.001	I, II
4961	−0.137	+ 0.047	−0.001	+0.460	+ 0.120	−0.004	I, II
5009	−0.037	+ 0.021	−0.004	−1.130	− 0.838	−0.007	I
5014	+0.687	− 4.786	−0.105	+0.230	− 0.022	+0.002	II
5037	+0.145	+ 0.502	+0.086	+1.162	+ 0.960	+0.043	I
5051	−0.574	+ 0.088	−0.000	+0.207	+ 0.023	−0.001	I, II
5062	+0.285	− 0.164	−0.002	−0.163	+ 0.080	+0.014	II
5141	−2.434	+ 0.335	+0.002	+0.415	− 0.065	+0.002	II
5146	+0.385	− 0.341	+0.007	+0.032	+ 0.007	−0.005	II
5163	−0.221	− 7.300	+0.152	+0.727	− 0.057	+0.002	II
5180	−0.881	+ 0.114	+0.001	−0.229	− 0.195	−0.001	
5219	0.157	− 0.039	−0.009	−1.259	− 0.141	−0.005	I
5321	+1.097	− 0.071	−0.000	+0.079	+ 0.027	−0.007	I, II
5344	−0.398	+ 0.108	−0.000	+0.079	− 0.006	+0.000	II
5346	+0.031	+ 0.012	+0.002	+2.948	− 0.664	+0.001	I
5433	−0.009	+ 0.026	−0.000	+0.292	− 0.182	+0.006	I
5455	−0.080	+ 0.006	+0.002	+0.915	+ 0.229	−0.016	I
5511	−0.233	− 0.062	+0.002	−4.216	+ 0.637	+0.001	I
5651	+0.066	− 0.033	+0.003	−0.188	− 0.037	−0.003	I, II
5654	+0.124	0.067	−0.005	+0.189	+ 0.143	+0.005	I
5742	−0.002	+ 0.000	−0.001	−2.089	− 0.196	−0.002	I
5874	−0.922	+ 0.129	+0.005	+0.183	+ 0.088	−0.010	II
5894	−0.468	+ 0.315	−0.007	−0.696	− 0.055	−0.001	II
5916	−0.219	+ 0.038	+0.011	−1.381	+ 2.227	−0.085	I
5976	−0.150	− 0.178	−0.001	+0.385	− 0.250	+0.013	
6077	−0.928	+ 0.081	+0.002	−0.522	− 0.058	−0.003	I, II

Distributing every apex to that plane requiring the smaller value of $(\partial\pi)_0$, the material correction is deduced. Star No. 4403 was rejected, being of full weight, and exerting an undue influence on the results. We find

$$\Sigma p = 6611 \qquad\qquad n = 115$$
$$\Sigma p\,(\partial\pi)_0 = 363''.197 \qquad \Sigma\,(\partial\pi)_1 = 0.903$$
$$(\partial\pi)_0 = 0''.055 \qquad\qquad (\partial\pi)_1 = 0.008$$

Hence we obtain as material correction

$$d\pi = 0''.055 + 0.008\,d\rho'$$

A $d\pi$ of $0''.1$ will therefore serve as a limit in selecting the stars. Table III contains the distribution of the stars as determined by the new elements. It is found that of the 116 stars, 62 belong to Plane I, and 57 to Plane II; 30 are common to both planes, and the remaining 27 belong to neither.

It is evident that the 30 stars common to both planes have to be neglected in solving the two groups of equations formed according to (26). In this solution the weight was limited to whole numbers from 1 to 10. The result is

Plane I : 32 stars

$$+5.206 + 0.802\cos H_0\,d\gamma - 2.942\,d\kappa = [pvv]$$
$$+0.802 + 54.521\cos H_0\,d\gamma + 15.285\,d\kappa = 0$$
$$-2.942 + 15.285\cos H_0\,d\gamma + 87.192\,d\kappa = 0$$

$$\cos H_0\,d\gamma = -0.0254 \pm 0.0571 \qquad d\kappa = +0.0382 \pm 0.0452$$
$$d\gamma = -4^\circ.76 \pm 3^\circ.95 \qquad\qquad = +2^\circ.19 \pm 2^\circ.59$$

Plane II : 27 stars

$$+12.552 + 1.325\cos H_0\,d\gamma - 3.166\,d\kappa = [pvv]$$
$$+1.325 + 80.691\cos H_0\,d\gamma - 21.002\,d\kappa = 0$$
$$-3.166 - 21.002\cos H_0\,d\gamma + 86.811\,d\kappa = 0$$

$$\cos H_0\,d\gamma = -0.0074 \pm 0.0811 \qquad d\kappa = +0.0347 \pm 0.0782$$
$$d\gamma = -0^\circ.66 \pm 7^\circ.23 \qquad\qquad = +1^\circ.99 \pm 4^\circ.48$$

where the given error is the mean error.

We will stop with this second approximation and consider these corrections as differentials, by means of which we deduce the final result according to the directions given in Section V. Computing the correction E and the coefficients W' and P' we have all the data for making use of the criterion (32). These data are given in Table IV.

TABLE IV

P.G.C. No.	E	$(\partial\pi)_0$	$(\partial\pi)_1$	E	$(\partial\pi)_0$	$(\partial\pi)_1$	Distributed to Plane
		''			''		
6	+0.837	+ 0.282	−0.007	−0.165	− 0.040	−0.035	II
12	−0.248	+ 0.048	−0.002	−0.509	− 0.049	−0.002	I, II
55	−0.419	− 1.462	−0.055	−0.377	− 0.242	+0.005	
74	−0.378	− 0.251	−0.007	+0.332	− 0.611	−0.017	
127	−0.639	−11.38	−0.611	−0.649	− 0.381	+0.014	
168	−0.141	+ 0.055	−0.004	−0.578	− 0.125	−0.004	I
171	−0.115	+ 0.027	+0.008	+0.963	+ 1.575	−0.003	I
244	−0.307	+ 0.311	+0.001	+0.851	− 0.360	+0.001	
259	−0.448	+ 0.078	+0.002	+0.101	+ 0.003	−0.002	I, II
264	+0.364	+ 3.379	−0.104	−0.943	−35.28	−0.161	
314	+0.214	− 0.019	−0.000	+0.112	+ 0.006	−0.001	I, II
313	−0.519	+ 0.339	+0.018	−0.604	− 0.166	−0.000	
321	+0.267	− 0.115	+0.010	−0.749	− 0.097	−0.001	I
372	+0.392	− 0.086	+0.004	+0.065	+ 0.009	−0.006	I, II
391	+0.444	− 0.647	+0.020	−0.676	+ 1.541	+0.021	
394	−0.001	+ 0.000	−0.001	+0.267	+ 0.046	−0.005	I, II
477	+0.486	− 0.042	+0.000	+0.750	+ 0.044	+0.000	I, II
514	+0.424	− 0.108	+0.002	+0.623	+ 0.610	−0.017	
530	−0.810	+ 1.349	+0.000	−0.447	− 0.583	+0.004	
588	+0.610	+ 0.841	−0.014	−0.254	− 0.137	−0.002	
710	+0.443	+ 0.555	−0.006	−0.194	− 0.035	−0.001	II

TABLE IV (*Continued*)

P.G.C. No.	E	$(\vartheta\pi)_0$	$(\vartheta\pi)_1$	E	$(\vartheta\pi)_0$	$(\vartheta\pi)_2$	Distributed to Plane
					″		
764	−0.226	− 0.067	−0.002	+0.614	+ 5.814	+0.000	I
814	−0.366	+ 1.387	+0.101	−0.879	− 3.636	+0.013	
825	−0.969	− 1.705	−0.010	−0.066	− 0.010	−0.000	II
848	+0.566	− 2.859	+0.052	−0.824	+ 1.601	+0.020	
984	−0.419	+ 0.225	+0.003	+0.082	+ 0.095	+0.001	II
1077	−0.268	+ 0.145	+0.000	−0.215	− 0.057	+0.001	II
1246	−0.051	+ 0.006	−0.001	−0.035	− 0.003	−0.001	I, II
1259	+0.231	− 0.284	+0.002	+0.021	+ 0.004	−0.001	II
1565	−0.285	+ 0.015	−0.002	−0.031	− 0.002	−0.000	I, II
1732	0.112	+ 0.088	+0.009	−0.417	− 0.559	+0.009	I
1809	−0.091	+ 0.018	−0.002	+0.958	+ 1.257	−0.007	I
1952	+0.646	+ 0.249	+0.005	−0.553	− 0.440	−0.002	
1979	−0.344	+ 0.018	−0.001	−0.913	− 0.157	−0.004	I
2023	+0.636	− 0.293	+0.002	+0.251	+ 0.032	−0.000	II
2031	−0.652	+ 0.163	−0.002	−0.673	− 0.130	+0.000	
2053	+0.322	+ 0.136	−0.001	+0.200	− 0.031	−0.001	II
2161	−0.526	+ 0.088	−0.002	+0.538	+ 0.285	−0.001	I
2199	−0.092	+ 0.030	+0.000	+0.890	− 0.131	−0.002	I
2380	+0.129	− 0.012	−0.002	−0.055	− 0.016	+0.003	I, II
2404	−0.496	+ 0.089	−0.006	−0.131	− 0.012	−0.000	I, II
2413	−0.595	+ 0.065	−0.000	+0.284	+ 0.060	−0.000	I, II
2469	−0.355	+ 0.139	−0.009	−0.193	− 0.065	−0.001	II
2552	−0.188	+ 0.036	−0.004	+0.125	+ 0.031	−0.001	I, II
2573	−0.533	+ 0.146	−0.003	+0.135	+ 0.037	+0.000	II
2632	+0.691	− 0.212	−0.001	−0.436	− 0.147	+0.001	
2681	+0.301	− 0.023	−0.001	+0.486	+ 0.087	−0.001	I, II
2734	+0.753	− 0.184	−0.000	+0.107	+ 0.069	+0.004	II
2742	+0.153	+ 0.013	+0.000	+0.176	+ 0.010	+0.000	I, II
2775	+0.232	− 0.240	−0.020	+0.879	+ 0.362	−0.000	
2806	−0.037	− 0.007	+0.003	−0.448	+ 0.595	+0.008	I
2935	−0.800	− 2.031	+0.006	+0.373	+ 0.306	+0.003	
2943	−0.561	− 0.126	+0.005	+0.015	+ 0.001	+0.001	II
2984	0.767	+ 0.456	−0.010	+0.121	+ 0.017	+0.003	II
3069	−0.464	− 0.080	+0.003	+0.055	+ 0.011	−0.000	I, II
3101	+0.139	− 0.012	−0.006	+0.035	+ 0.003	+0.003	I, II
3105	+0.296	− 0.194	−0.011	−0.231	− 0.178	+0.010	
3112	−0.109	− 0.046	+0.001	+0.558	+ 0.647	+0.001	I
3145	−0.057	− 0.030	+0.007	+0.988	− 1.351	+0.000	I
3279	+0.356	− 0.072	−0.005	−0.414	− 0.093	+0.001	I, II
3307	−0.267	− 0.082	+0.006	−0.236	− 0.021	+0.001	I, II
3383	−0.022	− 0.007	+0.011	−0.126	− 0.006	+0.001	I, II
3412	−0.246	− 0.132	+0.026	−0.804	− 0.200	−0.003	I
3424	+0.447	− 0.205	−0.008	−0.600	− 0.350	+0.003	
3448	−0.675	− 0.121	+0.011	+0.603	+ 0.350	+0.017	
3558	+0.150	+ 0.080	+0.012	+0.274	+ 0.038	+0.001	I, II
3650	−0.787	− 1.816	+0.009	−0.348	− 0.106	−0.002	
3662	0.094	+ 0.194	−0.015	+0.983	+ 5.161	−0.000	

TABLE IV (*Continued*)

P.G.C. No.	E	$(d\pi)_0$	$(d\pi)_1$	E	$(d\pi)_0$	$(d\pi)_1$	Distributed to Plane
3735	+0.724	+ 3.239	+0.055	−0.522	− 0.769	+0.016	
3813	−0.539	+ 0.222	−0.005	+0.126	+ 0.218	+0.012	
3895	+0.298	− 0.076	−0.001	+0.009	+ 0.008	+0.002	I, II
4042	−0.839	− 0.413	+0.002	−0.337	+ 0.081	+0.003	II
4055	−0.470	+ 0.203	−0.002	+0.495	+ 0.218	−0.000	
4090	+0.499	− 0.133	−0.007	−0.148	− 0.025	−0.004	II
4108	+0.207	+ 0.283	+0.057	−0.903	− 0.438	−0.003	I
4246	−0.438	− 0.207	+0.002	+0.388	+51.32	−0.257	
4370	−0.044	+ 0.015	−0.002	+0.849	+ 0.295	−0.009	I
4371	−0.001	+ 0.000	−0.001	+0.850	+ 0.283	−0.008	I
4372	−0.059	+ 0.018	−0.001	+0.945	+ 1.627	+0.047	I
4403	−0.749	− 8.514	−0.004	+0.686	+ 3.367	−0.001	
4497	−0.015	+ 0.031	−0.033	+0.635	+ 0.657	−0.023	I
4571	−0.663	+ 0.590	−0.018	+0.305	+ 0.126	−0.008	II
4638	+0.165	− 0.066	−0.000	+0.793	+ 0.187	−0.002	I
4656	−0.901	+ 0.166	+0.001	+0.448	− 0.064	+0.001	II
4665	+0.340	− 0.169	−0.002	−0.012	− 0.002	−0.000	II
4672	+0.150	− 0.045	−0.002	−0.923	+21.87	+0.032	I
4722	+0.076	− 0.031	−0.010	−0.821	− 0.412	−0.015	I
4753	+0.056	− 0.014	−0.001	−0.316	− 0.058	−0.001	I, II
4815	−0.978	+ 0.749	+0.001	+0.196	− 0.012	+0.001	II
4892	+0.585	+ 0.252	+0.002	+0.200	− 0.029	+0.001	II
4950	−0.067	+ 0.039	+0.000	−0.041	+ 0.006	+0.001	I, II
4961	−0.181	+ 0.063	−0.001	+0.392	+ 0.115	−0.005	I
5009	−0.045	+ 0.025	−0.004	−0.768	− 0.926	−0.007	I
5014	+0.592	+ 2.717	+0.959	+0.258	− 0.025	+0.002	II
5037	+0.101	+ 0.379	+0.090	+0.746	− 0.933	+0.043	I
5051	−0.535	+ 0.095	−0.000	+0.169	+ 0.019	−0.001	II
5062	+0.315	− 0.195	−0.002	−0.130	+ 0.063	+0.014	II
5144	−0.928	+ 0.341	+0.002	+0.364	− 0.060	+0.002	II
5146	+0.321	− 0.276	+0.006	+0.017	+ 0.004	−0.005	II
5163	−0.199	− 0.340	+0.009	+0.603	− 0.059	+0.002	II
5180	−0.627	+ 0.104	+0.001	−0.222	− 0.258	−0.002	
5219	−0.113	+ 0.029	−0.009	−0.773	− 0.138	−0.005	I
5321	+0.762	− 0.077	−0.001	+0.114	+ 0.039	−0.007	I, II
5344	−0.408	+ 0.136	−0.000	+0.047	− 0.003	+0.000	II
5346	+0.021	+ 0.008	+0.002	+0.957	− 0.760	+0.001	I
5433	+0.022	− 0.075	−0.001	+0.304	− 0.198	+0.006	I
5455	−0.124	+ 0.009	+0.002	+0.655	+ 0.241	−0.017	I
5511	+0.245	− 0.072	+0.003	−0.971	+ 0.677	+0.001	I
5651	+0.032	− 0.015	+0.002	−0.208	− 0.042	−0.003	I, II
5654	+0.156	− 0.084	+0.005	+0.213	+ 0.171	+0.004	I
5742	+0.036	− 0.006	−0.001	−0.892	− 0.192	−0.002	I
5874	−0.711	+ 0.143	+0.005	+0.403	+ 0.081	−0.010	II
5894	−0.383	+ 0.290	+0.008	−0.564	− 0.053	−0.001	II
5916	−0.216	+ 0.039	+0.015	−0.829	+ 1.975	−0.068	I
5976	−0.120	− 2.078	+0.024	+0.379	− 0.262	+0.013	
6077	−0.690	+ 0.082	+0.001	−0.495	− 0.063	−0.003	I, II

Proceeding as before we first compute the material correction. Once more neglecting star No. 4105 and taking the mean in all doubtful cases, we find

$$\Sigma p = 6558 \qquad n = 115$$
$$\Sigma p(\partial\pi)_0 = 383''.071 \qquad \Sigma(\partial\pi)_1 = 0.813$$
$$(\partial\pi)_0 = 0''.058 \qquad (\partial\pi)_1 = 0.007$$

from which we derive as material correction

$$d\pi = 0''.058 + 0.007\, d\rho'$$

Hereafter we will rigidly adhere to a maximum value for $d\pi$ of $0''.093$. The distribution of the stars founded on this value is found in Table IV. A comparison with the distribution given in Table III shows a change in 8 stars. Of the total 116, 58 belong to Plane I and 55 to Plane II; the number common to both is 26, while 29 belong to neither plane.

The solution of the two groups of conditional equations, rejecting the 26 stars common to both and weighting every equation strictly according to the rule given above, leads to the following result.

Plane I : 32 stars

$$\begin{aligned} +4.408 + 1.803 \cos H_0\, d\gamma - 0.181\, d\kappa &= [pvv] \\ +1.803 + 51.239 \cos H_0\, d\gamma + 7.014\, d\kappa &= 0 \\ -0.181 + 7.014 \cos H_0\, d\gamma + 93.350\, d\kappa &= 0 \end{aligned}$$

$$\cos H_0\, d\gamma = -0.0338 \pm 0.0519 \qquad d\kappa = +0.0045 \pm 0.0396$$
$$d\gamma = 2^\circ.31 \pm 3^\circ.59 \qquad = +0^\circ.26 \pm 2^\circ.27$$

Plane II : 29 stars

$$\begin{aligned} +12.845 + 2.777 \cos H_0\, d\gamma - 7.193\, d\kappa &= [pvv] \\ +2.777 + 86.865 \cos H_0\, d\gamma - 32.962\, d\kappa &= 0 \\ -7.193 - 32.962 \cos H_0\, d\gamma + 102.128\, d\kappa &= 0 \end{aligned}$$

$$\cos H_0\, d\gamma = -0.0060 \pm 0.0774 \qquad d\kappa = +0.0685 \pm 0.0714$$
$$d\gamma = -0^\circ.53 \pm 6^\circ.90 \qquad = +3^\circ.92 \pm 4^\circ.09$$

This solution may be considered as the definitive determination of the two velocity planes as deduced from the given list of 116 stars. The elements are therefore

$$\text{Plane I} \quad \gamma = 191^\circ.66 \qquad \kappa = +34^\circ.26$$
$$\text{Plane II} \quad \gamma = 151^\circ.47 \qquad \kappa = -16^\circ.08$$

The curves in the diagram correspond to these two poles.

Dudley Observatory, August, 1916.

VII

The mean errors of the final result are only slightly better than those of the second approximation, and about what might be expected from such a solution. Considering the small number of stars the determination of the velocity-planes is remarkably good. Three-quarters of the stars treated belong to one of the two planes. One node of the two planes lies close to the vertices of preferential motion for average and large proper-motion stars. It is interesting to note that apparently the stars belong nearly in equal number to the two planes, while from the graphical representation we might expect a smaller membership in Plane II.

The two planes are almost perpendicular. If we distinguish them by the indices 1 and 2, we have as a condition of perpendicularity

$$\tan \kappa_1 + \cot \kappa_2 \cos(\gamma_2 - \gamma_1) = 0$$

The value of this expression is

for the 2nd approximation	+ 0.0512
for the final result	− 0.0545

so that the final result nearly realizes this condition. The suggestion of a second plane of stellar distribution was pointed out by B. Boss at a meeting of the American Astronomical Society held in Pittsburgh, 1912. This statement was founded on a discussion of the distribution according to the Harvard Photometry. A full discussion will soon appear. The two phenomena bear a marked resemblance.

The mean values of magnitude, proper-motion, parallax and velocity for the two groups of stars are

	n	Mag.	μ	μ'	π	v km.
Plane I	32	4.7	1''.276	1''.046	0''.134	52.0
Plane II	29	4.8	0 .740	0 .614	0 .083	57.1

If we deduce the material correction from the two groups we find

$$\text{Plane I} \quad d\pi = 0''.038 + 0.010\, d\rho'$$
$$\text{Plane II} \quad d\pi = 0''.035 + 0.004\, d\rho'$$

We may therefore state that the value $0''.036$ represents the absolute error of an observed parallax, affected by the errors of proper-motion, solar motion, and by the perturbations of the apices.

Further investigation of the subject is in progress.

CONTENTS.

Editor, Benjamin Boss, Albany, N. Y.; Associate Editors, E. E. Barnard, Ernest W. Brown, F. R. Moulton and R. S. Woodward
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MICROMETRIC MEASURES OF DOUBLE STARS,

MADE AT THE YERKES OBSERVATORY,

By F. P. LEAVENWORTH

The following measures of double stars were made with the 40-inch and 12-inch refractors of the Yerkes Observatory during the summer of 1916. They are a continuation of the work done with the same instruments during the summer of 1914 as published in the *Astronomical Journal* No. 675.

Two observing lists were prepared: one of close and difficult doubles for nights of good definition. The second list for poorer nights was made up mostly of stars discovered by BURNHAM with the 6-inch refractor.

There were 25 nights on which the 40-inch was used. On one-half of these nights, stars were measured of 0″.35 separation or less. Therefore 0″.35 may be taken to represent the average condition of the nights for the whole series with the 40-inch.

Of the 112 double and multiple stars measured, 41 have a separation under 0″.5 and 10 have a separation of 0″.25 or less.

The numbers printed to the left are the numbers in BURNHAM'S *General Catalogue*.

The right ascensions are for 1880.

The magnitudes are the means of the estimates for each night.

In the fourth column is given the estimated value of the measure on a scale with 5 as perfect.

The fifth column gives the magnifying power.

The sixth column gives the aperture of the object-glass, as reduced by the iris diaphragm. When the number 12 occurs, the 12-inch refractor was used.

I am greatly indebted to Director EDWIN B. FROST for the privileges of the Observatory, which have made these measures possible.

The efficient aid of Mr. F. R. SULLIVAN at the 40-inch materially increased the number of measures.

61 β 255
0h 6m +27° 45′
8.0 8.7

	°	″			in.
.689	90.1	0.43	3	400	12
.700	89.5		2		40
1916.69	89.8	0.43			

Slow retrograde motion probable.

86 β 1027
0h 9m +20° 53′
7.5 9.8

.628	188.0	1.58	3	700	40
.637	187.6	1.68	4	940	40
1916.63	187.8	1.63			

No motion.

150 β 1015
0h 14m +11° 39′
8.5 8.8

	°	″			in
.618	136.0	0.36	2	1340	40
.628	141.0	0.39	3	940	40
.637	140.4	0.38	3	940	40
1916.63	139.1	0.38			

Angle increasing.

153 β 1093
0h 15m +10° 19′
7.5 8.7

.628	73.2	0.47	2	700	40
.637	79.2	0.54	2	940	40
.689	77.9	0.46	2	400	12
1916.65	76.8	0.49			

Angle increasing slowly.

208 β 1225
$0^h 21^m$ $+20° 26'$
8.2 11.2

					m
1916.628	189.4	1.28	3	700	40
.637	190.7	1.24	3	940	40
1916.63	190.0	1.26			

No motion.

276 β 1310
$0^h 26^m$ $+22° 32'$
AB and *C* 7.0 14.0

.642	296.9	16.74	2	420	40
.661	297.3	16.46	2	420	40
1916.65	297.1	16.60			

AB and *D*

.642	146.6		1	420	24
.661	146.3	96.01	2	420	40
1916.65	146.4	96.01			

314 Ho 212
$0^h 29^m$ $-4° 15'$
A and *B*

1916.618	286.2	0.27	2	940	40

Comparison with AITKEN's ephemeris, *Lick Bulletin*, No. 110.

$O - C$ $+12°.1$ $-0''.03$

AB and *C*

1916.618	46.5	..	1	940	40

728 Lv 1
$1^h 17^m$ $+1° 7'$
9.4 9.7

.618	168.9	0.66	2	700	40
.689	168.7	0.73	2	400	12
1916.65	168.8	0.70			

No motion.

926 Σ 158
$1^h 10^m$ $+32° 34'$
8.2 8.6

					m
1916.694	256.2	2.18	2	460	40
.700	258.6	2.11	2	460	40
1916.70	257.4	2.14			

Slow increase in angle.

956 β 1016
$1^h 13^m$ $+32° 29'$
8.5 9.0

1916.694	197.9	0.69	2	460	40

1070 OΣ 38
$1^h 57^m$ $+41° 45'$
A and *BC*

.694	62.7	10.24	2	460	25
.700	64.1	10.04	2	460	40
1916.70	63.4	10.14			

B and *C*

.694	111.0	0.56	1	460	25
.700	115.2	0.64	2	460	40
1916.70	113.1	0.60			

Comparison with HUSSEY's Ephemeris in *Lick Publications*, Vol. V.

$O - C$ $+3°.9$ $0''.00$

7211 Σ 1932
$15^h 13^m$ $+27° 16'$
6.2 6.4

.522	3.6	0.68	2	460	40
.612	11.8	0.59	2	400	12
.634	7.7	0.59	1	400	12
1916.59	7.7	0.62			

Binary.

7222 β 32
$15^h 15^m$ $+1° 9'$
5.0 10.0

1916.502	16.6	2.60	3	160	40
.634	16.7	2.80	1	400	12
1916.57	16.6	2.70			

No motion.

7293 β 33 and β 34
$15^h 25^m$ $-12° 35'$
A and *B* 8.0 10.0

1916.502	40.6	3.05	2	160	40

No motion.

C and *D* = β 34
11.0 11.1

1916.502	55.6	6.36	4	160	40

No motion.

A and *E* 8.0 12.5

1916.502	312.0	28.85	4	160	40

7487 Σ 1998
$15^h 58^m$ $-11° 3'$
A and *B* 5.0 5.2

.502	171.4	1.02	2	700	40
.522	171.3	0.91	2	460	18
.579	173.8	0.85	4	940	40
1916.53	172.1	0.93			

Comparison with AITKEN's Ephemeris in *Lick Publications*, Vol. XII:

O − C 0°.0 −0″.05

AB and *C* . . . 7.5

.502	59.7	7.68	2	700	40
.522	63.4	7.27	2	460	24
.689	60.9	7.23	2	400	12
1916.57	61.3	7.39			

7506 β 949
$16^h 2^m$ $-9° 47'$
7.5 8.0

.579	194.2	0.23	3	940	40
.637	196.4	0.17	3	1340	40
1916.61	195.3	0.20			

7763 β 43
$16^h 12^m$ $+2° 57'$
9.0 9.2

1916.522	213.3		1	160	40
.541	212.0	1.03	2	160	40
1916.53	212.6	1.03			

No motion.

7778 Σ 2106
$16^h 45^m$ $+9° 37'$
6.8 8.2

.560	286.0	0.22	2	1340	40
.579	280.5	0.27	3	940	40
.637	283.8	0.25	3	940	40
1916.59	283.1	0.25			

Angle and distance diminishing. Probably binary.

7885 β 1118
$17^h 3^m$ $-15° 34'$
4.0 4.6

.560	238.6	0.65	3	700	40
.574	241.8	0.51	3	400	12
.618	242.6	0.53	2	460	40
1916.58	241.0	0.56			

Angle decreasing.

7887 β 124
$17^h 4^m$ $-0° 36'$
7.3 10.0

.541	260.7	1.01	1	700	40
.634	266.0	1.12	3	400	12
.670	265.5	0.96	3	700	40
1916.62	264.1	1.03			

No motion.

7991 β 46
$17^h 18^m$ $+13° 31'$
8.0 10.9

.508	204.3	2.01	2	700	35
.513	205.0	1.99	3	700	40
1916.51	204.6	2.02			

No motion.

8011 β 129

$17^h\ 21^m$ $-25°\ 21'$

8.0 8.3

1916.689	105.0	0.85	2	400	12^m

Measures discordant.

8038 Σ 2173

$17^h\ 24^m$ $-0°\ 58'$

6.0 6.1

.574	159.6	0.59	2	400	12
.598	158.4	0.65	3	940	40
1916.59	159.0	0.62			

Comparison with Lohse's Ephemeris. *Potsdam Publications*, Vol. XX.

$O - C$ $+1°.4$ $-0''.24$

Aitken's elements make the difference in distance a little smaller.

8100 β 634

$17^h\ 34^m$ $-0°\ 35'$

7.4 7.2

.513	58.0	0.44	3	940	40
.527	55.9	0.42	3	940	40
1916.52	57.0	0.43			

Angle slowly decreasing.

8288 β 47

$17^h\ 55^m$ $-10°\ 14'$

8.7 10.5

1916.670	280.9	1.38	3	460	40

No motion.

8355 β 243

$18^h\ 1^m$ $-22°\ 17'$

8.0 8.5

.648	122.5		1	400	12
.689	122.5	0.66	2	400	12
1916.67	122.5	0.66			

Probably unchanged.

8356 β 244

$18^h\ 1^m$ $-27°\ 53'$

8.0 9.3

1916.689	260.0	2.50	2	400	12^m

8370 OΣ 524

$18^h 2^m$ $+19°\ 39'$

7.0 8.2

.579	38.6	0.20	3	1340	40
.598	37.2	0.19	2	1340	40
1916.59	37.9	0.20			

Decrease in angle and distance. Should be measured occasionally during next few years.

8371 β 245

$18^h\ 2^m$ $-30°\ 45'$

6.0 8.8

.626	352.0	3.68	2	250	12
.631	353.2	3.76	2	250	12
1916.63	352.6	3.72			

No motion.

8380 Σ 2281

$18^h\ 4^m$ $+3°\ 58'$

6.3 7.2

.513	77.6	0.48	3	940	40
.527	79.0	0.48	3	940	40
.541	74.9	0.45	2	940	40
1916.53	77.2	0.47			

8390 β 132

$18^h\ 4^m$ $-19°\ 52'$

7.0 7.0

.574	214.6	0.92	3	400	12
.623	214.9	0.88	2	400	12
1916.60	214.8	0.90			

Angle decreasing 0°.6 a year.

8403 Lv 7

18h 6m −15° 23′

8.5 11.5

	°				m.
1916.494	278.2	3.87	4	460	40
.546	280.8	3.86	4	460	40
1916.52	279.5	3.86			

No motion.

8414 β 131

18h 7m −15° 38′

A and *B* 8.0 10.0

.494	280.9	2.90	4	460	40
.546	279.7	2.94	3	460	40
1916.52	280.3	2.92			

No motion.

A and *C* 11.5

.494	285.4	7.88	3		40
.546	285.5	7.92	3	460	40
1916.52	285.3	7.90			

8433 Σ 2294

18h 8m +0° 9′

8.0 8.2

.513	100.3	0.47	4	940	40
.527	99.6	0.54	4	940	40
.541	98.5	0.55	2	940	40
1916.53	99.5	0.52			

All three measures placed angle in the second quadrant

8456 β 246

18h 11m −19° 13′

8.4 8.1

.574	107.5	0.44	2	400	12
.623	104.8	...	1	400	12
.634	96.0	0.53	3	400	12
.689	101.2	0.18	2	400	12
1916.63	102.4	0.48			

Measures discordant; no motion.

8488 β 48

18h 14m −19° 43′

8.0 10.3

					m.
1916.574	356.3	2.12	3	400	12
.623	357.5	2.00	2	400	12
.626	356.8	[1.48]	1	400	12
1916.61	356.9	2.06			

No motion.

8520 β 49

18h 17m −19° 38′

A and *B* 8.0 11.0

.574	44.8	8.43	3	400	12
.623	44.6	7.98	3	400	12
.689	44.7	8.36	2	400	12
1916.63	44.7	8.26			

No motion.

A and *C* 12.5

1916.689	302.6	23.16	1	400	12

Distance doubtful.

A and *D* ... 12.0

1916.689	149.7	24.10	2	400	12

No motion.

8535 A. Clark 11

18h 19m −4° 39′

7.0 7.2

.513	180.9	0.29	3	940	40
.526	179.2	0.26	3	940	40
1916.52	180.0	0.28			

8543 β 1203

18h 20m +0° 43′

7.7 7.9

.513	90.9	0.34	2	940	40
.527	88.6	0.31	2	940	40
.541	86.8	0.33	2	940	40
1916.53	88.8	0.33			

8617 β 247

$18^h\ 26^m$ $-9°\ 27'$

8.0 10.7

	°	"			m.
1916.670	167.2	7.83	1	460	40
.672	165.8	8.03	2	400	12
.683	167.7	7.84	2	400	12
1916.68	166.9	7.90			

No motion.

8655 Barnard 10

$18^h\ 30^m$ $-12°\ 5'$

9.0 10.5

1916.637	134.6	0.43	3	1340	40

Increase in distance.

8670 β 135

$18^h\ 31^m$ $-14°\ 6'$

7.0 12.0

.552	185.8	2.29	4	460	40
.628	186.6	2.29	4	460	40
1916.59	186.2	2.29			

No motion.

A 15 mag. star 65°, 20″.

8679 A 88

$18^h\ 32^m$ $-3°\ 18'$

7.0 7.3

.628	245.9	0.16	2	1340	40
.637	240.0	0.16	2	1340	40
1916.63	243.0	0.16			

Comparison with Aitken's orbit in *Lick Publications*, Vol. XII:

$$O-C \qquad -11°.3 \qquad +0''.05$$

AB and *C*

1916.628	114.1	....	3	111	40

8710 β 50

$18^h\ 34^m$ $+39°\ 29'$

A and *B* 8.3 12.7

	°	"			m.
1916.502	6.4	22.34	3	460	40
.522	6.5	21.67	3	460	40
.533	6.1	21.62	3	460	40
1916.52	6.3	21.88			

No motion.

A and *C* 9.2

.502	329.3	73.21	2	460	40
.522	329.4	72.79	2	460	40
.533	329.1	73.41	3	460	40
1916.52	329.3	73.14			

No motion.

C and *D* ... 11.8

.502	164.9	6.10	3	460	40
.522	165.2	6.04	2	460	40
.533	166.7	6.05	2	460	40
1916.52	165.6	6.06			

No motion.

8736 Σ 2367

$18^h\ 37^m$ $+30°\ 11'$

A and *B* 7.0 7.5

.560	70.9	0.31	2	1340	40
.598	72.8	0.33	3	1340	40
.631	70.0		2	400	12
1916.60	71.2	0.32			

AB and *C* ... 8.3

.612	193.2	14.71	2	400	12
.623	192.7	14.27	3	250	12
.631	193.0	14.44	3	400	12
1916.62	193.0	14.47			

No motion.

13127 A 858
$18^h\ 37^m$ $-0°\ 22'$
9.0 15.0

	°	″			m.
1916.637	322.4	1.17	3	950	10

No change in twelve years.

13128 A 859
$18^h\ 39^m$ $-0°\ 21'$
8.5 9.0

1916.637	15.5	0.26	3	1340	30

No change in twelve years.

8801 β 51
$18^h\ 42^m$ $+39°\ 34'$
A and *B* 9.0 10.5

.502	185.6	74.07	3	460	40
.522	184.7	73.38	2	460	40
1916.51	185.2	73.72			

B and *C* . . . 11.2

.502	296.7	6.26	2	460	40
.522	296.1	6.70	2	460	40
1916.51	296.4	6.48			

No change.

9032 Ho 95
$19^h\ 1^m$ $+27°\ 6'$
8.0 8.4

.579	216.3	0.33	2	940	40
.598	215.0	0.27	3	1340	40
1916.59	215.6	0.30			

No change.

9038 Σ 2454
$19^h\ 2^m$ $+30°\ 15'$
7.8 9.2

.598	255.7	0.90	3	700	40
.694	255.9	0.91	2	700	40
1916.65	255.8	0.90			

9059 Ho 98
$19^h\ 3^m$ $+26°\ 54'$
8.0 8.3

	°	″			m.
1916.598	133.7	0.21	3	1340	40

9095 β 1204
$19^h\ 6^m$ $+2°\ 25'$
A and *B* 7.3 7.6

.513	185.3	0.33	2	940	40
.527	184.8	0.29	2	940	40
.541	188.7	0.33	2	940	40
.546	186.3	0.36	2	940	40
1916.53	186.3	0.33			

No change. Angle placed in third quadrant on each night.

AB and *C* . . . 14.3

.513	194.4	12.90	2	940	40
.541	193.7	13.26	2	940	40
.546	194.7	13.14	2	700	40
1916.53	194.3	13.10			

No change.

AB and *D* . . . 14.7

.513	159.4	20.96	2	940	40
.541	157.4	22.00	2	700	40
.546	158.9	21.26	2	700	40
1916.53	158.6	21.41			

No change.

AB and *E* . . . 14.6

.546	317.6	26.82	3	700	40
.552	316.7	26.31	3	700	40
1916.55	317.2	26.56			

No change.

AB and *F* . . . 14.4

.546	291.0	27.52	3	700	40
.552	292.8	27.25	3	700	40
1916.55	291.9	27.38			

No change.

9106 β 138
$19^h\ 7^m$ $-14°\ 39'$
7.8 10.8

					m.
1916.552	295.0	1.42	2	700	40
.670	301.9	1.07	2	460	40
1916.61	298.1	1.24			

Slight increase in angle probable.

9191 β 248
$19^h\ 13^m$ $+22°\ 19'$
6.2 9.5

.505	125.6	2.02	2	700	30
.508	124.5	2.09	2	700	35
1916.51	125.0	2.06			

No change.

9253 β 141
$19^h\ 17^m$ $+22°\ 17'$
A and *B* 8.0 9.5

.505	82.6	0.96	1	700	40
.508	78.7	0.97	2	700	40
1916.51	80.7	0.97			

No change.

C and *D* 11.1 12.6

.505	182.1	5.33	2	700	40
.508	179.4	6.10	2	700	40
.522	175.7	5.96	2	460	40
.533	178.5	4.93	2	940	40
1916.52	178.9	5.58			

AB and *C* . . . 11.2

.550	332.4	27.75	2	700	40
.508	332.6	29.31	3	700	40
.522	333.3	27.71	2	460	40
1916.51	332.8	28.26			

No change.

AB and *E* 9.2

					m.
1916.505	90.5	50.31	2	700	40
.508	90.6	50.16	2	700	40
1916.51	90.6	50.24			

No change.

AB and *F* 12.8

.508	213.8	50.06	2	700	40
.522	212.5	49.50	2	460	40
.533	213.5	50.15	2	700	40
1916.52	213.3	49.90			

No change.

AB and *G* . . 15.2

.505	241.0	20.74	2	700	40
.508	236.4	20.94	2	700	40
1916.51	238.7	20.84			

No other measures.

9424 β 53
$19^h\ 30^m$ $+11°\ 11'$
8.8 10.0

.618	248.6	1.53	3	460	40
.648	254.0	1.41	1	400	12
.691	244.8	1.44	1	400	12
1916.65	249.0	1.48			

No change.

9466 β 249
$19^h\ 32^m$ $+0°\ 4'$
A and *B* 7.3 9.9

.612	129.7	1.18	2	400	12
.626	134.4	1.15	2	400	12
.670	135.2	1.30	3	700	40
1916.64	133.1	1.21			

No change.

A and *C* . . . 14.0

1916.670	115.7	18.13	3	700	40

Eight distant companions seen with the 40-inch. *C* is the nearest.

CONTENTS.

Editor, Benjamin Boss, Albany, N. Y. Associate Editors: E. E. Barnard, Ernest W. Brown, F. R. Moulton and R. S. Woodward.
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MICROMETRIC MEASURES OF DOUBLE STARS,

MADE AT THE YERKES OBSERVATORY,

By F. P. LEAVENWORTH.

The following measures of double stars were made with the 40-inch and 12-inch refractors of the Yerkes Observatory during the summer of 1916. They are a continuation of the results published in the *Astronomical Journal* No. 708.

9500 Σ 2556
$19^h\,34^m$ $+21°\,59'$
7.5 8.0

	°	″			in.
.552	127.1	0.37	2	940	40
.618	128.2	0.29	2	940	24
.628	124.9	0.39	2	1340	40
1916.60	126.7	0.35			

9524 β 145
$19^h\,37^m$ $+30°\,26'$
A and *B* 7.5 9.5

1916.566	264.3	1.04	2	700	40

No change.

AB and *D* . . . 13.5

.502	28.5	8.67	2	460	40
.566	29.3	8.71	2	700	40
1916.53	28.9	8.69			

No change.

AB and *D* . . 10.6

.502	156.6	26.88	2	460	40
.566	156.2	26.76	2	700	40
1916.53	156.4	26.82			

No change.

9574 A. G. Clark 10
$19^h\,39^m$ $+10°\,29'$
7.7 7.7

	°	″			in.
.513	141.5	0.30	2	940	40
.527	147.1	0.26	2	940	40
1916.52	144.3	0.28			

No motion.

9590 β 146
$19^h\,40^m$ $-20°\,10'$
8.0 10.0

1916.574	319.8	0.86	3	400	12

Measures very discordant.

9643 A.G. Clark 11
$19^h\,44^m$ $+18°\,51'$
5.5 6.0

.513	173.9	0.24	2	1340	40
.527	174.4	0.23	2	1340	40
.541	172.9	0.21	2	1340	40
1916.52	173.7	0.23			

Comparison with Van Biesbroeck's ephemeris, *A. J.*, No. 692:

$O-C$ $-4°.4$ $0''.00$

9881 β 57

$20^h\ 0^m$ $+15°\ 9'$

6.2 10.7

					m.
.522	118.7	2.79	2	460	40
.533	119.9	2.53	2	700	40
1916.53	119.3	2.66			

No motion.

10118 β 1259

$20^h\ 16^m$ $+30°\ 13'$

A and B 8.8 9.4

.579	176.3	0.41	2	700	10
.618	172.3	0.46	2	700	26
.694	176.2	0.44	2	700	10
1916.63	174.9	0.44			

AB and C . . . 13.0

1916.694	9.2	16.00	2	700	40

No other measures?

13525 A 1208

$20^h\ 17^m$ $+30°\ 52'$

9.1 9.4

.618	151.1	0.36	2	700	24
.694	151.5	0.44	2	700	40
1916.66	151.3	0.40			

No change.

10266 β 63

$20^h\ 25^m$ $+10°\ 30'$

A and B 6.1 8.5

.574	344.0	0.82	3	100	12
.604	347.8	0.66	1	700	18
.612	345.6	0.91	2	100	42
.618	346.5	0.78	2	700	20
1916.60	346.0	0.79			

No motion. Too frequently observed during last ten years.

AB and C 15.3

	°	''			m.
.604	348.0	16.56	2	460	40
.618	346.1	17.05	2	700	40
.642	348.3	17.14	2	460	40
1916.62	347.5	16.92			

10363 β 151

$20^h\ 32^m$ $+14°\ 11'$

A and B 4.0 5.0

.508	316.2	0.34	1	940	40
.513	316.2	0.36	2	940	40
.527	320.0	0.29	2	940	40
.541	315.4	0.30	2	940	40
1916.52	317.0	0.32			

Comparison with Aitken's elements in *Lick Publications*, Vol. XII:

$O - C$ $-0°.7$ $-0''.01$

AB and C . . . 13.1

.541	117.4	23.84	2	940	40
.546	119.4	23.15	3	460	40
.552	119.1	23.32	3	940	40
1916.55	118.6	23.44			

10127 β 267

$20^h\ 35^m$ $-4°\ 49'$

9.0 9.5

.522	62.3	.	1	460	40
.618	61.6	2.00	2	460	40
1916.57	62.0	2.00			

No change.

10187 β 64

$20^h\ 39^m$ $+12°\ 17'$

A and B 8.2 8.4

.527	200.9	0.27	2	940	40
.546	197.4	0.32	2	940	40
.552	195.7	0.32	2	940	40
.566	194.1	.	1	940	40
1916.55	197.0	0.30			

Slow increase in angle and decrease in distance.

AB and *D* . . 10.5

	°	″			in.
1916.560	118.5	62.64	2	700	40

No motion.

10500 β 153

$20^h\ 40^m$ $-26°\ 51'$

8.0 10.5

1916.574	270.6	1.50	2	400	12

Angle diminishing slowly.

10559 Σ 2729

$20^h\ 45^m$ $-6°\ 4'$

6.0 7.0

.546	334.5	0.44	2	700	40
.552	332.6	0.44	2	940	40
1916.55	333.6	0.44			

Comparison with Aitken's ephemeris in *Lick Publications*, Vol. XII:

$O-C$ $+1°.0$ $-0''.06$

10566 β 67

$20^h\ 46^m$ $+30°\ 28'$

6.7 9.0

1916.694	294.3	1.66	2	700	40

10601 Lv 8

$20^h\ 49^m$ $-11°\ 20'$

8.8 10.5

1916.574	301.1	0.91	2	400	12

10607 β 367

$20^h\ 50^m$ $+27°\ 38'$

A and *B* 8.0 8.5

.579	214.2	0.19	2	1340	40
.628	214.7	0.20	2	1340	40
1916.60	214.4	0.20			

Near periastron: should be measured frequently for next few years.

AB and *C* 12.0

	°	″			in.
.579	18.5	32.18	3	700	40
.585	18.1	32.21	2	700	40
.598	18.7	31.86	2	700	40
1916.59	18.4	32.08			

Angle and distance changing from proper-motion of *A*.

AB and *D* . . . 14.5

.579	91.1	28.77	2	700	40
.585	89.2	28.32	2	700	40
.598	90.6	28.78	2	700	40
1916.59	90.3	28.62			

Angle and distance changing from proper-motion of *A*.

10689 β 69

$20^h\ 57^m$ $+21°\ 13'$

A and *B* 8.0 8.9

.505	321.1	0.97	2	460	40
.508	317.1	0.92	2	700	40
.513	321.1	0.98	3	940	40
1916.51	319.8	0.96			

No change.

AB and *C* . . . 7.0

.502	239.7	77.26	3	460	40
.508	239.4	77.05	3	460	40
1916.50	239.6	77.16			

Decrease in distance probable.

C and *D* . . . 13.2

.502	157.8	19.04	2	460	40
.508	159.2	18.36	2	700	40
.513	156.9	18.82	2	940	40
1916.51	158.0	18.74			

10692 Lv 9

$20^h\ 57^m$ $+38°\ 45'$

9.3 10.7

1916.694	193.0	2.42	2	700	40

No motion.

10707 *β* 269
$20^h\ 59^m$ $+7°\ 17'$
8.1 10.5

					in.
.541	248.2		1	940	40
.560	250.4	1.26	2	940	40
.566	250.6		1	460	40
.604	251.3	1.01	2	700	40
1916.57	250.1	1.14			

No motion

10749 OΣ 527
$21^h\ 2^m$ $+4°\ 40'$
6.0 8.0

.527	267.4	0.40	2	940	40
.546	267.1	0.38	2	940	40
1916.54	267.2	0.39			

Very slow decrease in angle.

10782 Knott 2
$21^h\ 4^m$ $+9°\ 39'$
A and *B* 4.0 11.2

.533	269.8	2.48	2	460	40
.541	270.5	2.58	2	460	40
1916.54	270.2	2.53			

No change.

A and *C* . . . 12.0

.533	5.5	46.41	2	460	40
.541	5.3	46.14	2	460	40
1916.54	5.4	46.28			

Change due to proper-motion of *A*.

10787 *β* 251
$21^h\ 5^m$ $-31°\ 5'$
7.0 10.0

1916.689	232.3	2.75	2	100	12

No change.

10808 *β* 159
$21^h\ 6^m$ $+47°\ 12'$
A and *B* 6.8 9.5

		″			in.
.675	313.4	1.29	2	460	40
.694	315.3	1.43	2	700	25
1916.68	314.4	1.36			

A and *D* . . . 15.5

1916.675	144.4	15.44	2	460	40

No other measures.

A and *E* . . . 14.5

1916.675	337.0	. . . .	2	460	40

No other measures.

10818 *β* 270
$21^h\ 8^m$ $+6°\ 43'$
A and *B* 7.0 9.5

1916.560	338.4	0.33	2	940	40

Decrease in distance probable.

A and *C* . . . 13.5

.541	30.7	32.03	2	700	40
.560	31.1	32.26	2	700	40
.585	30.8	32.37	2	460	40
.604	30.6	32.44	2	700	40
1916.57	30.8	32.28			

No change.

10829 *δ Equulei*
$21^h\ 9^m$ $+9°\ 31'$
A and *B* 4.0 4.8

.527	196.9	0.29	2	940	40
.546	201.5	0.30	2	940	40
.552	199.5	0.31	3	940	40
.628	200.5	0.26	2	1340	40
1916.56	199.6	0.29			

Comparison with Aitken's elements in *Lick Publications*, Vol. XII:

$O - C$ $+1°.7$ $-0''.02$

10880 β 163
$21^h\ 13^m$ $+11°\ 4'$

1916.637 round.

13571 A 887
$21^h\ 20^m$ $+10°\ 50'$
8.5 9.2

	°	″			in.
1916.637	120.2	0.18	1	1340	40

11006 β 72
$21^h\ 24^m$ $-5°\ 55'$
8.7 11.0

.566	41.8	2.11	2	460	40
.618	42.1	1.82	2	460	40
.700	41.0	1.93	1	460	35
1916.63	41.6	1.95			

No motion.

11007 β 684
$21^h\ 24^m$ $-5°\ 57'$
9.1 9.5

.566	118.3	0.98	2	460	40
.618	129.0	1.10	2	460	40
.700	125.4	0.98	2	460	31
1916.63	124.2	1.02			

No motion.

11088 β 167
$21^h\ 31^m$ $+29°\ 31'$
6.8 11.0

.585	90.6	[1.58]	1	700	40
.642	88.8	. .	2	460	24
.675	87.2	2.17	2	700	40
1916.64	88.8	2.17			

No motion.

11125 β 1212
$21^h\ 34^m$ $-0°\ 31'$
A and *B* 6.5 6.9

	°	″			in.
.574	296.3	0.44	2	400	12
.612	291.9	0.48	2	400	12
.661	291.0	0.45	2	700	24
1916.62	293.1	0.46			

Direct motion in angle.

AB and *C*

.612	146.8	41.21	2	400	12
.661	146.4	41.43	2	700	40
1916.64	146.6	41.32			

Change due to proper-motion.

11190 Lv 10
$21^h\ 38^m$ $-11°\ 36'$
8.5 10.2

.566	282.8	1.26	2	460	40
.689	281.5	1.24	2	400	12
1916.63	282.2	1.25			

Slow direct motion in angle.

11210 Ho 166
$21^h\ 39^m$ $+27°\ 18'$

1916.598	51.6	0.32	2	940	40

Angle diminishing rapidly.

11214 μ *Cygni*
$21^h\ 39^m$ $+28°\ 12'$
A and *B* 4.0 5.6

.585	133.7	1.88	2	460	40
.675	134.3	1.62	2	700	27
.689	134.8	1.58	4	400	12
1916.65	134.3	1.69			

Motion still rectilinear.

A and C . 11.2

	°	″			in.
.675	275.1	46.06	2	700	40
.689	274.8	45.51	2	400	12
1916.68	275.0	45.78			

11222 *κ Pegasi*

$21^h\ 39^m$ $+25°\ 6'$

A and B 4.7 5.1

.546	122.5	0.25	2	1340	40
.560	122.3	0.26	2	1340	40
.579	120.4	0.23	3	1340	40
.637	124.5	0.22	3	1340	40
1916.58	122.4	0.24			

Comparison with Lohse's ephemeris in *Potsdam Publications*, Vol. XX:

$O - C \quad +3°.1 \quad -0''.05$ or $0''.00$

AB and C 4.0 10.8

.585	297.5	12.37	2	460	40
.604	297.9	12.76	2	460	40
1916.59	297.7	12.56			

11346 *β* 75

$21^h\ 50^m$ $+10°\ 19'$

8.0 8.8

1916.670	19.2	0.97	2	700	24

Motion so far rectilinear.

11512 *β* 170

$22^h\ 3^m$ $+19°\ 4'$

8.0 8.2

.574	56.9	1.37	3	400	12
.689	56.4	1.62	3	400	12
1916.63	56.6	1.50			

Slow decrease in angle probable.

11732 *β* 294

$22^h\ 22^m$ $+3°\ 55'$

A and B 8.0 8.6

	°	″			in.
.552	174.0	0.26	2	940	40
.618	170.9	0.34	2	940	40
.628	180.4	0.30	2	940	40
.637	181.0	0.26	2	940	32
1916.61	176.6	0.29			

Angle slowly increasing. The apparent change in distance may be due to errors of observation.

AB and C . 14.8

.618	125.4	30.68	2	940	40
.628	123.9	30.71	3	940	40
.637	123.8	30.78	2	940	40
1916.63	124.4	30.72			

No motion.

11763 37 *Pegasi*

$22^h\ 24^m$ $+3°\ 49'$

1916.618	282.2	0.23	2	1340	40

13621 Ho 982

$22^h\ 30^m$ $+14°\ 0'$

7.2 10.7

.579	212.0	0.88	2	700	40
.598	213.4	0.96	2	700	40
1916.59	212.7	0.92			

11888 *β* 277

$22^h\ 34^m$ $+40°\ 45'$

8.0 8.5

1916.689	208.0	0.53	2	400	12

11895 Ho 296

22h 35m +13° 55′

6.0 6.5

	°	″			m
.560	28.2	0.29	2	1340	40
.579	28.7	0.28	2	1340	40
.598	32.5	0.28	2	940	40
1916.58	29.8	0.28			

Angle probably decreasing.

11917 β 710

22h 37m +29° 5′

8.9 9.2

.598	239.1	0.44	2	940	40
.604	231.0	0.48	2	700	40
.642	228.4	0.38	2	700	24
.670	233.1	0.47	2	700	40
1916.63	232.9	0.44			

Apparently no change.

11921 *η Pegasi*

22h 37m +29° 36′

B and *C* 9.7 10.2

1916.598	62.7	0.35	1	940	40

A and *BC* 4.0 11.0

.642	338.8	91.12	2	460	25
.675	338.9	90.87	3	460	40
1916.66	338.8	91.00			

Change in distance due to proper-motion of A.

D and *E* 14.0 16.0

1916.642	181.4	5.70	2	940	40

No other measures.

BC and *D*

1916.642	320.4	62.86	2	940	40

No other measures.

12046 β 178

22h 49m −5° 38′

6.0 8.0

		″			m.
.579	323.7	0.36	2	700	40
.637	327.7	0.43	2	940	40
.689	326.3	0.44	2	400	12
1916.64	325.9	0.41			

The distance has apparently diminished in the last few years.

12094 OΣ 483

22h 53m +11° 5′

6.2 8.4

.560	233.1	1.12	2	700	40
.579	231.8	1.03	2	700	40
.585	234.9	1.04	2	700	40
.598	235.8	0.96	2	700	24
.604	234.2	0.81	2	700	18
1916.58	234.0	0.99			

12257 β 1220

B and *C* 23h 11m −9° 37′

9.4 9.2

1916.637	99.7	0.62	3	940	24

BC and *E*

1916.637	348.0	18.66	2	940	40

12276 β 79

23h 11m −2° 10′

8.0 9.8

.598	71.7	1.12	3	700	40
.670	71.9	. . .	2	460	40
.689	67.3	1.10	3	400	12
1916.65	70.3	1.11			

Angle slowly decreasing.

12401 β 1266

$23^h 24^m$ $+30° 10'$

A and *B*

1916.601 round with power 700 40-inch.

AB and *C* 7.0 9.0

	°				in.
.604	203.4	18.96	3	700	40
.661	203.2	18.70	2	460	25
1916.63	203.3	18.83			

AB and *D* . 14.5

1916.661	7.7	36.82	2	460	40

No other measures.

12432 β 720

$23^h 28^m$ $+30° 40'$

6.0 6.1

.604	188.8	0.37	2	940	40
.642	187.3	0.32	1	940	15
.661	189.2	0.38	1	700	25
1916.64	188.4	0.36			

13652 A 1240

$23^h 31^m$ $+31° 46'$

9.0 12.0

1916.642	350.2	1.93	2	700	24

12523 β 279

$23^h 36^m$ $-15° 12'$

5.0 9.8

.689	85.0	5.86	2	400	12
.700	87.0	5.51	2	460	40
1916.69	86.0	5.68			

No motion.

12655 Σ 3047

$23^h 52^m$ $+56° 43'$

A and *B* 9.0 9.5

	°				in.
1916.694	71.6	1.05	2	460	40

β 280 *AB* and *C* . . . 13.0

1916.694	196.1	8.31	2	460	40

Measures discordant.

12686 *AG* 299

$23^h 54^m$ $+26° 15'$

1916.628 round 40

12701 85 *Pegasi*

$23^h 56^m$ $+26° 27'$

A and *B* 6.0 10.0

.628	138.1	0.89	2	1340	40
.637	143.9	0.81	2	940	40
1916.63	141.0	0.85			

Comparison with Lohse's ephemeris in *Potsdam Publications*, Vol. XX:

$$O-C \quad -8°.4 \quad +0''.03$$

Comparison with Bowyer's and Furner's elements, *M. N.*, Vol. LXVI:

$$O-C \quad -5°.8 \quad +0''.03$$

AB and *C* . . . 8.0

.661	334.4	54.31	2	460	40
.670	334.8	54.42	3	700	40
.675	334.8	54.46	3	700	40
1916.67	334.7	54.40			

AB and *D* . . . 14.0

.675	295.2	105.02	2	460	40
.694	295.0	104.02	2	460	40
1916.68	295.1	104.52			

University of Minnesota, Minneapolis, January 3, 1917.

CONTENTS.

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THE ORBIT OF *EPSILON EQUULEI*.

By HENRY NORRIS RUSSELL.

ε Equulei. [BURNHAM'S *G. C.* 10643, Σ 2737, 20ʰ 54ᵐ.1, +3° 55′ (1900)].

This close and nearly equal pair, (5ᵐ.8 and 6ᵐ.3) is a fine example of a binary system whose orbit is turned almost edgewise toward the *Earth*. The distance has increased from 0″.35 in 1835 to 1″.05 in 1873, and diminished again to less than 0″.10 in 1916, while the angle has changed but a few degrees.

A list of all the observations published up to 1903 is given by LEWIS, (*Mem. R. A. S. 56*, 626–7). To these have been added the observations listed in Table VI, of which those up to 1912 have been kindly furnished by Professor DOOLITTLE from his card catalogue of recent measures, while the very important measures of Professor AITKEN in 1914–16 have been generously communicated by him in advance of publication.

TABLE VI

Date	p	s	Obs.	
1888.43	288°.2	0″.89	SCHIA.	10n
1898.89	284 .7	0 .77	SCHIA.	9n
1904.80	284 .2	0 .67	MAW.	2n
1908.70	284 .3	0 .47	MAW.	5-3n
1908.73	281 .5	0 .42	LAU.	2n
1908.72	284 .8	0 .62	OLIV.	2n
1908.72	281 .6	0 .46	R. E. WILSON	2n
1908.73	284 .2	0 .47	W. N. NEFF	2n
1909.76	276 .7	0 .37	BRY.	1n
1910.64	278 .0	0 .28	BRY.	1n
1910.83	290 .7	0 .43	DOB.	4n
1911.79	274 .8	0 .35	BRY.	2n
1912.75	286 .9	0 .29	BRY.	2n
1914.63	279 .3	0 .16	AITKEN	2n
1915.83	266 .9	0 .14	AITKEN	1n
1916.59	238 ±	0 .10 ±	AITKEN	1n
1916.58				
1916.64	Too close to measure (AITKEN)			

Professor AITKEN writes "The angle on 1916.59 is, of course, subject to considerable uncertainty. The distance probably did not *exceed* 0″.10; it may have been 0″.01 or 0″.02 less."

The available observations have been combined into normals, (giving the results of each observer equal weight, to avoid any possibility of prejudice) with the results shown in Table VII, in which the second column shows the number of individual results of different observers combined into each normal.

TABLE VII. NORMAL PLACES, *ε Equulei*

Date	Obs.	p	s	O — C	
1837.37	3	291°.9	0″.12	+ 0°.5	0″.00
1842.55	3	289 .6	0 .65	− 0 .4	+0 .05
1847.41	2	287 .5	0 .68	− 1 .5	−0 .06
1852.84	4	286 .3	0 .85	− 2 .0	−0 .01
1856.01	3	287 .8	0 .90	− 0 .3	−0 .02
1860.33	4	287 .2	1 .00	− 0 .6	+0 .02
1866.07	4	287 .2	1 .04	− 0 .2	+0 .01
1871.85	5	286 .3	1 .07	− 0 .6	+0 .02
1877.72	6	287 .6	1 .01	+ 1 .1	−0 .03
1886.13	4	285 .7	0 .94	− 0 .3	−0 .04
1890.51	4	285 .4	0 .98	− 0 .2	+0 .06
1893.82	4	286 .2	0 .87	+ 0 .8	+0 .01
1896.43	1	286 .0	0 .81	+ 0 .9	0 .00
1899.67	6	284 .5	0 .71	− 0 .3	−0 .03
1903.70	4	283 .8	0 .62	− 0 .1	+0 .01
1908.72	5	283 .9	0 .49	+ 1 .2	+0 .04
1910.41	3	281 .8	0 .36	− 0 .6	−0 .03
1912.27	2	280 .8	0 .32	− 0 .5	+0 .02
1914.63	1	279 .3	0 .16	+ 1 .3	−0 .03
1915.83	1	266 .9	0 .14	− 9 .3	+0 .01
1916.59	1	238 ±	0 .10 ±	(−31 ±)	+0 .01

Upon plotting these distances against the time (Figure 2) it appears that the maximum distance of 1″.05 was reached about 1873.5, and that the increase

and decrease of distance are sensibly symmetrical with respect to the time. It follows that the line of apsides of the true orbit must be practically coincident with the line of nodes, that is, that λ must be 0° or 180°. This fortunate circumstance considerably simplifies the further computations.

The mean of all the observed position angles from 1852 to 1896, while the distance was greater than 0''.8, is 286°.26, which must be very nearly the position angle of the maximum elongation, or of the line of nodes. The projections of the individual distances on this line are sensibly equal to the observed distances themselves. We have now to determine from these distances the eccentricity, major axis, and period. If we were obliged to depend upon geometrical relations alone BURNHAM's remark (*B. G. C.*, p. 918) "The period will necessarily remain wholly unknown until the companion is at its greatest distance in the first or second quadrant" would be true; but the dynamical relations between the distance and the time permit the solution of the problem. From a plot of the observations, it is found that the maximum distance, 1''.05, was reached on 1873.50, and the distance 0''.40 on 1910.05, 36.55 years later. If now E denotes the eccentric anomaly at the latter date, and n the mean motion, we have the equations

$$a(1+e) = 1''.05; \quad a(\cos E - e) = -0''.40;$$
$$M = E - e\sin E; \quad 36.55\,n = 180^\circ - M$$

which determine the other elements as functions of the assumed eccentricity, on the assumption, (which will soon be justified) that the observed elongation coincides with the apastron. With these elements, the curve giving the relation between the distance and the time may be plotted and compared with the observations. The curves given in Figure 2 correspond to the assumptions:

(a)	$e = 0.00$	$a = 1''.05$	Period = 194.4 years
(b)	$e = 0.75$	$a = 0''.60$	Period = 95.5 years
(c)	$e = 0.90$	$a = 0''.55$	Period = 87.8 years

It is evident from the figure that the eccentricity is not far from 0.75, with elongation at apastron. The curves for $e = 0.00$ and $e = 0.90$ deviate from the observed points in opposite directions, and that for a smaller orbit is wholly inconsistent with the rapid decrease of the distance in the last few years.

Though the assumption that $e = 0.75$ gives already an excellent representation of the observations, a least-squares solution was attempted for its improvement. In this solution λ was kept equal to zero, since there will be no real chance of determining its deviation from this value until the companion has returned to the fourth quadrant, while a, e, T and n were varied. In the equations of condition, which are given in Table VIII,

$$\alpha = \frac{da}{a}, \quad \beta = a\,de, \quad \gamma = na\,dT, \quad \delta = 50\,a\,dn$$

TABLE VIII

0.42 α	+2.04 β	−1.05 γ	+0.75 δ	=	0''.00
0.60	+1.77	−0.81	+0.50	=	+ .05
0.74	+1.55	−0.64	+0.33	=	− .06
0.86	+1.35	−0.48	+0.19	=	− .01
0.92	+1.25	−0.40	+0.13	=	− .02
0.98	+1.14	−0.30	+0.07	=	+ .02
1.03	+1.04	−0.16	+0.02	=	+ .01
1.05	+1.00	−0.03	+0.00	=	+ .02
1.04	+1.01	+0.09	+0.01	=	− .03
0.98	+1.13	+0.28	+0.07	=	− .04
0.92	+1.24	+0.38	+0.13	=	+ .06
0.86	+1.33	+0.47	+0.18	=	+ .01
0.82	+1.43	+0.54	+0.25	=	− .01
0.74	+1.55	+0.64	+0.33	=	− .03
0.62	+1.74	+0.78	+0.48	=	.00
0.45	+2.00	+1.00	+0.70	=	+ .04
0.39	+2.08	+1.08	+0.80	=	− .03
0.30	+2.15	+1.18	+0.93	=	+ .02
0.18	+2.21	+1.35	+1.12	=	− .02
0.12	+2.19	+1.42	+1.22	=	+ .02
0.07	+2.14	+1.47	+1.27	=	+ .02

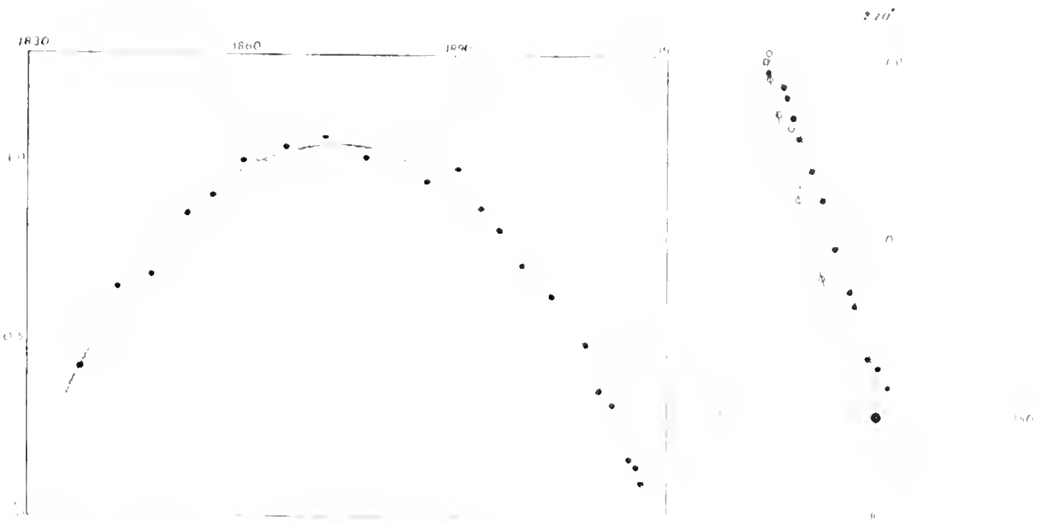
The resulting normal equations are

11.42 α	+19.62 β	+ 1.57 γ	+ 3.68 δ	=	−0''.006
19.62	+36.66	+14.41	+18.56	=	+0 .082
1.57	+14.41	+14.00	+ 7.09	=	+0 .058
3.68	+18.56	+ 7.09	+ 7.77	=	+0 .049

Whence

$\alpha = +0.015$, weight 0.15 $\qquad \gamma = 0.000$ weight 6.13
$\beta = -0.016$, weight 0.18 $\qquad \delta = +0.038$ weight 0.05

Although there is a serious loss of weight during the solution of the equations, the elements are nevertheless determined with considerable accuracy, as appears from the probable errors given below. To complete the determination of the orbit, a solution for the elements γ and Ω was made from the position angles, which under the peculiar circumstances depend almost entirely upon these elements alone. The preliminary



values assumed were ℧ = 286°.0 γ = 85°.35, and the equations of condition took, with sufficient accuracy the simple form

$$d\,\Omega + (p - ℧)\,\frac{d\gamma}{90° - \gamma} = \Delta p$$

where p is the computed position angle and Δp the excess of the observed over the computed value. The equations were given weights proportional to the distances, in order to take account of the loss of accuracy of the measures of angle when the pair was very close. The estimated angle of 1916 was disregarded, and that of the previous year given half weight.

The final elements derived for this system are as follows:

Major semi-axis	a	0″.61	±0″.04
Eccentricity	e	0 .72	±0 .09
Apastron passage	T	1873.5	±0 .22
Period	P	97.4	±5.0 yrs. Angles decreasing
Inclination	γ	85°.5	±0°.3
Node	Ω	106°.8	±0°.2
Periastron minus Node	λ	0°.0	(assumed)

Probable error of one normal distance, ±0″.022

Probable error of one normal angle, at distance 1″.0 ±0°.62 = ±0″.011

The residuals from this orbit are given in the last two columns of Table VII.

Although the whole motion in angle has been only 25° (excluding the last uncertain estimate) the orbit of this system may fairly be described as already pretty well determined. This is due to the accuracy of the distance measures, and especially to the vitally important data secured by Professor Aitken in the last few years. For the next ten years the pair will be very close, and measures should be secured with the largest telescopes. An ephemeris, computed from preliminary elements which agree with those given above far within their probable errors, is given in Table IX.

Table IX

Date	p	s
1916.5	266°.9	0″.08
1917.5	232	0 .03
1918.5	133	0 .04
1919.5	115 .8	0 .10
1920.5	109 .7	0 .14
1921.5	108 .0	0 .15
1922.5	104 .5	0 .12
1923.5	91 .9	0 .07
1924.5	62	0 .02
1925.5	314	0 .05
1926.5	301 .8	0 .11
1927.5	297 .4	0 .16

The measures of the next ten or twelve years should suffice to give a very accurate orbit.

There is a distant companion, of magnitude 7.2, which shares the proper-motion of the close pair, — which, according to Boss, is 0″.191 toward 221°. A least square solution of the measures of this star from

1824 ... 1915 give the following formula for the relative positions, the angles being corrected for precession:

$$\theta = 75°.5 + 0°.11 \ldots 0°.052 \pm 0°.001 \ (t - 1871.4)$$
$$\rho = 10''.74 \pm 0''.02 + 0''.0011 \pm 0''.0007 \ (t - 1871.4)$$

The relative motion is at the rate of 0''.99 ± 0''.08 per century. This is of the order of magnitude which might be expected if the stars are physically connected. If the mass of the distant companion was half that of the close pair, its period, if moving in a circular orbit at a distance of 10''.7, would be 5,900 years, and its orbital motion would be at the rate of 1''.14 per century, if at right angles to the line of sight. Since however the radial components of the distance and velocity are unknown, it can only be said that orbital motion appears very probable.

If the mass of the close pair is twice that of the *Sun*, — a reasonable estimate, in view of its spectrum *F5*, — its parallax would be 0''.023, the absolute magnitudes of the components 2.6 and 3.1, and their luminosities 8 and 5 times the *Sun's*, — values which are quite normal for this spectral class. The relative orbital velocity of the components at periastron would be 20 km., almost all in the line of sight. As the components are nearly equal in brightness, the chances are favorable for a spectroscopic determination of the parallax about the time of periastron passage in 1922.

In conclusion, the writer desires to express his best thanks to Professors Barnard, Aitken, Doolittle and Adams, whose liberality in supplying unpublished material has alone made possible the determination of the orbits and motions of these interesting systems.

Princeton University Observatory, 1917 January 12.

PRELIMINARY PARALLAX OF BARNARD'S STAR OF LARGE PROPER-MOTION.

By S. A. MITCHELL.

Three values of the parallax of this star have already been given by Adams, Russell, and Schlesinger. In view of the great interest attaching to this star, a fourth may not come amiss.

The present value is the result of the measures of seven photographs taken with the 26-inch McCormick refractor of the University of Virginia. Two exposures were made on each of these plates with the exception of the first plate taken, on which there is but one exposure. The parallax star was measured with respect to five comparison stars, and the reductions were carried out by the method of dependences. The parallactic shifts in right ascension only were measured.

The most satisfactory value of the parallax of the star at the present time is obtained by assuming a value of the proper-motion. The best value so far published, seems to be that of Schlesinger, obtained by the comparison of the positions on four plates taken at Harvard College Observatory in the years 1890 to 1893, with two plates made at the Allegheny Observatory in 1916. The proper-motion in right ascension derived is −0''.74 per year.

Measures on the McCormick plates were carried out in terms of millimeters. For convenience, the measures are given below in seconds of arc, obtained by multiplying the measured values by the scale-value of the photographs, one millimeter = 20''.8.

Plate	Date	Hour Angle	Observer	Parallax Factor	Measured Solution	Correction for Proper-Motion	Solution Corrected for Proper-Motion	Wt.	Residual
2728	June 18	0.1	L	+0.017	0.333	0.156	−0.189	0.7	+0.019
2729	18	0.4	M	+ .017	.297	.156	.453	0.9	− .012
2911	Aug. 20	0.3	A	− .868	.840	.029	.869	0.9	− .010
2912	20	0.9	A	− .868	.853	.029	.882	0.9	+ .002
2916	25	0.1	A	− .907	.869	.019	.888	1.0	− .010
2917	25	0.5	A	− .907	.867	.019	.886	1.0	− .012
2953	Sept. 3	0.5	M	− .962	−0.953	0.000	−0.953	1.0	+0.029

M = S. A. Mitchell; A = H. L. Alden; L = R. C. Lamb.

The measured solutions were corrected for proper-motion. From these corrected solutions, the following normal equations were found:

$$6.1 \quad c - 4.3112\,\pi = -5''.0534$$
$$-4.3112c + 3.9271\,\pi = +3\ .8841$$

Their solution yields: $\pi = +0''.17 \pm 0''.01$

The probable error of the measure of a plate of unit weight is ± 0.0006 millimeters, or $\pm 0''.012$.

SCHLESINGER obtained two values for the parallax of this star, $0''.50$ from measures in right ascension, and $0''.13$ from declination measures. The former value is the more accurate one. Giving the right ascension determination twice the weight of that in declination, the value obtained is $0''.47$.

In view of the small number of plates on which the present solution depends, which are separated by only a 77-day interval, the agreement with the value obtained by SCHLESINGER is excellent.

Leander McCormick Observatory, University of Virginia, January 24, 1917.

THE ORBIT AND PROBABLE SIZE OF A VERY FAINT ASTEROID.

BY SETH B. NICHOLSON AND HARLOW SHAPLEY.

The direct motion of Asteroid A on the dates given in the preceding note suggested that its orbit might be of special interest; accordingly, further observations were secured.

Observations

No.	G. M. T. 1916	α (1916.0)	δ (1916.0)	log. $p\Delta$ α	log. $p\Delta$ δ	O — C $\cos\delta\,\Delta\alpha$	O — C $\Delta\delta$
		h m s	° ′ ″			″	″
(1)	Sept. 6.8809	2 20 56.33	+12 49 22.4	9.394n	0.532	+1.2	+0.1
(2)	Sept. 6.9997	2 20 59.13	+12 49 23.4	9.049	0.510	−0.3	+0.5
(3)	Sept. 22.7600	2 23 27.04	+12 25 41.6	9.628n	0.601	−1.2	−0.3
(4)	Sept. 23.8642	2 23 15.96	+12 22 12.6	9.229n	0.524	0.0	0.0
(5)	Sept. 24.9684	2 23 01.98	+12 18 30.2	9.204	0.524	−0.9	+1.1
(6)	Oct. 18.6941	2 09 08.72	+10 16 20.4	9.597n	0.609	0.0	+0.3

A circular orbit based upon positions (1) and (2), which are separated by an interval of less than three hours, was used to predict the position for September 22. The preliminary general orbit computed from (3), (4), and (5) permitted the determination of a place for October 18; this solution was then corrected on the basis of positions (2) and (6). The adopted elements are as follows:

$$\begin{aligned} \text{Epoch} &= 1916 \text{ Sept. } 23.8588 \text{ G.M.T.} \\ M &= 11^\circ\ 20'\ \ 5'' \\ \left.\begin{aligned} \Omega &= \ \ 7\ \ 31\ \ 14 \\ \omega &= 187\ \ 44\ \ 20 \\ i &= \ \ 2\ \ \ 2\ \ 44 \end{aligned}\right\} & \ 1916.0 \\ \varphi &= 13\ \ 12\ \ \ 4 \\ \mu &= 976''.30 \\ \log a &= 0.37362 \end{aligned}$$

CONSTANTS FOR THE EQUATOR (1916.0)

$$x = r[0.00000] \sin\,(\ 90^\circ\ 13'\ 26'' + v)$$
$$y = r[9.96892] \sin\,(\ \ 0\ \ 19\ \ 42\ \ + v)$$
$$z = r[9.56262] \sin\,(359\ \ 32\ \ 28\ \ + v)$$

The orbit does not prove very exceptional, the direct motion on September 6 being accounted for by the position of perihelion and the rather large mean motion and eccentricity. In the matter of dimensions, however, this faint asteroid is out of the ordinary. As the brightness and size of such objects have been only roughly estimated heretofore, it is of interest to make a closer determination in the present case. The result, though quite probable, can be considered only approximate, even though the distance and brightness are definitely known, for the albedo and color-index of small asteroids must be inferred from analagous planetary data which are none too certain.

Through the intercomparison with stars of known brightness in the vicinity of the *Orion* nebula, the photographic magnitude of the asteroid on September 21 was found to be 17.1. On that date the distance from the *Sun* in astronomical units was 1.84, and from the *Earth* 0.93. Correcting for phase and color-index the adopted value of *g* (visual) is 14.3. Assuming an albedo similar to that of *Mars* and the brighter asteroids, the magnitude observed corresponds to a diameter of between 2 and 3 miles. Such dimensions are probably the smallest yet assigned, on the basis of a definite determination, to any planetary body in the solar system.

Mount Wilson Solar Observatory, January, 1917.

INVESTIGATION OF A NEW SCREW MEASURING MACHINE AT THE YERKES OBSERVATORY,

By OLIVER J. LEE and HANNAH B. STEELE.

This machine is one of several of the same type that have recently been completed by Wm. Gaertner & Co., of Chicago, for astrometric purposes at various observatories in this country. The general design of the machine is due to Dr. FRANK SCHLESINGER, and the details of the design were laid out by the Gaertner Company.

The use of this instrument at the Yerkes Observatory was made possible by a generous grant of six hundred and forty dollars to Professor FROST by the Directors of the Gould Fund of the National Academy of Sciences.

The details of the investigations by the writers will appear shortly in Vol. IV of the *Publications of the Yerkes Observatory*. The present note gives the chief results.

The machine has a screw 18mm. in diameter, with 249 threads of millimeter pitch. Its periodic errors were derived in the usual way, by measuring an interval of 0.20mm. with successive fifths of a revolution of the screw. Two series of measures were made by each of us at three points on the screw, viz.: at readings 40, 100 and 160 on the attached scale. The errors therefore are derived from the appropriate means from twelve revolutions of the screw. The average deviation of a complete measure from the mean is 0.00017 millimeter. The errors are represented by the following equation:

$$f(\theta) = +0.000036 \cos\theta - 0.000011 \sin\theta$$
$$-0.000013 \cos 2\theta - 0.000033 \sin 2\theta.$$

The maximum error from this source is about 0.00013mm, which is negligible in all work with stellar images.

Some difficulty was experienced in getting observations of uniform accuracy over the whole length of the screw for the determination of the progressive errors. Satisfactory measures were obtained, however, by means of a long glass scale, silvered on the under side, and adjusted so that a beam of light sent into it from a mirror mounted on the microscope showed the shaded diamond rulings of the scale most distinctly. Several accordant series of measures were made with this apparatus in December, 1916, and from the mean of them the progressive errors were derived for every centimeter of the screw from readings 5 to 195 on the attached scale. The arrangement of the observations is similar to that described in Vol. V of *Travaux et Memoires du Bureau International des Poids et Mesures*. The resulting symmetrical system of equations was solved in the manner employed by P. A. HANSEN. The range of temperature during the period of measurement was about 1°.3 C. The final means, involving double measures of 190 distances on screw and scale, were used in deriving the errors at 18 points on the screw, the errors at readings 15 and 185 having been assumed to be zero. The values of the corrections for progressive errors are as follows:

Readings	Corrections
5	−0.0004mm
15	.0000
25	+ .0010
35	+ .0007
45	+ .0002
55	+ .0007
65	+ .0012
75	+ .0012
85	+ .0014
95	+ .0011
105	+ .0005
115	− .0004
125	− .0011
135	− .0006
145	− .0002

Readings	Corrections
155	− .0007mm
165	− .0004
175	− .0010
185	.0000
195	−0.0001

The corrections run as if the cause of them were the pressure of the cutting tool when the threads were made. They are small to a gratifying degree and may safely be neglected in much astrometric work.

The relative coefficient of expansion of the screw and the glass scale is 0.24×10^{-5}. Hence the ordinary range of temperature in which measures are comfortably made can have no appreciable effect upon the results.

The errors of the V-way were obtained from measures on a long diamond ruling on plate glass, viewed directly and as seen through the glass. The maximum corrections out of the 38 that were derived are −0.0006 and +0.0006 millimeters at readings 74 and 136, respectively, on the attached scale. Errors from this source are also negligible in most work with stellar images.

The microscope is constructed to permit of varying the magnification from about 8.5 to 17 diameters. In all of our test work an auxiliary eyepiece and reticle were used which gave about 75 diameters.

Various slight alterations have from time to time been made in our shop to improve the illumination and to correct slight matters that had been overlooked in the final fitting together of parts.

Altogether, the results of the definitive tests of the machine show it to be an excellent instrument, and it is proper to express appreciation of the painstaking work of Mr. Gaertner and of his desire to meet every requirement of accuracy sought for on the part of the Observatory.

Yerkes Observatory, February, 1917.

OBSERVATIONS OF THE NINTH SATELLITE OF *JUPITER*,

By SETH B. NICHOLSON and HARLOW SHAPLEY.

The following positions of *Jupiter's* Ninth Satellite have been obtained from photographs with the 60-inch reflector. The time given is the observed Greenwich mean time of mid-exposure. The positions are referred to the mean equinox and equator of the beginning of the year (1916.0). The magnitude of the satellite on Plates 3484 and 3487 is 18.3. (*Publications of the Astronomical Society of the Pacific*, 281, 1916)

Observations

Gr. Mean Time 1916	α (1916.0)	δ (1916.0)	Red. to App. α	Red. to App. δ	Log $p\Delta$ α	Log $p\Delta$ δ	Exp.	Plate	Stars
d h m	h m s	° ′ ″	s	″			m		
Sept. 6 21 8.5	2 20 14.52	+12 56 27.2	+3.99	+23.3	9.394*n*	0.532	37	3443	1, 2, 3, 4
Sept. 6 23 59.5	2 20 13.75	+12 56 20.2	+3.99	+23.3	9.049	0.510	45	3451	1, 2, 3, 4
Oct. 18 17 30.5	2 6 43.73	+11 28 07.5	+4.74	+27.7	9.502*n*	0.570	32	3484	5, 6, 7
Oct. 19 16 54.0	2 6 15.69	+11 25 17.5	+4.75	+27.7	9.565*n*	0.586	32	3487	5, 6, 7

Mean Places of Comparison Stars for 1916.0

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	2 19 59.32	+12 38 20.0	*A.G. Leipzig* I 697	5	2 5 37.24	+11 43 04.1	*A.G. Leipzig* I 641
2	2 20 35.26	+13 1 17.6	*A.G. Leipzig* I 703	6	2 6 59.43	+11 29 23.4	*A.G. Leipzig* I 648
3	2 20 43.34	+12 57 58.8	*A.G. Leipzig* I 704	7	2 7 15.38	+11 22 03.4	*A.G. Leipzig* I 649
4	2 21 42.81	+13 8 22.3	*A.G. Leipzig* I 711				

The trails of three seventeenth magnitude asteroids were found on these plates. Their positions were measured and are given below.

Asteroid A

Plate	Gr. Mean Time 1916	α (1916.0)	δ (1916.0)
	d h m	h m s	° ′ ″
3443	Sept. 6 21 8.5	2 20 56.33	+12 49 22.4
3451	Sept. 6 23 59.5	2 20 59.13	+12 49 23.4

Mount Wilson Solar Observatory, December, 1916.

Asteroid B

Plate	Gr. Mean Time 1916	α (1916.0)	δ (1916.0)
	d h m	h m s	° ′ ″
3484	Oct. 18 17 30.5	2 7 33.6	+11 21 01
3487	Oct. 19 16 54.0	2 6 31.4	+11 20 59

Asteroid C

Plate	Gr. Mean Time 1916	α (1916.0)	δ (1916.0)
	d h m	h m s	
3487	Oct. 19 16 54.0	2 6 08.8	+11 49 16

The approximate daily motion of Asteroid *C* as determined from the trail was −51ˢ in α and −4′.6 in δ.

SCHAUMASSE COMET

Announcement is made of the discovery of a comet by SCHAUMASSE, April 25.6430 G. M. T., at right ascension 23ʰ 5ᵐ 11ˢ, declination +10° 19′. The comet is visible in a small telescope. Slow motion to northeast.

NEW ASTRONOMICAL WORK

Connaissance des Temps, Pour l'an 1918. Published by the Bureau des Longitudes, Paris 1916.

Tables Auxiliaries Pour la Resolution de l'Equation de Gauss. Published by Thadee Banachiewicz. 1916.

Publications of the Astronomical Observatory of the University of Michigan, Vol. II. Ann Arbor. 1916.

Catalogue de 14263 Etoiles. Photographic Zone +16° to +24°). Published by the Abbadia Observatory, Hendaye. 1915.

Observaciones en la Zona −52° a −56°. Tomo III. Published by the La Plata Observatory, La Plata. 1916.

The American Ephemeris and Nautical Almanac 1919. Published by the Nautical Almanac Office, Washington, D. C., 1917.

Tables Giving the Times of Rising and Setting of the Sun and Moon, 1917 and 1918. Published by the Nautical Almanac Office, Washington, D. C., 1917.

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ORBIT OF *KRUEGER 60*.

By HENRY NORRIS RUSSELL.

Most of the published methods of determining the orbits of visual binaries are based mainly, if not exclusively, upon the purely geometrical relation that the observed positions of the companion must lie upon an ellipse. Although COMSTOCK, many years ago, pointed out that of the three data connected by the ordinary record of an observation, the time is known with very much greater accuracy than the others, the dynamical relations connecting the coördinates and the time are rarely utilized. The calculations here given illustrate what advantages may sometimes be gained by taking these relations into account.

I. KRUEGER 60, (BURNHAM'S *Gen. Cat.* 11761, $22^h\ 24^m.5$, $+57°\ 12'$, 1900)

This system has long been remarkable on account of its large proper-motion, large parallax, and rapid orbital motion. Since 1900 it has been very assiduously observed by BARNARD, upon whose long and homogeneous series of measures the present orbit is principally based.

A. The Orbit.

The position angles and distances were plotted separately against the time, and smooth curves drawn to represent them. These curves were then adjusted by trial and error so that the condition of constant areal velocity was satisfied. For the interval 1900–1915 very little adjustment was required, the curves being sharply defined by the observations, and the constant of areas $s^2\frac{dp}{dt}$ being 41.0 if the distance is taken in seconds of arc and the angular velocity $\frac{dp}{dt}$ in degrees per year. In the case of BURNHAM's isolated measure of 1890, however, it was evident that the distance required to be sensibly increased to be consistent with the observed motion in angle, and the curves were drawn accordingly.

Upon plotting the points derived from these curves it was found that they all lay within $0''.02$ of an ellipse, which was adopted as the preliminary orbit called *B* below. This ellipse was very easily determined analytically by reading from the plot on coördinate paper the values of the y coördinate ($s \cos p$) corresponding to equidistant values of the x coördinate, ($s \sin p$) and taking the equation of the ellipse in the form

$$y = a + b(x - x_0) \pm c\sqrt{(x_0 - x)(x - x_1)}$$

where x_0 and x_1 are the maximum and minimum values of x for points on the ellipse. The coördinates of the centre, and the lengths and directions of a pair of conjugate axes, follow immediately from this equation, and the transition to the elements of the true orbit may then easily be made on LOHSE's modification of ZWIERS' method*.

In the case now under discussion, x_0 was $+3''.12$, and, except for the measure of 1890, all the values of x exceeded $2''.3$. The constant b was therefore rather poorly determined; but, when a value of this was adopted, the remaining constants were well determined, with the aid of the early position. Three orbits were therefore computed, corresponding to the values 0.00, −0.10 and −0.175 of b. These orbits are given in Table I, and the comparison of the observed and computed positions in Table II. The observations up to 1915 are taken from BARNARD's papers, (*M. N. 68*, 629, 1908, and *76*, 592, 1916.) BARNARD's observations of 1916, and DOOLITTLE's later measures, given in the lower part of the table, were very kindly communicated in manuscript by the authors. These were not used in determining the orbits.

The observations of 1916 show that the shortest period (Orbit A) is too short. Either of the other two will still do, and all three represent the observation of 1890 within its limits of uncertainty. The

*Zwiers, *A. N. 139*, 369, 1896; Lohse, *Potsdam Publications, 20*, 73, 1909.

TABLE I. ORBITS OF KRUEGER 60

		A	B	C
Apparent Orbit.				
Length of projection of major semi-axis	α	2″.09	2″.44	2″.80
Length of conjugate semi-axis	β	2″.07	2″.15	2″.26
Position angle of projection of periastron	X_1	305°.4	306°.8	308°.6
Position angle of conjugate axis	X_2	37°.0	41°.2	46°.6
Star from centre		1″.21	0″.87	0″.51
True Orbit.				
Major semi-axis	a	2″.55	2″.49	2″.86
Eccentricity	e	0.590	0.357	0.182
Inclination	γ	35°.0	25°.1	39°.0
Node	☊	40°.3	102°.8	113°.6
Periastron minus node	λ	274°.0	153°.8	161°.0
Period	P	37.8	46.0	54.9 yrs.
Periastron passage	T	1921.8	1925.6	1929 .3

Angles Decreasing.

TABLE II. COMPARISON WITH OBSERVATIONS

Date	Obs.	n	p	s	O − C Orbit A		O − C Orbit B		O − C Orbit C	
1890.79	β	1	178°.8	2″.32	−0°.9	−0″.23	−2°.9	−0″.18	−4°.5	−0″.14
1898.45	D	5	140 .7	3 .19	−2 .0	− .02	−2 .3	+ .02	−2 .4	+ .05
1900.84	B, D		133 .6	3 .22	+0 .5	− .07	+0 .3	− .05	+0 .2	− .02
1901.79	B	7	130 .7	3 .28	+1 .0	− .04	+0 .9	− .02	+0 .9	.00
1902.76	B	4	127 .0	3 .36	+0 .9	+ .03	+0 .9	+ .04	+0 .9	+ .05
1903.56	B	15	123 .5	3 .37	+0 .5	+ .04	+0 .5	+ .04	+0 .5	+ .04
1904.58	B	19	119 .8	3 .38	+0 .5	+ .05	+0 .5	+ .05	+0 .5	+ .05
1905.91	B	2	115 .3	3 .31	+0 .9	.00	+0 .9	.00	+0 .9	.00
1906.56	B	7	112 .2	3 .32	+0 .2	+ .03	+0 .2	+ .02	+0 .2	+ .02
1907.58	B	12	107 .2	3 .28	−0 .8	+ .03	−0 .8	+ .02	−0 .8	+ .02
1908.54	B	12	103 .9	3 .18	−0 .6	− .04	−0 .5	− .04	−0 .4	− .04
1909.63	B	16	98 .8	3 .14	−1 .4	− .01	−1 .3	− .01	−1 .2	− .01
1910.71	B	21	94 .8	3 .10	−0 .7	+ .02	−0 .7	+ .03	−0 .7	+ .04
1911.71	B	10	90 .8	2 .99	−0 .4	− .01	−0 .4	.00	−0 .4	+ .01
1912.70	B	11	86 .0	2 .87	−0 .4	− .03	−0 .4	− .02	−0 .4	− .02
1913.55	B	8, 7	81 .2	2 .85	−0 .8	+ .05	−0 .8	+ .05	−0 .8	+ .05
1915.68	B	18, 17	69 .1	2 .54	−0 .3	+ .04	−0 .3	+ .01	−0 .3	− .01
1916.61	B	17, 18	63 .2	2 .45	+0 .5	+ .15	−0 .1	+ .05	−0 .3	.00
1906.98	D	2	111 .7	3 .37	+1 .3	+ .09	+1 .3	+ .08	+1 .3	+ .08
1907.21	D	2	106 .2	3 .30	−3 .3	+ .03	−3 .3	+ .02	−3 .3	+ .02
1908.83	D	5	103 .3	3 .24	−0 .1	+ .01	0 .0	+ .04	+0 .1	+ .04
1909.75	D	3	99 .3	3 .00	−0 .3	− .14	−0 .2	− .14	−0 .1	− .14
1911.73	D	5	90 .4	3 .00	−0 .7	.00	−0 .7	+ .01	−0 .7	+ .02
1913.71	D	5	78 .3	2 .97	−3 .4	+ .19	−3 .4	+ .19	−3 .4	+ .19
1915.70	D	7	69 .3	2 .56	+0 .2	+ .06	+0 .2	+ .03	+0 .2	+ .01

In the column headed "Obs." β denotes BURNHAM; B, BARNARD, and D, DOOLITTLE.

ephemerides given in Table III show that it will be possible to determine the period and the remaining elements within narrow limits after five or six years; further observations are available.

TABLE III. EPHEMERIDES

Date	Orbit B		Orbit C	
1916.5	64°.1	2″.41	64°.3	2″.46
1917.5	56 .5	2 .27	57 .2	2 .34
1918.5	47 .9	2 .12	49 .5	2 .24
1919.5	38 .0	1 .97	40 .9	2 .15
1920.5	26 .9	1 .83	31 .6	2 .09
1921.5	14 .0	1 .71	22 .0	2 .04
1922.5	359 .1	1 .62	11 .9	2 .01
1923.5	342 .8	1 .57	1 .7	2 .00
1924.5	325 .4	1 .55	351 .5	2 .02
1925.5	308 .6	1 .57	341 .8	2 .08

It may be worth while to add that much time was saved in computing the ephemerides by starting with equidistant values of the eccentric anomaly, and computing one way to the coördinates, and the other to the time, and plotting the results,— thus avoiding the solution of KEPLER's equation.

B. The Mass Ratio.

The measures of the distant optical companion C now afford material for a determination of the ratio of the masses of the components of the binary system which should entirely supersede that published by the writer in 1910.* In the present solution, the observations from 1890 to 1916 are employed, as summarized in Table IV. In this table the first column gives the date, the next two the rectangular coördinates x_B and y_B of the binary companion relative to the principal star, computed from Orbit B, according to the relations $x = s \cos (p - 59°)$, $y = s \sin (p - 59°)$, and the fourth and fifth the corresponding rectangular coördinates of the distant star C with reference to A, (already corrected for a parallax of 0″.25). A preliminary examination of these quantities shows that they are very nearly represented by the expressions

$$x_c = \quad 44''.25 + 0''.900\ (t - 1910) + 0.40\ x_B$$
$$y_c = -\ 0''.50 + 0''.090\ (t - 1910) + 0.40\ y_B$$

The excesses of the observed values of x_c, y_c above the amounts computed by these formulæ are given in the sixth and seventh columns, under the headings Δx, Δy.

The equations of condition for determining corrections to the constants just written are of the form

$$x + (t - 1910)\ \mu\ + z x_B = \Delta x$$
$$y + (t - 1910)\ \mu' + z y_B = \Delta y$$

and may be immediately written down. They were assigned weights as shown in the last column of the table, in proportion to the number of nights of observation. DOOLITTLE's measure of 1898, which shows a very large but unexplainable residual in distance, was rejected. The resulting solutions were found to depend to a considerable degree upon the single measure of 1890,— the normal equations and solutions, according as this measure was included or rejected, being as follows:

Including the Measure of 1890

$$\begin{aligned} 23.0\,x &\ - 52.4\,\mu &+ 43.90\,z &= -0''.02 \\ -52.4 &\ +755.0 &- 21.76 &= -0\ .31 \\ +43.90 &\ - 21.76 &+ 95.21 &= -0\ .39 \end{aligned}$$

$x = +0''.37$ weight 0.38
$\mu = +0\ .021$ weight 99
$z = -0\ .17$ weight 1.79

$$\begin{aligned} 23.0\,y &\ - 52.4\,\mu' &+ 49.75\,z &= +0''.21 \\ -52.4 &\ +755.0 &-207.0 &= -5\ .40 \\ +49.75 &\ -207.0 &+126.34 &= +1.\ 45 \end{aligned}$$

$y = -0''.11$ weight 1.37
$\mu' = -0\ .003$ weight 167
$z = +0\ .05$ weight 4.93

Excluding it

$$\begin{aligned} 22.5\,x &\ - 42.8\,\mu &+ 44.58\,z &= -0''.16 \\ -42.8 &\ +571.0 &- 34.94 &= +2\ .47 \\ +44.58 &\ - 34.94 &+ 94.27 &= -0\ .20 \end{aligned}$$

$x = +0''.23$ weight 0.19
$\mu = -0\ .015$ weight 71
$z = -0\ .11$ weight 0.88

$$\begin{aligned} 22.5\,y &\ - 42.8\,\mu' &+ 48.7\,z &= +0''.26 \\ -42.8 &\ +571.0 &-186.8 &= -6\ .36 \\ +48.7 &\ -186.8 &+124.11 &= +1\ .55 \end{aligned}$$

$y = +0''.42$ weight 0.18
$\mu' = -0\ .058$ weight 155
$z = -0\ .24$ weight 0.59

* *Astrophysical Journal*, 32, 363, 1910.

It appears from these solutions that the value of z determined from the x coördinates, though of relatively small weight, is little influenced by the inclusion or exclusion of the early measure, but that the value derived from the y coördinates depends almost wholly on this measure. The x measures appear to be considerably more accurate than the y's, and to deserve about double the weight, (which is not surprising, as the former depend on the measured distances of star C and the latter on the position angles.) If, finally, the equations from the x and y coördinates are solved together, with weights as stated, the results are

	Solution I Including the Observation of 1890		Solution II Excluding it	
x =	$+0''.086$	$\pm0''.050$	$+0''.304$	$\pm0''.058$
y =	$+0\ .066$	$\pm0\ .043$	$+0\ .238$	$\pm0\ .019$
μ =	$+0\ .004$	$\pm0\ .003$	$+0\ .019$	$\pm0\ .003$
μ' =	$-0\ .014$	$\pm0\ .004$	$-0\ .039$	$\pm0\ .005$
z =	$-0\ .041$	$\pm0\ .022$	$-0\ .139$	$\pm0\ .027$
P.E. of unit wt.		$\pm0\ .065$		$\pm0\ .041$

The residuals from both these solutions are given in Table IV. It is difficult to decide between them. The weighted sum of the squares of the residuals for the observations from 1900 to 1916 is 0.1543 for Solution I and 0.0993 for Solution II, — so that the rejection of the early observation very decidedly improves the representation of the later ones; but for this early measure Solution II gives residuals of $+0''.45$ in distance and $-1^\circ.67$ in angle, — the last of which appears inadmissibly large. For the present it seems best to retain both solutions, waiting for the observations of the next ten years to decide the matter.

Since the distance of the center of gravity of the binary system is at the fraction $0.40+z$ of the distance from Star A to Star B, the values of the proper-motion, of the system and of the mass ratio, resulting from the two solutions, are; —

	Solution I		Solution II	
μ =	$+0''.904$	$\pm0''.003$	$+0''.919$	$\pm0''.003$
μ' =	$+0\ .076$	$\pm0\ .004$	$+0\ .051$	$\pm0\ .006$
$B:A$ =	$0\ .56$	$\pm0\ .06$	$0\ .35$	$\pm0\ .05$

Proper-motion $0''.907$ toward $243^\circ.8$ (I)
$0''.920$ toward $242^\circ.2$ (II)

It seems therefore certain that the fainter star is considerably less massive than the brighter component of the binary. The contrary result obtained in 1910 can now be seen to be due entirely to the overwhelming influence of the isolated measure of 1890 upon the results then obtainable. In a few years more a good determination of the mass ratio can be made independently of this uncertain observation.

Table IV. Measures of $A\,C$

							Solution I O—C		Solution II O—C		
Date	x_b	y_B	x_c	y_c	Δx	Δy	x	y	x	y	Weight
	$''$	$''$	$''$	$''$	$''$	$''$	$''$	$''$	$''$	$''$	
1890.79	−1.37	+2.11	+26.70	−1.18	+0.29	−0.10	+0.24	−0.34	(+0.15)	(−0.78)	½
1898.45	+0.33	+3.15	34.61	−0.15	+ .63	+ .13	(+ .61)	(+ .03)	(+ .58)	(− .09)	0
1900.84	+0.88	+3.14	36.36	+0.02	.00	+ .08	− .01	+ .02	− .02	− .08	1
1901.77	+1.09	+3.14	37.23	+0.19	− .04	+ .18	− .04	+ .13	− .04	+ .06	1
1902.76	+1.29	+3.07	38.25	+0.21	− .04	+ .13	− .04	+ .09	.00	+ .04	1
1903.63	+1.46	+2.99	39.12	+0.30	+ .02	+ .11	+ .03	+ .08	+ .04	+ .04	3
1904.62	+1.66	+2.89	40.03	+0.19	− .04	+ .01	− .03	− .01	− .01	− .05	3
1905.91	+1.88	+2.72	41.21	+0.37	− .11	+ .15	− .10	+ .14	− .08	+ .13	½
1906.56	+1.99	+2.64	41.83	+0.21	− .12	− .04	− .11	− .04	− .09	− .05	1
1907.57	+2.13	+2.46	42.84	+0.21	− .07	− .05	− .06	− .05	− .04	− .04	1
1908.36	+2.23	+2.33	43.66	+0.22	.00	− .06	+ .04	− .05	+ .02	− .03	1
1909.63	+2.36	+2.07	44.87	+0.25	+ .04	− .05	+ .02	− .03	+ .03	− .01	2
1910.71	+2.46	+1.82	45.89	+0.22	+ .02	− .07	+ .03	− .05	+ .03	− .03	2
1911.70	+2.51	+1.59	46.74	+0.28	− .04	− .01	− .03	+ .01	− .03	+ .03	1
1912.70	+2.56	+1.33	47.70	+0.25	.00	− .02	.00	+ .01	.00	+ .03	1
1913.55	+2.57	+1.10	48.48	+0.25	.00	− .01	.00	+ .02	.00	+ .04	1
1915.67	+2.49	+0.47	50.38	+0.13	+ .03	− .07	+ .02	− .04	− .03	− .02	2
1916.50	+2.40	+0.21	51.17	+0.17	− .11	.00	+ .09	+ .04	+ .02	+ .04	1

C. The Proper-motion.

The measures of the other faint stars nearby, which are given in BARNARD's papers, have been corrected for parallax, reduced to rectangular coördinates referred to the axes used in the preceding section, and solved by least squares to determine the proper-motion. In these solutions the center of gravity of the system has been taken as 0.26 of the way from Star *A* to Star *B*, in accordance with Solution II above. The resulting values of the proper-motion (including for convenience that previously found from Star *C*) are given in Table V, which gives also the magnitudes of the reference stars, the number of years in which observations were obtained, and the interval covered by these observations. The probable error of the position of the center of gravity has been taken into account in determining the probable errors of the proper-motions.

TABLE V. PROPER-MOTION

Star	Mag.	Years	Interval	P. M.			
C	9.5	16	15.7	0″.920	±0″.003	towards 242°.1	±0°.3
D	15.2	6	11.3	0 .881	±0 .008	towards 241 .7	±0 .8
E	12.6	6	12.7	0 .918	±0 .006	towards 241 .7	±0 .5
F	14.1	8	13.9	0 .928	±0 .007	towards 242 .2	±0 .4
Mean				0 .912		towards 241 .9	

The values obtained from the four stars are in excellent agreement, indicating that these stars have little proper-motion of their own. According to KAPTEYN's latest estimates (*Groningen Publications 24*, 15) the mean parallax of these four stars should be 0″.002, and their mean parallactic motion therefore 0″.006 towards 88°. If allowance is made for this, the true proper-motion of the binary becomes 0″.907 towards 241°.9. This is, however, still somewhat uncertain on account of the uncertainty of the position of the center of gravity of the system. If this is taken according to Solution I, the proper-motion comes out 0″.893 towards 243°.5.

D. Astrophysical Data.

The system of KREUGER 60 is exceptional in many ways. Its parallax is remarkably large, and very well determined. Adopting the value 0″.256 ±0″.005 (the mean of the concordant determinations of BARNARD, SCHLESINGER and the writer) and the visual magnitude 9.43 for the combined light of the pair, as measured at Harvard, the absolute magnitude corresponding to the combined light is 11.5, — to which corresponds a luminosity 0.0022 that of the *Sun*, (taking the visual magnitude of the latter as −26.72, and its absolute magnitude as +4.85). The relative brightness of the components has never been accurately measured. DOOLITTLE estimated them as 9.1 and 10.5; BARNARD, in 1900–01, as 9.3 and 10.5, but in 1915–16 as 9.7 and 12.2. Taking the difference of magnitude as 1.7, (the mean of these discordant estimates) the magnitudes of the components come out 9.6 and 11.3, their absolute magnitudes 11.7 and 13.4, and their luminosities 1/550 and 1/2600 that of the *Sun*. The spectrum of the brighter component, according to ADAMS, is of class *Mb*.* That of the fainter component is unknown, and may be beyond the reach of present observation; but its color index should be determinable, and would be of much interest.

With the parallax 0″.256, the actual mean distance is 9.7 astronomical units, according to orbit *B*, and 10.9 according to orbit *C*, while the mass of the system comes out 0.44 times the *Sun's* mass in the first case, and 0.47 in the second. The orbit *A*, rejected for discordance with the latest observations, gives the larger value 0.70 for the mass. All the orbits, however, which are consistent with the observations to date give very nearly the same value for the mass.

This phenomenon often appears in the case of double stars with poorly determined orbits but well known parallaxes, and admits of a simple explanation. The observations, extending over but a portion of the circumference of the apparent orbit, suffice to determine with precision the apparent distance of the companion at some date near the middle of the interval covered by the observations, the apparent velocity of the orbital motion at that time, and its curvature, — that is to say, the apparent acceleration of the companion towards the principal star. These are not the true distances, velocity and acceleration, but the projections of the latter upon the "plane of the sky." If θ is the angle which the true radius vector makes with this plane at the time considered, and if s and s', a and a', denote the true and apparent distances and

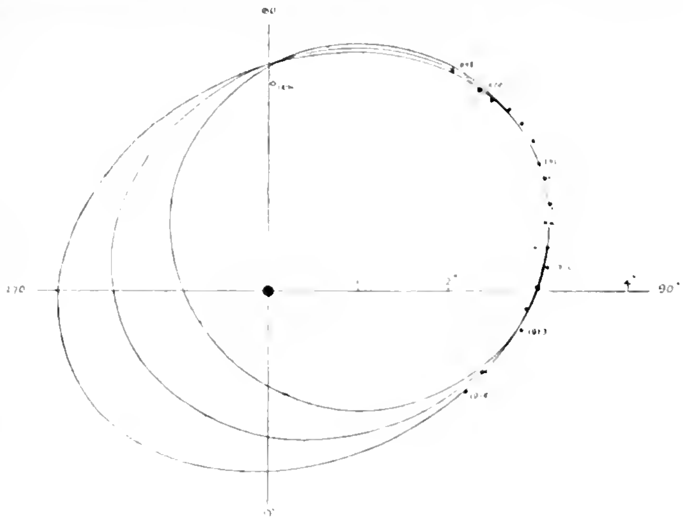
* *Publications of the Astronomical Society of the Pacific*, *26*, 258, 1915.

accelerations, we have $s' = s \cos\theta$, $\alpha' = \alpha \cos\theta$. The mass of the system is evidently proportional to $s^2\alpha$; but this equals $s'^2\alpha' \sec^3\theta$. Now any orbit which represents the observations must reproduce closely the observed values of s' and α'; but the assumed position of the orbit plane, and hence the angle θ, will in general be different for the various possible orbits, and the computed masses will vary as $\sec^3\theta$. If the computed position of the node falls near the middle of the observed arc, the value of θ will almost always be small, and the computed mass will also be near its smallest possible value; but, if the node be far from this position, θ will be a considerable angle, (unless the inclination is small) and the computed mass will be greater.

The present instance affords an excellent illustration of these principles. The accurately observed arc extends from position angle 140° to 63°. Orbits *B* and *C* place the node respectively in 103° and 113°, near the middle of this arc, and lead to nearly the same value for the mass. Orbit *A* places the node in 40°, with inclination 35°, and makes the value of θ for the middle of the observed arc about 30°. It might therefore be expected that the mass computed from this orbit would be greater than that from the others in about the ratio $\sec^3 30° : 1$, or 1.5 : 1, and this indeed is the case.

It may safely be concluded that the mass of the system is 0.45 that of the *Sun*, with a probable error of about ten per cent. The masses of the components *A* and *B* then come out 0.29 and 0.16 according to Solution I above, and 0.33 and 0.12 according to Solution II. The fainter component is therefore probably between one-sixth and one-eighth as massive as the *Sun*, — which is by far the smallest mass which has so far been assigned to a visible star on the basis of reliable data of any sort, and tends to support the opinions expressed by ADAMS and by the writer to the effect that such very faint stars are probably of small mass.

It is noteworthy, however, that these small stars are very massive, in proportion to their luminosity. If the two components were equal in density and surface brightness to the *Sun*, their diameters would be 0.69 and 0.49 of the *Sun's*, and their luminosities 260 and 620 times the observed values. Even if their mean densities are assumed to be ten times that of the *Sun*, or fourteen times that of water, (surely an extreme assumption) their diameters would be 0.32 and 0.23 times the *Sun's* and to make them as faint as they are, their surface intensities would have to be only 1/53 and 1/135 of the *Sun's*. Such values are however entirely consistent with the presumably



very low temperature indicated by the spectral type of even the brighter component. It is obvious that these stars exhibit every characteristic which might be supposed typical of bodies at the very end of their evolutionary history, and on the verge of extinction.

The radial velocity of the brighter component, according to four Mt. Wilson plates, (which has very kindly been communicated to the writer by Professor ADAMS) is −24 km. The radial component of the orbital velocity can hardly at present exceed 0.5 km. so that this may be taken as the radial velocity of the system. With the parallax and proper-motion adopted above, the component of velocity at right angles to the line of sight is 16.8 km. It follows that the velocity of the system in space, relatively to the *Sun*, is 29 km., directed towards $14^h\,50^m$, −57°, and making an angle of 145° with the present direction of the star from the *Sun*. With this motion, it will continue to approach us for 106,000 years, and at that time will be in *Aquila*, [$19^h\,10^m$, +16°, 1900)], will be of magnitude 8.2, have a proper-motion of 2″.76, and a parallax of 0″.45. Taking the solar motion as 19.5 km. towards 18^h, +30°, the motion of the binary system, referred to the stars as a whole, comes out 33 km. towards $16^h\,27^m$, −26°. A superficial search among other stars of large parallax and proper-motion failed to find any whose motion was sufficiently similar to suggest any suspicion of physical connection.

Figure 1 shows the observed positions of the companion *B*, (BARNARD's observations being represented by dots, DOOLITTLE's by crosses, and BURNHAM's by an open circle) and the three apparent orbits *A*, *B*, *C*. The next ten years' observations will make this one of the best known systems in the heavens.

Princeton University Observatory, 1917, January.

ON THE SECULAR CHANGES IN THE PROPER-MOTIONS AND OTHER ELEMENTS OF CERTAIN STARS,

By FRANK SCHLESINGER.

The discovery in recent years of several stars having large parallaxes and large proper-motions has brought into the realm of practical astronomy what was formerly only of theoretical interest; namely, the secular changes in the proper-motion, the parallax and the radial velocity of a star caused by the star's movement in space. In what follows we assume that all the stars and the *Sun* continue to move with uniform linear velocity.

Let μ, π and V be respectively the annual proper-motion in arc of a great circle, the parallax, and the radial velocity in kilometers a second, counted positive if the star is receding from us; let R and T be the radial and tangential (or cross) motion in a year, and d the distance of the star, all of these being expressed in kilometers. We have then

$$R = V \times 3.156 \times 10^7$$

$$T = \frac{\mu}{\pi} \times 1.495 \times 10^8$$

$$d = \frac{3.083}{\pi} \times 10^{13}$$

After the lapse of t years the distance becomes $d + t \cdot R$, neglecting the very small increase due to the cross motion. Considering the reciprocal of the new distance we see that the parallax will be altered by

$$-\frac{t \cdot R \cdot \pi^2}{3.083} \times 10^{-13}$$

or $$-t \cdot V \cdot \pi^2 \times 1.024 \times 10^{-6}$$

This gives the change in seconds of arc. It is very small for every known star. For the 8th magnitude star Cordoba Zones 5^h 243, whose proper-motion, parallax and radial velocity are respectively $8''.7$, $0''.32$ and $+242$ kilometers, the parallax diminishes at the rate of $0''.0025$ a century. The 10th magnitude star in *Ophiuchus* recently discovered by BARNARD has a proper-motion of $10''.3$ per annum and a radial velocity of about -100 km. The writer's preliminary parallax for this star, $0''.50$, has been confirmed by MITCHELL. With these data it comes out that the parallax is increasing at the same rate with which that of C. Z. 5^h 243 is decreasing. For all other known stars the change is smaller.

The radial velocity of a star changes on account of the variation in the angle between the line of sight and the direction of motion of the star. This angle is given by

$$\tan \theta = \frac{T}{R} = 4.74 \frac{\mu}{\pi \cdot V}$$

The angle subtended by two positions of the star separated by the interval t is approximately $t \cdot \mu$, and consequently the new radial velocity per second will be

$$V \cos (t \cdot \mu) + 4.74 \frac{\mu}{\pi} \sin (t \cdot \mu)$$

Neglecting terms in t^2 and beyond, the change in velocity is

$$+ 2.30 \frac{\mu^2}{\pi} t \times 10^{-5}$$

It will be noticed that the velocity itself does not appear in this expression. The change is always positive; in other words, the radial velocities of all stars are always increasing, a statement that may seem startling at first sight, but only at first sight.

This change is also very small. For either of the two stars mentioned above it amounts to half a kilometer after the lapse of a century. It is greatest in the case of 1830 GROOMBRIDGE, whose proper-motion is $7''.1$, and whose parallax is $0''.10$. But even in this case the change in a century is only a little more than one kilometer a second.

We next consider the proper-motion. Let us denote by σ the angle between two positions of the star with an interval of t years. This angle is given by the rigorous equation

$$\tan \sigma = \frac{t \cdot T}{d + t \cdot R}$$

Very approximately

$$\sigma = 206265 \frac{t \cdot T}{d}\left(1 - \frac{t \cdot R}{d}\right),$$

or

$$\sigma = t \cdot \mu - t^2 \cdot \mu \cdot V \cdot \pi \times 1.024 \times 10^{-6}$$

If we differentiate this with respect to t we obtain the proper-motion at the time t:

$$\mu - t\,\mu\,V\,\pi \times 2.05 \times 10^{-6}$$

The secular term in the proper-motion amounts to considerable in the case of both C. Z. 5^h 243 and of the star in *Ophiuchus*; for the former it is $-0''.138$ a century and for the latter $+0''.105$. That is if the annual change in position of the latter is say just $10''.30$ at the beginning of the 20th century, then the annual change at the beginning of the next century will be $10''.405$.

Returning to the expression for σ just derived, let us consider the effect of the quadratic term on the position of the star in *Ophiuchus*. The effect is greater in the case of C.Z. 5^h 243, but as the former is visible at moderate zenith distances from every observatory, it has been better observed in the past and is apt to be better observed in the future. Let us assume that the proper-motion of this object was $10''.30$ in 1900. Then its position for any other date, referred to its position for 1900 will be

$$\sigma = +10''.30\,t + (t - 1900)^2 \times 0''.00053$$

Thus the position in 1920 will differ by $0''.2$ from the position that would have been reached had the proper-motion of 1900 been maintained; in 1950 the difference will be $1''.3$ and in 2000 no less than $5''$. The presence and sign of this quadratic term can unquestionably be demonstrated from accurate observations covering the next ten years, if indeed the data for this purpose do not already exist. To determine the quadratic term accurately, say with a probable error of five per cent, is another matter, and would require observations covering several decades. For it must be remembered that the proper-motion itself would have to be considered an unknown and that therefore the maximum deviations from uniform motion are not the quantities quoted above, but only a fraction of them.

If the positions of this and similar stars are accurately observed for sufficiently long periods of time and the quadratic terms determined in this way, then we shall be in position to determine the radial velocities of these stars independently of the spectroscope and with an excellent degree of precision. For example, suppose that observations of this kind should show that the quadratic term in the case of the star in *Ophiuchus* amounts to $-0''.00050$ with a probable error of $0''.00001$, and suppose that the definitive parallax of the star comes out $0''.50$ with a probable error of $0''.005$; then by substituting these and the value of the proper-motion in the algebraic expression for the quadratic term, we find the radial velocity to be -95 kilometers a second, with a probable error of 3 kilometers.

Allegheny Observatory of the University of Pittsburgh,
19 February 1917.

PARALLAX OF BARNARD'S PROPER-MOTION STAR,

By OLIVER J. LEE.

The following preliminary value of the parallax of the "Runaway" star has been obtained from nine plates taken with the forty-inch telescope within the last eight months:

$$+0''.52 \pm 0''.02$$

Yerkes Observatory, 19 February 1917.

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Editor, Benjamin Boss, Albany, N. Y. Associate Editors: E. E. Barnard, Ernest W. Brown, F. R. Moulton and R. S. Woodward.
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THE ORBIT OF ζ *HERCULIS*.

Σ 2084 Burnham, *G. C.*, 7747 $\alpha = 16^h 37^m.6$ $\delta = 31° 47'$

By GEORGE C. COMSTOCK.

This binary star has been well observed over more than two revolutions in its orbit, and elements of its motion have been derived by a considerable number of astronomers, but none of these elements that have come under my observation represent the observed places in a satisfactory manner. This failure to represent the data has been the subject of comment by Lohse (Publikationen, Potsdam No. 58) who suggests as its explanation, systematic errors of observation affecting the data; and by Lewis (Memoirs R. A. S. Vol. 56, p. 462 *et seq.*) who as the result of an elaborate investigation suggests substantial changes in the orbit itself, *viz.*, a progressive increase in the major axis and eccentricity and an oscillating value of the periodic time. I endeavor in the following pages to examine the suggestions of both investigators and to determine how far they are needed or will suffice to represent a considerable body of subsequent data as well as the material utilized by them and other investigators.

I proceed first to the question of systematic errors of observation, or more properly systematic differences of a personal character, which render the data obtained by different observers not comparable *inter se* and ill adapted to combination. I have chosen to make the investigation of such possible differences depend solely upon observations of ζ *Herculis*, rather than to attempt to reduce to a common standard the heterogeneous mass of observations of many double stars made by many observers, and by comparison with this standard to determine for each observer peculiarities which might be assumed to affect his observations of a given star. To this end I have selected from the observations of ζ *Herculis* collected by Lewis (*loc cit*) and See (*Evolution of the Stellar Systems*, I) all observations which satisfy the following conditions and have added to these all accessible later observations included under the same criteria, *viz.*, (*a*) There must be available observations made by the same observer under substantially similar circumstances in at least five different years. (*b*) These observations must not be too widely separated in point of time. The criterion *a* is intended to secure an amount of data sufficient in quantity to afford some index to the peculiarities of the observer. It certainly does not err in demanding too much, while insistence on more observations would have limited unduly the available material. The criterion *b* is intended to guard against the effect of changed habits of observing, such as are suspected to accrue with lapse of time. Fortunately it has not been found necessary to define precisely the time limits here contemplated. No observer has been consciously omitted whose work satisfies these two criteria, but in a single case, specially noted below, the criteria have been slightly relaxed in favor of a well known observer.

All of the available material falls between the epochs 1826 and 1916 A.D. and covers approximately two and a half revolutions in the orbit. By adding to the earlier dates and subtracting from the later dates of observation an approximate value of the periodic time, 34.47 years, I have condensed all this data within the period covered by a single revolution in the orbit, and with the corrected times as abscissæ the observed position angles (reduced for precession to the epoch 1900) and the observed distances have been separately plotted upon an open scale. A smooth curve drawn through the plotted points is assumed to represent the best available approximation to a normal system of observations of this star and the deviation of any plotted point from this curve (measured along an ordinate) becomes a measure of the peculiarity attaching to the observation in question. The mean of such deviations for an individual observer is then adopted as his systematic peculiarity or systematic

error. The following objections to this procedure have been constantly borne in mind and are believed not to affect seriously its validity:

a. An error in the assumed periodic time will affect with errors of opposite sign the peculiarities attributed to the early and late observations. But the peculiarities resulting from the curve show no marked tendency to present opposite signs in the early and late observations and the assumed periodic time used in the reduction differs by only four days from that finally adopted.

b. Any real disturbance in the motion of the star, *e.g.*, such as might be produced by an invisible companion, would be interpreted as an error of observation. Unless such disturbance have a period which is approximately a sub-multiple of 34.47 years, its effect will be considerably mitigated by the dependence of each point of the curve upon observations made in two or three different revolutions of the companion in its orbit. If there be a disturbance having as its period a sub-multiple of 34.47 years, the curve furnishes an excellent means of detecting it. No appreciable sinuosities which can be attributed to such a source are displayed by the curve.

c. I anticipated, *a priori*, that the peculiarities of each observer would be dependent to a considerable extent upon the angular distance separating the *comes* from its primary. With this in mind I have reduced each error in position angle to its equivalent in arc of a great circle and have tabulated for each observer the mean of these equivalents, together with the mean of the distance errors directly read from the curve. While a certain possible dependence of these errors upon the distance between the stars is not wholly excluded by the data, such dependence is certainly not a conspicuous feature of the data, and I have ignored it in the further discussion.

The results of the examination thus conducted may be summarized as follows: The systematic errors are generally small (less than 0″.1 in each coördinate) but they are of appreciable magnitude for about one-half of the observers considered and in a few cases they are so conspicuously variable as to be extremely embarrassing. In respect of both systematic and accidental error, observations of position angle are decidedly superior to observations of distance, or at least, they are more consistent among themselves. The earliest observations utilized, *viz.*, those of W. STRUVE, OTTO STRUVE, MAEDLER and DAWES, appear to be inferior in quality to more modern ones, but upon the whole this difference is not so great as to require special treatment for them.

The individual results of this investigation are shown in Table I, whose several columns appear to be sufficiently explained by their headings and the following remarks: In the column n are given the number of annual mean results included in each group for which the adopted systematic correction is given; the number of individual results may usually be assumed as from 2 to 4 times n. The column r_1 gives the computed probable error of a single annual mean derived upon the assumption of a constant value for the correction C − O. On account of paucity of data the individual values of r_1 must be considered very uncertain, but their totality conveys a fair idea of the average quality of the material employed and emphasizes the contrast above drawn between the quality of position angles and distances. Usually a single systematic correction in each coördinate has been derived for each observer, but in three cases it has been found necessary to depart from this rule, as is set forth in the following notes concerning individual observers.

NOTES CONCERNING INDIVIDUAL OBSERVERS

MAEDLER. The corrections to the position angles fall into two distinct groups while the distance measures show no such difference. The latter are, however, affected by an extraordinary systematic error, which appears to be the chief cause of the small size of the orbit of ζ *Herculis*, as found by LEWIS from the observations of 1826-56. There can be no doubt as to the sign and the large magnitude of the correction to the distances here found, since each of the 18 residuals is included between the limits +0″.05 and +0″.46. That this peculiarity is chargeable to MAEDLER rather than to the star's motion is apparent from its non-appearance in the simultaneous observations of Σ, DAW, OΣ, SEC, and DU.

OTTO STRUVE. I have used the data marked *Corrigée* and have divided it into three groups in accordance with OΣ's own suggestion (Obs. Poulk. Vol. IX, p. (113)) that his systematic errors are apparently constant within these periods but differ from one period to another. The result is very disappointing, particularly in the third period, where the error varies capriciously from year to year and large discordances are common. This condition could be somewhat improved by breaking the interval 1853–88 into two or more sections, but I have not done this, preferring to rely upon a mean value of the error. The situation would not be sensibly improved by using the data marked, *Observée*.

Table I. Adopted Systematic Corrections

Observer	Symbol	Epoch	Position Angle C − O	n	r_1	Distance C − O	n	r_1
W. Struve	Σ	1826.10	+0.05	8		+0.04	8	
Maedler	Ma	1836.46	− .03	7		+ .29	18	±.082
Maedler	Ma	1846.73	+ .05	15	±.025			
O. Struve	OΣ	1836.13	− .06	5		+ .01	1	
O. Struve	OΣ	1843.53	− .07	8		+ .11	9	
O. Struve	OΣ	1853.88	+ .02	24	.064	− .07	22	.068
Dawes	Daw	1840.66	− .02	14	.026	− .04	14	.082
Dembowski	Δ	1855.78	.00	19	.039			
Dembowski	Δ	1867.71				+ .08	1	
Dembowski	Δ	1871.78				− .13	8	
Secchi	Sec	1855.60	.00	6		+ .15	6	
Duner	Du	1867.75	− .04	9		− .06	9	
Schiaparelli	Sp	1875.98	+ .03	20	.027	.00	21	.055
Hall	Hall	1876.91	− .04	14	.016	+ .01	13	.046
Burnham	β	1878.81	− .04	5		− .03	6	
H. Struve	HΣ	1885.95	.00	10	.030	− .09	9	
Comstock	C	1888.16	− .03	22	.037	− .09	21	.065
Maw	Maw	1888.04	− .01	11	.026	− .03	11	.083
Lewis	Lw	1894.11	+ .01	15	.058	.00	15	.073
Aitkin	A	1897.14	+ .01	13	.024	.00	13	.058
Bowyer	Bow	1898.12	+ .01	14	.036	− .01	14	.086
Bryant	Bry	1898.12	+ .01	10	.043	+ .13	10	.017
Doolittle	Doo	1900.10	.00	6		− .06	6	
Biesbroeck	Bies	1903.15	.00	9		+ .10	9	

Dembowski. I have not used Δ's estimates of distance made prior to 1867. The later measured distances seem to fall into two sharply distinguished classes, all residuals in the first class being positive, all in the second class negative.

Secchi. There is some indication that Secchi's residuals steadily increase during the period here covered, but the data are insufficient to establish more than a mean value of the correction.

Hall. The correction to the observed position angles is remarkably constant. Each of the 14 annual means requires a negative correction.

Burnham. β's observations were made with different instruments of very diverse character. Although it is doubtful if they are homogeneous, I have taken a mean value of the residuals.

Bryant. The considerable correction to Bryant's distances seems well established. Each of the 10 annual means gives a positive residual included between 0″.04 and 0″.17.

By use of the systematic corrections tabulated above, the raw material available for an investigation of the orbit was combined into a set of normal places as follows: To each annual mean result of each observer, when based upon from 2 to 10 nights observing, there was assigned unit weight; a similar result based upon more than 10 nights was given weight 2; and to a single observation was assigned weight 0.5. A weighted mean result, corrected for systematic error and reduced to the epoch 1900, was then found for each year or, in case of abundant material, more than one such result was formed for the year. Table II shows these normal places, their dates, the observers upon whose work they are based, and under the rubrics $\Delta\theta$, Δs the systematic corrections which are incorporated in the printed values of position angle, θ, and distance, s. Under the rubric Wt. is given the weight of each normal place and the later columns of the table give results of the discussion that are explained below. With the time as abscissae these position angles and distances have been plotted as

Table II

Observer	Date	Wt	Δθ	θ	Δs	s	O − C Δθ	O − C Δs
Σ	1826.63	1	+3.1	26.0	+.01	0.95	1.0	+0.01
Σ	32.75	0.5	+3.5	223.6	+.01	0.85	− 1.1	− .09
Σ	34.15	1	+3.1	206.2	+.01	0.95	− 0.5	− .10
Σ	35.15	1	+2.6	199.1	+.01	1.13	+ 2.4	+ .01
Σ Ma OΣ	36.62	3 4	+0.8	186.3	+.01	1.13	+ 0.2	+ .01
Σ	37.47	1	+2.6	177.7	+.01	1.11	− 0.6	.00
Σ	38.44	1	+2.8	171.0	+.01	1.07	+ 1.1	− .08
Σ OΣ Daw	39.72	2	−1.0	158.9	−.01	1.19	− 0.7	+ .01
Σ OΣ Daw	40.61	2.5	−1.0	154.0	.00	1.29	+ 2.2	+ .08
Ma OΣ Daw	41.56	3	−1.7	144.6	+.09	1.29	− 0.1	+ .05
Ma OΣ Daw	42.54	3	−2.0	139.6	+.09	1.16	+ 2.1	− .11
Ma Daw	43.65	3	−1.4	128.3	+.18	1.22	− 1.4	− .09
Ma OΣ	44.50	2	−2.1	122.3	+.20	1.28	− 2.0	− .06
Ma OΣ	45.50	3	−2.1	117.6	+.23	1.31	− 0.6	− .04
Ma OΣ	46.59	3			+.20	1.11	.....	+ .01
Ma OΣ Daw	46.66	1	+0.6	111.7		...	+ 0.3	.....
Ma OΣ Daw	47.52	3.5	+1.0	107.6	+.19	1.53	+ 0.8	+ .06
Ma OΣ Daw	48.59	3	−1.0	100.4	+.12	1.51	− 0.7	− .00
Ma OΣ Daw	50.01	2		....	+.12	1.62	...	+ .07
Ma OΣ Daw	50.11	2.5	−0.5	94.3			+ 0.5	
Ma OΣ Daw	51.77	3	+0.5	86.9	+.12	1.57	+ 0.8	− .02
Ma OΣ	52.63	2	−0.1	83.1	+.20	1.56	+ 0.9	− .03
Ma Daw	53.10	2	+0.5	78.9	+.12	1.57	+ 0.2	− .02
Ma OΣ	53.74	2	−0.3	76.8	+.11	1.45	− 0.5	− .13
Ma OΣ	54.66	2	.		+.11	1.55	.	− .01
Ma OΣ Δ	54.86	3	+0.6	72.5			+ 0.6	..
OΣ Daw Sec	55.58	2.5	0.0	69.7	+.02	1.60	+ 1.2	+ .06
OΣ Δ Sec	56.51	1	+0.3	64.3			+ 0.6	
OΣ Sec	56.57	2			+.04	1.49	...	+ .01
Ma OΣ Sec	57.53	3		.	+.12	1.10	...	− .02
Ma OΣ Sec Δ	57.59	4	+0.6	59.5	.	.	+ 1.6	
Ma OΣ Sec	58.58	1 3	+0.7	54.5	+.12	1.37	− 0.3	+ .06
Ma Sec	59.51	2	+1.4	43.2	+.22	1.22	− 1.8	+ .04
OΣ Daw	59.62	2	0.0	13.9	−.06	1.26	− 0.1	+ .09
OΣ Sec	61.04	2.5			+.02	1.00	...	+ .06
Ma OΣ Sec	61.16	3.5	+1.3	25.5			− 2.3	...
OΣ Δ	62.61	1.5	+0.3	355.0			− 2.6	.
OΣ	62.74	0.5	.		−.07	0.93	....	[+ .33]
Δ	63.49	0.5	0.0	342.8			[+13.1]	..
OΣ Daw Δ	66.70	4	−0.3	233.9		.	+ 2.1	
OΣ Daw	66.78	3			−.05	0.92		+ .02
Δ Du	67.62	2	−1.2	222.1	+.01	0.93	+ 2.0	− .05
OΣ Δ Du	68.56	3	−0.4	208.4	−.09	0.99	− 1.9	− .05
Δ Du	69.60	2	−0.1	200.7	+.01	1.09	+ 0.7	+ .01
Δ Du	70.54	2	−0.9	190.9	+.01	1.17	+ 0.0	+ .06
Δ	71.12	2	0.0	184.1	−.13	1.28	− 1.8	+ .15
OΣ Du	71.57	2.5	−1.6	184.3	−.06	1.24	− 0.3	+ .14

TABLE II — *Continued*

Observer	Date	Wt.	Δθ	θ	Δ		O − C Δθ	O − C Δs
OΣ Δ Dυ	72.51	1	−0.3	172.9	−.10	1.16	− 0.2	+ .02
OΣ Δ Dυ	73.56	1	0.3	161.7	−.10	1.25	+ 0.2	+ .09
OΣ Δ Dυ	74.57	3.5	+0.1	158.3	−.10	1.27	+ 2.1	+ .08
Δ Dυ Sp	75.57	4	−0.6	147.0	−.06	1.24	− 1.1	+ .02
Sp Hall	76.53	2	−0.3	140.2	.00	1.25	− 0.8	.00
OΣ Δ	76.59	2	+0.1	141.5	−.10	1.21	+ 3.8	− .01
Δ Sp	77.56	2	+0.6	132.5	−.06	1.25	− 1.1	− .01
OΣ Hall	77.58	2	−0.5	131.0	−.03	1.17	+ 0.5	− .12
OΣ Δ Sp β	78.54	3.5	+0.1	126.8	−.06	1.31	− 0.2	− .02
Hall β . . .	79.46	2	−1.5	119.8	−.01	1.50	− 1.1	+ .13
Sp β	79.64	2	−0.2	120.7	−.01	1.27	+ 0.6	− .10
Sp β	80.54	2	−0.2	113.0	−.01	1.35	− 1.9	− .06
OΣ Hall β . .	81.53	3	−0.8	106.8	−.03	1.13	− 2.5	− .02
Sp HΣ	82.47	3 2.5	+0.7	103.5	00	1.47	− 0.3	− .02
OΣ Hall . . .	82.52	2	−0.1	102.0	−.03	1.41	− 1.9	− .05
OΣ Hall . . .	83.60	2	−0.4	98.9	−.03	1.42	+ .01	− .10
Sp	83.60	2	+1.1	97.6	00	1.52	− 0.8	− 00
OΣ Sp Hall	84.59	3	+0.1	94.3	−.02	1.60	− 2.5	+ .05
Sp Hall HΣ	85.54	3	−0.1	90.1	−.03	1.58	+ 0.8	+ .01
Sp Hall HΣ	86.60	3	−0.1	86.4	−.03	1.53	+ 2.0	− .06
Hall HΣ . .	87.56	2	−0.7	82.0	−.04	1.61	+ 1.9	+ .02
Sp	87.65	2	+1.1	80.4	00	1.55	+ 0.7	− .04
Sp Hall C . . .	88.56	3	−0.6	75.6	−.03	1.60	+ 0.1	+ .03
OΣ HΣ Maw	88.69	2.5	0.0	74.5	−.06	1.65	− 0.5	+ .08
Sp Hall	89.53	2	−0.1	72.3	00	1.58	+ 1.3	+ .02
HΣ C Maw	89.62	3	−0.6	71.0	−.07	1.60	+ 0.3	+ .05
Hall HΣ	90.55	2	−0.8	66.1	−.01	1.48	+ 0.1	− .03
Sp Maw.	90.74	2	+0.3	65.2	−.01	1.56	− 0.0	+ .06
Hall HΣ Maw .	91.57	3	−0.7	60.5	−.04	1.41	− 0.1	− .04
Sp HΣ C Maw . .	92.59	3.5	−0.1	56.0	−.05	1.35	+ 1.3	− .01
Sp	93.68	1	+1.1	48.7	.00	1.42	+ 1.3	+ .19
Maw Lw	94.58	2	0.0	39.2	−.01	1.18	− 0.6	+ .09
Sp HΣ C	95.51	1	+0.4	30.5	−.05	0.93	+ 1.6	+ .00
Sp C Lw	96.51	2.5	+0.3	13.1	−.04	0.65	+ 0.9	− .08
Sp Lw A	97.50	2.5			00	0.55		− .01
Sp C Lw A . .	97.53	3	+0.8	350.8			+ 8.7	
Sp Lw A	98.47	3	+1.8	304.2	00	0.58	+ 2.2	+ .07
A Bry Bow . .	98.72	2.5	+1.2	289.6	+.02	0.50	− 1.7	− .03
Lw A Doo	99.34	3	+0.8	266.9	−.02	0.52	− 2.9	− .10
C Lw Bow	99.54	3	−0.3	262.2	−.03	0.59	− 1.9	− .06
Maw A Doo	1900.50	3	0.0	241.8	−.03	0.71	− 1.5	− .09
Lw Bry	00.61	2	+0.9	236.7	+.06	0.78	− 1.2	− .04
Lw A	01.27	2	+0.7	228.7	00	0.83	− 1.9	− .07
C Lw Bow	01.57	3	−0.2	223.5	−.03	0.94	− 3.0	+ .01
Maw Bry	01.76	2	0.0	225.6	+.05	0.87	+ 1.6	− .08
C Lw A	02.50	3	−0.2	216.2	−.03	0.99	+ 0.2	− .02
Maw Bry Bow . .	02.62	3	+0.2	216.6	+.03	1.08	+ 1.8	+ .06

TABLE II — *Continued*

Observers	Date	Wt	$\Delta\theta$	θ	Δs	s	O − C $\Delta\theta$	O − C Δs
Lw A Bry Doo	03.39	3	+0.1	201.0	+.02	1.13	− 2.7	+ .07
C Maw Bow Brs	03.60	3.5	−0.5	203.1	.00	1.08	− 0.9	+ .01
Lw Bow Brs	04.47	2	+0.5	193.4	+.04	1.21	− 3.0	+ .11
C Maw	04.64	2	−1.0	196.8	−.06	1.06	+ 2.0	− .04
A	05.44	1	+0.5	191.0	.00	1.14	+ 1.1	+ .03
C Bow Brs	05.59	3	−0.3	188.3	.00	1.22	+ 2.5	+ .10
A Bry	06.40	2	+0.6	177.1	+.06	1.12	− 1.3	− .02
C Lw Bow	06.55	3	−0.2	175.8	−.03	1.17	− 1.4	+ .03
Lw Bry	07.48	2	+0.6	167.0	+.06	1.12	− 2.2	− .04
C Bow Brs	07.57	2.5	−0.3	171.1	−.02	1.29	+ 2.6	+ .12
C Lw Brs	08.55	3	−0.5	163.2	−.02	1.22	+ 1.4	+ .05
Lw Bry Bow	08.66	3	+0.5	161.5	+.04	1.20	+ 2.1	+ .02
Doo Brs	09.49	2	0.0	152.8	+.02	1.27	+ 0.3	+ .06
C Bow	09.60	2	−0.5	154.5	−.05	1.24	+ 2.8	− .00
C A Doo	10.47	2.5	−0.5	144.7	−.06	1.25	− 3.2	+ .01
Bow Brs	10.70	2	+0.3	141.0	+.04	1.22	+ 0.8	− .02
Bow Brs	11.47	2	+0.3	137.2	+.05	1.24	− 0.4	− .06
C Lw Bry	11.62	3	−0.1	138.6	+.01	1.18	+ 2.0	− .10
C Bow A	12.55	3	−0.1	130.0	−.03	1.26	− 0.3	− .05
Bry Brs	12.60	2	+0.3	129.1	+.12	1.27	− 0.6	− .04
C	13.57	1	−1.2	123.2	−.09	1.33	− 0.3	− .02
C A	14.50	2	−0.4	116.5	−.04	1.31	− 1.3	− .08
C Brs	15.63	2	−0.6	111.5	.00	1.45	+ 0.2	+ .02
C	16.15	1	−1.0	106.1	−.09	1.56	− 0.8	+ .09

ordinates upon a fairly open scale (1° = 1.3 mm, 1″ = 168 mm) and through the plotted points there have been drawn smooth curves (hereafter called the θ-curve, the s-curve) from which the coordinates of the companion at any time may be read with a precision presumably greater than that of any one normal place. Since the θ-curve covers more than two revolutions in the orbit, to every point in this [illegible] must correspond another point having the same value of the function, θ, and the difference of the abscissae of any two such points furnishes a determination of the periodic time in the orbit. As a test upon the suspected variability of this period I have determined it from a considerable number of [illegible] points uniformly distributed over the [illegible] curve, using for this purpose as initial [illegible] beginning of each calendar year from 1833 [illegible] the years 1864-5-6, in which [illegible] the course of the [illegible] Subsequent to [illegible] data, since the [illegible] angle of that epoch. It seems needless to present each result thus obtained, but I have united them in groups of five and give below the mean value of the periodic time derived from each such group:

Initial Years	P
1833 – 1837	34.42 years
38 – 42	.50
43 – 47	.50
48 – 52	.42
53 – 57	.40
58 – 63	.52
1867 – 1871	34.46
72 – 76	.44
77 – 82	.47
Mean	34.46

The variations in the value of P here shown appear to me to be fortuitous and to lend no appreciable support to the hypothesis of a variable periodic time

advanced by LEWIS, *loc. cit.*, I therefore adopt, provisionally, the hypothesis of undisturbed motion and from the measured slope of the θ-curve combined with the ordinates of the s-curve derive at intervals of two years the value of the, presumably constant, areal velocity, $s^2 \frac{d\theta}{dt}$. The resulting values (degrees per annum × seconds squared) are shown, in Fig. 1, plotted as a function of the time, the values from 1860 to 1866 being necessarily omitted for lack of data. Two values of the areal velocity that are peculiarly ill determined are here represented by triangles. The several epochs of periastron passage are denoted by the letter P. Whether the values here plotted shall be construed as indicating a variable areal velocity, approximately represented by the curve drawn through them, or whether their discordances shall be regarded as fortuitous errors accidentally grouped, must depend upon one's estimate of the precision of the data. Reserving judgment upon this point for a later stage of the discussion, I note that each plotted point might be brought to agreement with the mean value of all by slightly modifying the course of the s-curve, the amount of such change from the adopted curve amounting to so much as $0''.05$ in only a single case, and that one largely dependent upon one discordant observation by OTTO STRUVE. Without making such modification I assume that the data read directly from the curves may be made the basis of an approximate orbit. If there be a disturbing body in the system, its effect at any given date is shown by Fig. 1 not to exceed a very few hundredths of a second of arc and in the mean of two or three revolutions this effect at any point of the orbit will tend to a still smaller value if it be not wholly eliminated. By a graphical method I have derived from the data furnished by the θ-curve and s-curve, provisional elements of the orbit and have compared these with each normal place given in Table II. All of the resulting θ residuals corresponding to positions of the *comes* falling between 0° and 45° of mean anomaly were united into a single mean value, which became the absolute term of an equation for correction of the provisional values of the elements, Ω, i, λ, e, T. Similar equations were formed for each other octant of mean anomaly and a least square solution of them furnished definitive corrections to the provisional elements enumerated. Similarly all of the s residuals were used for correcting the provisional value of the semi-axis major. The periodic time $P = 34.46$ years is adopted from the discussion given above. I thus obtain the following elements which appear to me exhaustive of the data as here discussed:

DEFINITIVE ELEMENTS

$$
\begin{array}{llll}
P = 34.46 \text{ years} & \Omega = 231^\circ.6 & \\
T = 1898.77 & i = 47^\circ.5 & \Big\} 1900.0 \\
e = 0.458 & \lambda = 66^\circ.7 & \\
a = 1''.35 & n = -10^\circ.447 &
\end{array}
$$

From these elements I have computed the following ephemeris, Table III, whose argument, M, is the mean anomaly of the *comes*, given by the relation,

$$M = 10^\circ.447\ (T - T_0)$$
$$T_0 = 1829.85 \text{ or } 1864.31 \text{ or } 1898.77$$

TABLE III

Ephemeris of ξ Herculis

M	θ		s	
180	109.1		1.45	
190	104.0	5.1	1.49	4
200	99.1	4.9	1.52	3
210	94.5	4.6	1.55	3
220	90.0	4.5	1.57	2
230	85.6	4.4	1.59	2
240	81.3	4.3	1.59	0
250	76.9	4.4	1.58	1
260	72.5	4.4	1.56	2
270	67.9	4.6	1.53	3
280	63.1	4.8	1.48	5
290	57.8	5.3	1.41	7
300	51.9	5.9	1.31	10
310	45.1	6.8	1.18	13
320	36.0	9.1	1.03	15
330	23.7	12.3	0.84	19
340	3.8	19.9	0.66	18
340	363.8		0.66	
342	358.5	5.3	0.62	4
344	352.5	6.0	0.59	3
346	345.8	6.7	0.56	3
348	338.5	7.3	0.54	2
350	330.6	7.9	0.52	2
352	322.3	8.3	0.51	1
354	313.7	8.6	0.50	1
356	305.2	8.5	0.51	1
358	296.9	8.3	0.52	1
0	289.1	7.8	0.54	2
2	281.8	7.3	0.56	2
4	275.2	6.6	0.59	3
6	269.2	6.0	0.62	3
8	263.7	5.5	0.65	3
10	258.8	4.9	0.68	3

Ephemeris of ζ Herculis — Continued

M	θ		s	
10	258.8		0.68	
		19.1		15
20	239.7		0.83	
		14.4		11
30	225.3		0.94	
		10.5		8
40	214.8		1.02	
		9.9		5
50	204.9		1.07	
		9.4		3
60	195.5		1.10	
		8.8		2
70	186.7		1.12	
		8.6		2
80	178.1		1.14	
		8.3		1
90	169.8		1.15	
		8.1		2
100	161.7		1.17	
		7.8		3
110	153.9		1.20	
		7.5		2
120	146.4		1.22	
		7.1		3
130	139.3		1.25	
		6.8		4
140	132.5		1.29	
		6.4		4
150	126.1		1.33	
		6.0		4
160	120.1		1.37	
		5.6		4
170	114.5		1.41	
		5.4		4
180	109.1		1.45	

$$M = 10^{\circ}.447\,(T - T_0)$$

$T = 1829.85$

1864.31

1898.77

Table II shows, under the rubric O − C, the difference between each normal place and the corresponding position interpolated from this ephemeris. In no case do these residuals present an intolerable magnitude and in general they are quite as small as could be expected. Nevertheless I have deemed it proper to group them in octants of mean anomaly, as in the formation of the equations for correcting the provisional elements, and present in Table IV for each observed revolution of the *comes* the outstanding error in each octant of its orbit; for $\Delta\theta$ as well as Δs the residuals are expressed in hundredths of a second of arc of a great circle. Each residual is followed by its approximate weight, and successive revolutions in the orbit are designated I, II, and III.

An examination of these residuals suggests that in some cases they are systematic in character, the same residual occurring in successive revolutions. In other cases they seem fortuitous, but always they are of the same order of magnitude as the discordances shown in the areal velocity, Fig. 1, which were provisionally ignored as of minor significance.

I revert therefore to Fig. 1 and call to the attention of the reader that the curve there drawn through the plotted points is not a graph of the points themselves but represents the theoretical value of the areal velocity, as it would be if the *comes*, in consequence of the

Table IV. Mean Residuals

M	0° — 45°		45° — 90°		90° — 135°		135° — 180°		180° — 225°		225° — 270°		270° — 315°		315° — 360°	
Mean s	0″.7		1″.1		1″.2		1″.35		1″.5		1″.6		1″.3		0″.7	
									O − C in θ							
	r	p	r	p	r	p	r	p	r	p	r	p	r	p	r	p
															−2	1
I	−2	0.5	+1	7	+2	10	−2	12	0	9	+2	14	+1	16	−4	5
II	−3	6	−1	15	+2	15	−1	14	−3	18	+2	16	+1	11	+2	17
III	−2	24	−1	19	+2	24	−1	7	−2	1						
Mean	−1	30	1	30	+2	49	−1	33	−2	28	+2	30	+1	27	+4	23
									O − C in s							
	r	p	r	p	r	p	r	p	r	p	r	p	r	p	r	p
															+4	1
I	−9	0.5	3	15	0	10	−4	11	+4	8	−2	13	+3	15	+2	2
II	−1	5	−1	5	+1	15	−3	14	−4	17	+2	16	+1	11	+2	17
III	4	24	+4	19	0	24	−3	7	+9	1	...		...	..	..	
Mean	−4	29	−3	39	+1	49	−3	32	+1	26	0	29	+2	26	+2	20

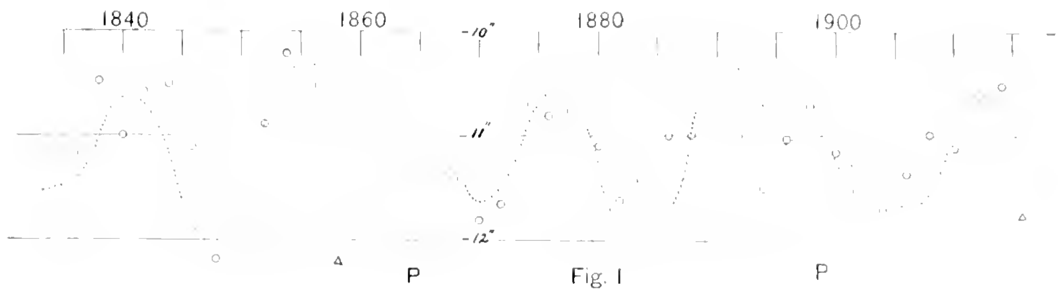

Fig. I

presence of a dark companion, were constrained to move in a circular orbit whose plane is perpendicular to the line of sight, whose radius equals 0″.04, and in which the periodic time is approximately 18 years. The analysis upon which this curve is based is a purely geometrical one in which no heed is given to the perturbations that would be produced, by the principal star, in the motion of such a *comes* relative to its assumed dark companion. Also, a more plausible assumption than that here made would place the orbit plane of the dark companion in or near the plane of the orbit above found for ζ *Herculis* and its companion; nevertheless the assumptions here made seem permissible as a first approximation, and they lead to the following analysis:

Let θ, s represent as above, the polar coördinates of the *comes* referred to ζ *Herculis* as origin; let ψ, r be the corresponding coördinates of the center of gravity of the assumed system and represent by v and a the coördinates of the visible *comes* referred to the center of gravity of itself and its dark companion. We readily find from the geometrical relations involved,

$$s^2 = r^2 + a^2 + 2ar\cos(v-\psi)$$

$$\theta = \psi + \frac{a}{s}\sin(v-\psi)$$

Treating a/r and a/s as quantities whose squares are negligible, we obtain by differentiation,

$$s^2\frac{d\theta}{dt} = r^2\frac{d\psi}{dt} + ar\cos(v-\psi)\left\{\frac{dv}{dt}+\frac{d\psi}{dt}\right\} - a\sin(v-\psi)\frac{dr}{dt}$$

By hypothesis a, $\frac{dv}{dt}$ and $r^2\frac{d\psi}{dt}$ are constant quantities and a is so small that in the last two terms of the equation we may write θ in place of ψ, and s in place of r, without sensible error. Making these substitutions, replacing $\frac{d\psi}{dt}$ by its value K/s^2, and writing k as an abbreviation for $\frac{dv}{dt}$ we obtain,

$$s^2\frac{d\theta}{dt} = K + a\left(ks + \frac{K}{s}\right)\cos(v-\theta) - a\sin(v-\theta)\frac{ds}{dt}.$$

The term K is very approximately the mean value of the observed areal velocities, *viz.*, -11.17, (expressed in square seconds × degrees per annum of angular motion.)

In accordance with the hypothetical orbit above suggested we assume for the sake of illustration that the motion of the visible star in this orbit is defined by the elements,

$$v = 20^\circ(t - 1906.0) \qquad a = 0''.04$$

and find in terms of the unit above defined,

$$s^2\frac{d\theta}{dt} = -11.17 + H\cos(v-\theta) - 2.3\,s'\sin(v-\theta)$$

Here s', the rate of change of s in seconds per annum, is to be read from the s-curve. The coefficient of $\cos(v-\theta)$ is a function of s whose value, corresponding to the constants $a = 0.04$, $k = 20.0$, $K = -11.17$, is here tabulated:

VALUES OF H

s	H	s	H
0″.4	−0.80	1″.0	+0.35
.5	− .49	.1	.48
.6	− .26	.2	.59
.7	− .08	.3	.69
.8	+ .08	.4	.80
.9	+ .22	.5	.90
1 .0	+0.35	1 .6	+1.00

The curve shown in Fig. 1 has been computed from these elements, and upon the whole its conformity to the observed values of the areal velocity seems to me something more than fortuitous agreement. If the radius of the hypothetical orbit were ten times greater than that employed above, there would be little hesitation in adopting the hypothetical companion as a real influence in the observed motion. But current ideas with regard to the accuracy of micrometer observations may well induce some skepticism as to an orbit having a radius of less than a twentieth of a second of arc. If the supposed orbit is real the observations

are intrinsically better than has been currently believed, even when all due allowance is made for the fact that the smooth and flowing form given to the u- and s-curves has tended to obscure the influence of the companion and to diminish the amplitude of its motion.

I have examined the temporary stability of the hypothetical companion above defined, using for this purpose the method expounded by MOULTON, *Astr. Jour.* No. 161, and find that under plausible suppositions as to mass its continued existence as a satellite of the *comes* seems entirely feasible, but a detailed presentation of the matter seems uncalled for in its present status.

Deferring to future consideration the status of the hypothetical companion above considered, the definitive results of the present investigation may be summarized as follows: The best observations of ζ *Herculis* are affected by small but appreciable personal errors of a systematic character. Approximate values of these errors have been determined, and when their effect is duly taken into account the hypothesis of undisturbed motion leads to an orbit, whose elements are given above, that satisfies the observations within limits of error commonly deemed satisfactory. The areal velocity in this orbit presents small irregularities which may be fortuitous but which are very well represented during the entire period of the available data, three-quarters of a century, as the effect of an invisible companion having a periodic time of eighteen years and an amplitude less than 0″.1.

Washburn Observatory, Madison, Wis.

OBSERVATIONS OF COMET *a* *1917* (*MELLISH*),

MADE WITH THE 16-INCH REFRACTOR AND FILAR MICROMETER OF GOODSELL OBSERVATORY,

By H. C. WILSON.

Date Gr. M.T.	$\Delta\alpha$	$\Delta\delta$	Comp.	App. α	App. δ	Red. to App. $\Delta\alpha$	Red. to App. $\Delta\delta$	*
March 20.5932	+0ᵐ 34ˢ.79	−0′ 11″.3	10, 4	2ʰ 09ᵐ 10ˢ.13	+14° 25′ 13″.0	+0ˢ.65	+5″.7	1
21.5681		−8 07 .5	3	. .	+11 45 27 .5		+5 .7	2
21.5731	+0 42 .95		10	2 09 14 .95	. .	+0 .63		2
23.5779	+0 00 .17	−3 16 .6	6, 6	2 08 05 .60	+15 27 55 .7	+0 .62	+5 .6	3
27.5752	−0 35 .91	+0 36 .0	9, 6	2 04 13 .76	+16 50 53 .2	+0 .59	+5 .6	4
29.5742	−0 23 .23	−1 18 .2	10, 6	2 01 05 .67	+17 29 15 .3	+0 .57	+5 .6	5

Mean Places of Comparison Stars for 1917.0

*	α	δ	Authority
1	2ʰ 09ᵐ 04ˢ.69	+11° 25′ 18″.5	Bordeaux Plate 608, 60.
2	2 08 31 .37	+14 53 29 .2	Boss *P. G. C.* 502.
3	2 08 04 .81	+15 24 33 .5	Bordeaux Plate 350, 49.
4	2 04 49 .68	+16 50 11 .6	Bordeaux Plate 335, Ref. star, *B.D.* +16° 247.
5	2 01 28 .33	+17 30 27 .8	Bordeaux Plate 335, 7.

CONTENTS.

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REMARKS ON THE METHOD OF LEAST SQUARES AND ITS APPLICATION TO THE DETERMINATION OF THE SOLAR MOTION.

By E. S. MANSON, Jr.

In all demonstrations of the validity of the ordinary method of determining the most probable values of the unknowns by the formation of Normal Equations the derivation of the exponential Law of Error has, so far as I am aware, been considered a necessary preliminary step. The following demonstration which does not make use of the Law of Error is presented as satisfactorily proving the legitimacy of that method.

Let us start with the assumption that in observations of a single unknown, the observations being of equal weight, the most probable value of that unknown is given by the arithmetic mean of the several observations. We shall also make use of the principle that in weighted observations the best value of the unknown is given by the expression $\frac{\Sigma pm}{\Sigma p}$ where the m's represent the results of the several measurements and the p's the respective weights. We shall also use the principle, which may be demonstrated without reference to the Law of Error, that weights are inversely proportional to the squares of the Mean Errors.

Now let us consider observations of multiples of a single unknown; the equations which represent the observations being of the form:

$$(1)\qquad \begin{aligned} a_1z &= m_1 \\ a_2z &= m_2 \\ &\text{etc.} \end{aligned}$$

We will suppose that the different observations are of equal weight. If we divide both sides of each equation by the coefficient of z in that equation, equations (1) become:

$$(2)\qquad \begin{aligned} z &= m_1/a_1 \\ z &= m_2/a_2 \\ &\text{etc.} \end{aligned}$$

But the Mean Error of the value of z given by the first equations of (2) is equal to the Mean Error of an observation divided by a_1. Hence this value of z should be given the weight a_1^2. Similarly the second value of z should have the weight a_2^2 and similarly with the other values. The most probable value of z is therefore given by the expression:

$$z = \frac{a_1^2\left(\frac{m_1}{a_1}\right) + a_2^2\left(\frac{m_2}{a_2}\right) + a_3^2\left(\frac{m_3}{a_3}\right) + \cdots}{a_1^2 + a_2^2 + a_3^2}$$

or

$$z = \frac{a_1m_1 + a_2m_2 + a_3m_3 + \cdots}{a_1^2 + a_2^2 + a_3^2 + \cdots} = \frac{\Sigma am}{\Sigma a^2}$$

Or, in words, the equation giving the most probable value of z is obtained by multiplying each of the original equations by the coefficient of z in that equation and adding.

Let us now consider the case where there are two unknowns and where we have a series of observation equations of the form:

$$\begin{aligned} a_1z_1 + b_1z_2 &= m_1 \\ a_2z_1 + b_2z_2 &= m_2 \\ &\text{etc.} \end{aligned}\qquad (3)$$

Let us by transposition write equations (3) as follows:

$$\begin{aligned} a_1z_1 &= m_1 - b_1z_2 \\ a_2z_1 &= m_2 - b_2z_2 \\ &\text{etc.} \end{aligned}\qquad (4)$$

Consequently proceeding as in the case last considered where there was but one unknown and considering z_1 as the unknown we have as the most probable value of z_1

$$z_1 = \frac{\Sigma am - z_2 \Sigma ab}{\Sigma a^2}$$

or this may be written, $z_1(\Sigma a^2) + z_2(\Sigma ab) = \Sigma(am)$

Next write equations (3) in the form.

$$\begin{aligned} b_1 z_2 &= m_1 - a_1 z_1 \\ b_2 z_2 &= m_2 - a_2 z_1 \\ &\text{etc.} \end{aligned} \tag{5}$$

Treating z_2 as the unknown its most probable value is given by the equation:

$$z_2 = \frac{\Sigma bm - z_1 \Sigma ab}{\Sigma b^2}$$

which may be written:

$$z_2(\Sigma b^2) = \Sigma bm - z_1 \Sigma ab$$

or
$$z_1(\Sigma ab) + z_2(\Sigma b^2) = \Sigma bm.$$

Hence for the determination of the most probable values of z_1 and z_2 we have the simultaneous equations:

$$\begin{aligned} z_1(\Sigma a^2) + z_2(\Sigma ab) &= \Sigma am \\ z_1(\Sigma ab) + z_2(\Sigma b^2) &= \Sigma bm \end{aligned}$$

or adopting the notation usually employed in Least Squares

$$\begin{aligned} (aa)\, z_1 + (ab)\, z_2 &= (am) \\ (ab)\, z_1 + (bb)\, z_2 &= (bm) \end{aligned}$$

The process can easily be extended to any number of unknowns and gives us the following and usual rule for forming Normal Equations:

Form a Normal Equation for each unknown by multiplying each observation equation by the coefficient of that unknown in that equation and adding.

Since the exponential Law of Error has not been used in the derivation of the above rule it is interesting to consider whether problems ever occur where the method of solution by forming Normal Equations according to that rule leads to useful results even though the Law of Error may not hold. Consider first the special case where there is only one unknown and where the Method of Least Squares becomes simply the taking of the arithmetic mean. Is an average value of a series of measured quantities ever of importance even though deviations from the mean do not follow the Law of Error and where the average value may not necessarily be the most probable value? I am going to take as an example to show that such may be the case the following astronomical problem because it is a satisfactory introduction to a more complex astronomical problem that I wish to consider afterwards.

Let us consider the case of a number of stars so nearly in the same part of the sky or in diametrically opposite parts of the sky that they may be considered as lying on a straight line. We will suppose the stars to be of equal mass. Let us suppose that we wish to measure the velocity with which the center of mass of this group of stars is approaching or receding from the *Sun*; or in other words the velocity with which the *Sun* is receding from or approaching the center of mass of the group of stars. Let us consider the direction of motion as lying along the x-axis. Let X represent the velocity of the *Sun* with respect to the center of mass of the group of stars. Let Δx represent the velocity of an individual star with respect to this center of mass and let dx represent the velocity, as may be determined spectroscopically, of the star with respect to the *Sun*.

Then
$$dx = \Delta x - X$$

but if the center of mass is that of all the stars, the *Sun* included, $\Sigma\Delta x + X = 0$; or if n represents the total number of stars excluding the *Sun* $\Sigma dx + (n+1)\,X = 0$ or $X = \dfrac{-\Sigma dx}{n+1}$.

If we let the center of mass be that of all the stars with the exception of the *Sun* $X = \dfrac{-\Sigma dx}{n}$. Hence the arithmetic mean of the observed radial velocities gives us the velocity of the *Sun* with respect to the center of mass of all the stars, distributed as above described, regardless of any law that may govern the radial velocities. Also the average velocity is not necessarily the most probable velocity.

Let us now go back to ordinary Normal Equations and let us suppose that there are three unknowns involved. The equations are:

$$\begin{aligned} x(\Sigma a^2) + y(\Sigma ab) + z(\Sigma ac) &= \Sigma am \\ x(\Sigma ab) + y(\Sigma b^2) + z(\Sigma bc) &= \Sigma bm \\ x(\Sigma ac) + y(\Sigma bc) + z(\Sigma c^2) &= \Sigma cm \end{aligned} \tag{6}$$

the equations of condition being of the form:

$$ax + by + cz = m. \tag{7}$$

Ordinarily the quantities represented by each of the symbols x, y, and z have the same numerical value in all of the equations, so that if all the observations were perfect all the equations of condition would be exactly satisfied. In such cases the problem is to de-

termine from the somewhat discordant observation equations the most probable values of the unknowns.

Let us now suppose that the numerical values of x, y, and z are not the same in the different equations of condition and let us consider whether under any circumstances the solution of Normal Equations, formed according to the ordinary rule, will give *average values* of these unknowns.

Let the average values of x, y, and z be represented by x_0, y_0, and z_0 and let the differences between the actual values and the average values in the several equations be represented by x', y', and z'; so that $x = x_0 + x'$; $y = y_0 + y'$; $z = z_0 + z'$. The equations of condition are of the form

$$a(x_0 + x') + b(y_0 + y') + c(z_0 + z') = m$$

where the quantities represented by the primes do not have the same numerical values in the different equations. It follows that if we should form Normals their solution would not give values of $(x_0 + x')$, $(y_0 + y')$ and $(z_0 + z')$ which would exactly satisfy the observation equations even though all of the observations were perfect.

The Normal Equations (6) become

$$\begin{aligned} x_0(\Sigma a^2) + y_0(\Sigma ab) + z_0(\Sigma ac) + \Sigma a^2x' + \Sigma aby' + \Sigma acz' &= \Sigma am \\ x_0(\Sigma ab) + y_0(\Sigma b^2) + z_0(\Sigma bc) + \Sigma abx' + \Sigma b^2y' + \Sigma bcz' &= \Sigma bm \\ x_0(\Sigma ac) + y_0(\Sigma bc) + z_0(\Sigma c^2) + \Sigma acx' + \Sigma bcy' + \Sigma c^2z' &= \Sigma cm \end{aligned} \tag{8}$$

We see that if

$$\begin{aligned} \Sigma a^2x' + \Sigma aby' + \Sigma acz' &= 0 \\ \Sigma abx' + \Sigma b^2y' + \Sigma bcz' &= 0 \\ \Sigma acx' + \Sigma bcy' + \Sigma c^2z' &= 0 \end{aligned} \tag{9}$$

Equations (8) become

$$\begin{aligned} x_0(\Sigma a^2) + y_0(\Sigma ab) + z_0(\Sigma ac) &= \Sigma am \\ x_0(\Sigma ab) + y_0(\Sigma b^2) + z_0(\Sigma bc) &= \Sigma bm \\ x_0(\Sigma ac) + y_0(\Sigma bc) + z_0(\Sigma c^2) &= \Sigma cm \end{aligned} \tag{10}$$

and their solution gives average values of x, y, and z.

To illustrate this principle by a numerical example consider the following case where there are two unknowns and three equations. Suppose that the values of x in the three equations are respectively $+2$, $+3$, and $+7$. Let the corresponding values of y be $+2$, $+6$, and $+1$. Let the observation equations be

$$\begin{aligned} +2.00000x - 3.00000y &= -2.00000 \\ -1.00000x + 2.00000y &= +9.00000 \\ -1.15233x + 2.17663y &= -5.88968 \end{aligned}$$

The first of these equations is satisfied by the first values of x and y, $+2$ and $+2$. The second is satisfied by the second values of x and y, $+3$ and $+6$; and the third is satisfied by the third values of x and y, $+7$ and $+1$.

Forming Normal Equations we have:

$$\begin{aligned} +\ 6.32786x - 10.50820y &= -\ 6.21315 \\ -10.50820x + 17.73772y &= +11.18035 \end{aligned}$$

The solution of these equations gives to four decimal places the values:

$$x = 4.00000 \quad ; \quad y = 3.00000.$$

Using the above notation, $x_0 = 4$; $y_0 = 3$. The values of x' are respectively -2, -1 and $+3$. The corresponding values of y' are -1, $+3$ and -2. Also

$$\begin{aligned} \Sigma a^2x' + \Sigma aby' &= -5.0164 + 5.0164 = 0 \\ \Sigma abx' + \Sigma b^2y' &= +6.4754 - 6.4754 = 0 \end{aligned}$$

The conditions given by equations (9) are thus satisfied and consequently the solution of the Normals would be expected to give average values of x and y as is the case.

We will now apply these principles to the problem of obtaining the direction and velocity of the *Sun* with regard to the geometrical center of mass of the system of the stars, meaning by geometrical centre of mass the centre of mass on the assumption that the masses of individual stars are equal. The problem is solved if we can find the values of the three rectangular components of the *Sun's* motion; and these are equal to the average of the rectangular components of the motions of the individual stars. For, using our previous notation,

$$\begin{aligned} dx &= \Delta x - X. \\ X &= \Delta x - dx. \\ nX &= \Sigma\Delta x - \Sigma dx. \end{aligned}$$

but $\quad \Sigma\Delta x = 0$

consequently $\quad X = -\dfrac{\Sigma dx}{m}$

similarly

$$Y = -\frac{\Sigma dy}{n}; \qquad Z = -\frac{\Sigma dz}{n}$$

Let us now consider briefly the method of Bravais which does not make use of Least Squares. Let Δx, Δy, Δz represent the x, y and z components of an individual star's motion with respect to the centre of mass

of the system. Let dx, dy and dz represent the x, y and z components of the velocity of the star with respect to the moving *Sun*. Let X, Y and Z represent the components of the velocity of the *Sun* with respect to the center of mass of the stars.

Then

$$\begin{aligned} \Delta x &= dx + X \\ \Delta y &= dy + Y \\ \Delta z &= dz + Z \end{aligned} \tag{11}$$

Let the plane of the equator be taken as the XY plane and let the axis of x be directed toward the vernal equinox. Then

$$\begin{aligned} x &= r \cos\alpha \cos\delta \\ y &= r \sin\alpha \cos\delta \\ z &= r \sin\delta \end{aligned} \tag{12}$$

where α, δ and r represent the star's right ascension, declination and distance from the *Sun*.

Differentiating, we obtain

$$\begin{aligned} dx &= -r \sin\alpha \cos\delta\, d\alpha - r \cos\alpha \sin\delta\, d\delta + \cos\alpha \cos\delta\, dr \\ dy &= r \cos\alpha \cos\delta\, d\alpha - r \sin\alpha \sin\delta\, d\delta + \sin\alpha \cos\delta\, dr \\ dz &= r \cos\delta\, d\delta + \sin\delta\, dr \end{aligned} \tag{13}$$

but

$$\Sigma \Delta x = 0 \quad ; \quad \Sigma \Delta y = 0 \quad ; \quad \Sigma \Delta z = 0$$

hence from the first equations of (11) and (13)

$$nX - \Sigma r \sin\alpha \cos\delta\, d\alpha - \Sigma r \cos\alpha \sin\delta\, d\delta + \Sigma \cos\alpha \cos\delta\, dr = 0 \tag{14}$$

where n represents the number of stars; $d\alpha$ and $d\delta$ represent respectively the proper-motions in right ascension and declination and dr represents the radial velocity of the star with respect to the moving *Sun*. Let Δr represent the radial velocity of the star with respect to a fixed origin (*i.e.* one maintaining a constant distance and direction with regard to the centre of mass) so taken that from it the star has the same right ascension and declination as from the *Sun*. Then

$$\Delta r = dr + X \cos\alpha \cos\delta + Y \sin\alpha \cos\delta + Z \sin\delta \tag{15}$$

In BRAVAIS' method the assumption is made that

$$\begin{aligned} \Sigma \Delta r \cos\alpha \cos\delta &= 0 \\ \Sigma \Delta r \sin\alpha \cos\delta &= 0 \\ \Sigma \Delta r \sin\delta &= 0 \end{aligned} \tag{16}$$

or that the sums of the components of the radial velocities with regard to a fixed origin resolved along the rectangular axes are equal to zero. Making this assumption and substituting (16) into (14) we obtain

$$dr = \Delta r - X \cos\alpha \cos\delta - Y \sin\alpha \cos\delta - Z \sin\delta \tag{17}$$

Simplifying and making the assumption that all the distances are equal this becomes:

$$nX - X\,\Sigma \cos^2\alpha \cos^2\delta - Y\Sigma \sin\alpha \cos\alpha \cos^2\delta - Z\Sigma \cos\alpha \sin\delta \cos\delta = \Sigma\{r \sin\alpha \cos\delta\, d\alpha + r \cos\alpha \sin\delta\, d\delta\}, \tag{18}$$

and by a similar process we also obtain the other two equations:

$$\begin{aligned} X\Sigma\{1 - \cos^2\alpha \cos^2\delta\} - Y\Sigma \sin\alpha \cos\alpha \cos^2\delta - Z\Sigma \cos\alpha \sin\delta \cos\delta &= r\Sigma\{\sin\alpha \cos\delta\, d\alpha + \cos\alpha \sin\delta\, d\delta\} \\ -X\Sigma \sin\alpha \cos\alpha \cos^2\delta + Y\Sigma\{1 - \sin^2\alpha \cos^2\delta\} - Z\Sigma \sin\alpha \sin\delta \cos\delta &= -r\Sigma\{\cos\alpha \cos\delta\, d\alpha - \sin\alpha \sin\delta\, d\delta\} \\ -X\Sigma \cos\alpha \sin\delta \cos\delta - Y\Sigma \sin\alpha \sin\delta \cos\delta + Z\Sigma \cos^2\delta &= -r\,\Sigma \cos\delta\, d\delta \end{aligned} \tag{19}$$

These equations solved for X, Y, and Z give us the three rectangular components of the solar motion.

Consider next CAMPBELL's method, which makes use of radial velocities. If we solve equations (13) for dr we find that

$$dr = dx \cos\alpha \cos\delta + dy \sin\alpha \cos\delta + dz \sin\delta$$

According to this method a Least Square solution is made taking as equations of condition the expressions for dr as if the radial motions of the stars were wholly due to the solar motion. That is X, Y, and Z are substituted for $-dx$, $-dy$ and $-dz$ and consequently the equations of condition are of the form

$$X \cos\alpha \cos\delta + Y \sin\alpha \cos\delta + Z \sin\delta = -dr.$$

The Normals become:

$$
(20)\qquad
\begin{aligned}
X\{\Sigma\cos^2\alpha\cos^2\delta\} &+ Y\Sigma\{\sin\alpha\cos\alpha\cos^2\delta\} + Z\Sigma\{\cos\alpha\sin\delta\cos\delta\} = -\Sigma dr\ \cos\alpha\cos\delta\\
X\{\Sigma\sin\alpha\cos\alpha\}\cos^2\delta &+ Y\Sigma\{\sin^2\alpha\cos^2\delta\} + Z\Sigma\{\sin\alpha\sin\delta\cos\delta\} = -\Sigma dr\ \sin\alpha\cos\delta\\
X\{\Sigma\cos\alpha\sin\delta\}\cos\delta &+ Y\Sigma\{\sin\alpha\sin\delta\cos\delta\} + Z\Sigma\{\sin^2\delta\} = -\Sigma dr\ \sin\delta
\end{aligned}
$$

The use of Least Squares might at first thought seem questionable in this case for the peculiar motions of the stars are not necessarily small in comparison with the solar motion; but let us apply the criterion given by equations (9). If we remember that $dx = \Delta x - X$; $dy = \Delta y - Y$; $dz = \Delta z - Z$, where $-X$, $-Y$ and $-Z$ are the average values of dx, dy and dz respectively, we see that the quantities corresponding to the x', y', and z' of equations (9) are Δx, Δy and Δz; and consequently the conditions under which equations (20) will give average values of dx, dy, and dz or in other words, X, Y, and Z are given by the equations

$$
(21)\qquad
\begin{aligned}
\Sigma\cos^2\alpha\cos^2\delta\,\Delta x &+ \Sigma\sin\alpha\cos\alpha\cos^2\delta\,\Delta y + \Sigma\cos\alpha\sin\delta\cos\delta\,\Delta z = 0\\
\Sigma\sin\alpha\cos\alpha\cos^2\delta\,\Delta x &+ \Sigma\sin^2\alpha\cos^2\delta\,\Delta y + \Sigma\sin\alpha\sin\delta\cos\delta\,\Delta z = 0\\
\Sigma\cos\alpha\sin\delta\cos\delta\,\Delta x &+ \Sigma\sin\alpha\sin\delta\cos\delta\,\Delta y + \Sigma\sin^2\delta\,\Delta z = 0
\end{aligned}
$$

but since

$$\Delta r = \Delta x\cos\alpha\cos\delta + \Delta y\sin\alpha\cos\delta + \Delta z\sin\delta$$

these equations become:

$$
(22)\qquad
\begin{aligned}
\Sigma\Delta r\cos\alpha\cos\delta &= 0\\
\Sigma\Delta r\sin\alpha\cos\delta &= 0\\
\Sigma\Delta r\sin\delta &= 0
\end{aligned}
$$

which equations are identical with equations (17) and represent the assumption made in the method of Bravais with regard to radial velocities. In fact it is easily seen that equations (20) follow at once from equations (15) and (17). It follows then that the assumption made in the method of Bravais is the same as that made in the method which uses radial velocities.

Let us now consider Airy's method. This method makes use of proper-motions but agrees with the method of Campbell in treating the peculiar motions of the stars as if they were accidental errors; that is, it treats the proper-motions, as Campbell's method treats the radial velocities, as if they were wholly due to the motion of the *Sun*. If we solve equations (13) for $d\alpha$ and $d\delta$ we obtain the values:

$$
(23)\qquad
\begin{aligned}
-r\cos\delta\,d\alpha &= dx\sin\alpha - dy\cos\alpha\\
-r\,d\delta &= dx\cos\alpha\sin\delta + dy\sin\alpha\sin\delta - dz\cos\delta
\end{aligned}
$$

Substituting X, Y and Z for $-dx$, $-dy$ and $-dz$ as in Campbell's method we have the equations of condition of the form:

$$
(24)\qquad
\begin{aligned}
X\sin\alpha - Y\cos\alpha &= r\cos\delta\,d\alpha\\
X\cos\alpha\sin\delta - Y\sin\alpha\sin\delta - Z\cos\delta &= r\,d\delta
\end{aligned}
$$

Consequently the Normals are, after simplification:

$$
(25)\qquad
\begin{aligned}
&X\Sigma\{1-\cos^2\alpha\cos^2\delta\} - Y\Sigma\sin\alpha\cos\alpha\cos^2\delta - Z\Sigma\cos\alpha\sin\delta\cos\delta = r\Sigma\{\sin\alpha\cos\delta\,d\alpha + \cos\alpha\sin\delta\,d\delta\}\\
&-X\Sigma\sin\alpha\cos\alpha\cos^2\delta + Y\Sigma\{1-\sin^2\alpha\cos^2\delta\} - Z\Sigma\sin\alpha\sin\delta\cos\delta\\
&\qquad = r\Sigma\{-\cos\alpha\cos\delta\,d\alpha + \sin\alpha\sin\delta\,d\delta\}\\
&-X\Sigma\cos\alpha\sin\delta\cos\delta - Y\Sigma\sin\alpha\sin\delta\cos\delta + Z\Sigma\cos^2\delta = -r\Sigma\cos\delta\,d\delta
\end{aligned}
$$

Remembering as before that $dx = \Delta x - X$; $dy = \Delta y - Y$; $dz = \Delta z - Z$ where $-X$, $-Y$ and $-Z$ are respectively the average values of dx, dy and dz we see from equations (9) that the conditions under which equations (25) may legitimately be used to obtain average values of dx, dy and dz or values of

X, Y, and Z are given by the equations

$$
(26)\qquad
\begin{aligned}
&\Sigma\Delta x\{1-\cos^2\alpha\cos^2\delta\} - \Sigma\Delta y\sin\alpha\cos\alpha\cos^2\delta - \Sigma\Delta z\cos\alpha\sin\delta\cos\delta = 0\\
&-\Sigma\Delta x\sin\alpha\cos\alpha\cos^2\delta + \Sigma\Delta y\{1-\sin^2\alpha\cos^2\delta\} - \Sigma\Delta z\sin\alpha\sin\delta\cos\delta = 0\\
&-\Sigma\Delta x\cos\alpha\sin\delta\cos\delta - \Sigma\Delta y\sin\alpha\sin\delta\cos\delta + \Sigma\Delta z\cos^2\delta = 0
\end{aligned}
$$

but since
$$\Sigma\Delta x = 0 \quad ; \quad \Sigma\Delta y = 0 \quad ; \quad \Sigma\Delta z = 0$$

these become

$$
(27)\qquad
\begin{aligned}
&\Sigma\Delta x\cos^2\alpha\cos^2\delta + \Sigma\Delta y\sin\alpha\cos\alpha\cos^2\delta + \Sigma\Delta z\cos\alpha\sin\delta\cos\delta = 0\\
&\Sigma\Delta x\sin\alpha\cos\alpha\cos^2\delta + \Sigma\Delta y\sin^2\alpha\cos^2\delta + \Sigma\Delta z\sin\alpha\sin\delta\cos\delta = 0\\
&\Sigma\Delta x\cos\alpha\sin\delta\cos\delta + \Sigma\Delta y\sin\alpha\sin\delta\cos\delta + \Sigma\Delta z\sin^2\delta = 0
\end{aligned}
$$

and since
$$\Delta r = \Delta x\cos\alpha\cos\delta + \Delta y\sin\alpha\cos\delta + \Delta z\sin\delta$$

Equations (27) further reduce to

$$
(28)\qquad
\begin{aligned}
\Sigma\Delta r\cos\alpha\cos\delta &= 0\\
\Sigma\Delta r\sin\alpha\cos\delta &= 0\\
\Sigma\Delta r\sin\delta &= 0
\end{aligned}
$$

which are the same as the assumptions made in the method of BRAVAIS.

We see from the foregoing discussion that if it is justifiable to make BRAVAIS' assumption that the components along the three rectangular axes of the radial motions of all of the stars with regard to a fixed origin tend to be eliminated when added together the application of the method of Least Squares according to the methods of AIRY and of CAMPBELL is legitimate. It gives us average values of the components of the stellar motions relative to the *Sun* which are what we need in order to determine the solar motion. It is not necessary to assume that the proper-motions and radial motions are principally due to the motion of the *Sun* or that the exponential Law of Error is followed by the peculiar motions of the stars used in the solution.

It should be mentioned that WEERSMA in his discussion of the solar motion by the method of BRAVAIS (*Publications of the Astronomical Laboratory at Groningen*, No. 21) calls attention to the fact that the Normal Equations used by CAMPBELL may be derived from the assumption made by BRAVAIS and that the Normal Equations of AIRY'S method are the same as the final Equations in the method of BRAVAIS.

My object has been not so much to emphasize these facts as to show that, without questioning that in most cases where the Method of Least Squares is applicable the errors follow the exponential law, it is not necessary to assume the exponential law in order to demonstrate the validity of the ordinary method of solution by the formation of Normal Equations; that cases may arise where the solution by Least Squares gives a useful result even though the Law of Error does not necessarily hold; and that the solution for the *Sun's* motion by the methods of AIRY and CAMPBELL give examples of such cases since it may be demonstrated that both of these methods are legitimate if the assumptions made in the method of BRAVAIS, which does not make use of Least Squares, may be considered as true.

Columbus, Ohio, March 10, 1917.

OBSERVATIONS OF THE SIXTH SATELLITE OF *JUPITER*.

MADE WITH THE 26-INCH EQUATORIAL OF THE U. S. NAVAL OBSERVATORY.

BY H. E. BURTON

[Communicated by Captain J. A. HOOGEWERFF, U. S. Navy, Superintendent.]

Date Wash. M. T.	✶	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$ α	log $p\Delta$ δ	Red. to App. Pl. α	Red. to App. Pl. δ
d h m s			m s	′ ″	h m s	° ′ ″			s	″
Apr. 23 14 12 17	1	30 , 10	−0 38.95	+0 8.6	16 49 13.82	−21 49 40.6	7.310	0.884	+1.67	−11.4
May 13 14 54 29	2	30 , 10	−1 31.87	−7 34.0	16 41 30.20	−21 23 16.9	9.316	0.868	+2.10	−12.2
May 17 14 22 3	3	30 , 10	−1 3.87	−3 48.7	16 39 14.29	−21 17 24.5	9.249	0.872	+2.18	−12.1
May 20 13 23 50	2	30 , 10	−1 37.98	+2 41.4	16 38 24.25	−21 13 1.5	8.926	0.879	+2.26	−12.2
June 8 12 27 4	1	30 , 10	−1 1.76	+8 43.6	16 30 1.94	−20 46 45.0	9.142	0.874	+2.48	−13.1

OBSERVATIONS OF THE SIXTH SATELLITE OF *JUPITER* (*Continued*)

Date Wash. M.T.	*	Comp	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$ α	log $p\Delta$ δ	Red. to App. Pl. α	Red. to App. Pl. δ
1913 h m s			m s	′ ″	h m s	° ′ ″			s	″
June 5 14 52 9	5	14, 3	−4 25.09	+0 17.7	19 14 59.82	−22 51 10.7	8.861	0.886	+2.82	−2.7
June 9 14 58 32	6	30, 6	−1 24.14	−8 30.4	19 13 10.69	−22 56 39.5	9.092	0.884	+2.95	−3.1
June 10 14 24 23	7	30, 6	−1 2.36	+2 32.9	19 12 12.88	−22 57 58.8	8.779	0.887	+2.98	−3.1
June 30 11 50 43	8	29, 6	−2 20.04	−4 3.2	19 1 12.39	−23 23 47.0	8.891*n*	0.888	+3.38	−3.2
July 7 13 17 3	9	20, 4	−4 13.74	−8 2.0	18 56 36.43	−23 31 18.6	9.246	0.884	+3.52	−3.4
1914										
Sept. 14 11 4 30	10	19, 5	−2 44.01	−0 43.8	21 7 38.12	−17 12 38.3	9.264	0.855	+4.11	+11.3
Sept. 20 9 54 24	11	18, 4	−2 33.42	+2 53.2	21 6 28.97	−17 52 7.6	8.937	0.863	+4.05	+10.7
Sept. 22 11 21 42	12	20, 4	−3 4.36	+2 46.1	21 6 9.30	−17 55 0.2	9.441	0.842	+4.03	+10.6
1915										
Oct. 27 9 44 51	13	15, 5	+0 53.89	+1 33.2	23 21 1.41	− 5 37 59.9	8.956	0.788	+4.16	+24.8
Oct. 28 8 6 20	14	8, 8	+0 2.90	+1 18.6	23 20 43.77	− 5 38 58.6	8.991*n*	0.788	+4.16	+24.8
Oct. 28 8 58 51	13	30, 10	+0 35.64	+0 31.4	23 20 43.16	− 5 39 1.7	7.840	0.789	+4.16	+24.8
Oct. 29 9 4 32	15	30, 10	−1 55.08	+1 35.7	23 20 24.93	− 5 40 0.4	8.429	0.789	+4.17	+24.8

The comparisons on Oct. 28 (1st date) were made with the driving clock running. All of the other comparisons were made by transits.

Mean Places of Comparison Stars for the Beginning of the Year

*	α	δ	Authority
	h m s	° ′ ″	
1	16 49 51.10	−21 49 37.8	$\frac{1}{2}$ (*A.G.* Algiers 6899 + same star comp. with *A.G.* Algiers 6895)
2	16 42 59.97	−21 15 30.7	*A.G.* Algiers 6852
3	16 43 45.98	−21 13 23.7	$\frac{1}{3}$ (*A.G.* Algiers 6857 + same star comp. with *A.G.* Algiers 6826 + same star compared with *A.G.* Algiers 6852)
4	16 31 4.22	−20 55 15.5	*A.G.* Algiers 6799
5	19 19 22.09	−22 51 55.7	Cordoba A. 13510
6	19 14 31.85	−22 48 6.0	Cordoba A. 13447
7	19 13 42.26	−23 0 28.6	Cordoba A. 13436
8	19 3 29.05	−23 19 40.6	Cordoba A. 13303
9	19 1 16.62	−23 23 13.2	Cordoba A. 13269
10	21 10 18.02	−17 42 5.8	*A.G.* Wash. 8006
11	21 8 58.34	−17 55 11.5	*A.G.* Wash. 7994
12	21 9 9.63	−17 57 56.9	*A.G.* Algiers 9097
13	23 20 3.36	− 5 39 57.9	*A.G.* Straszburg 8047
14	23 20 36.71	− 5 40 42.0	*A.G.* Straszburg 8051
15	23 22 15.84	− 5 42 0.9	$\frac{1}{2}$ (*A.G.* Straszburg 8055 + *A.G.* Wien-Ottakring 8318)

Some of the stars used were compared with other stars. Following are the comparisons:

Date	*A.G.* Algiers	$\Delta\alpha$	$\Delta\delta$
1914		m s	′ ″
April 17	6899 — 6895	+0 28.18	−5 27.5
April 18	6852 — 6826	+5 11.91	−4 56.6
May 16	6852 — 6826	+5 11.96	−4 56.3
May 16	6857 — 6826	+5 57.97	−2 49.6
May 16	6857 — 6852	+0 46.01	+2 6.7
June 17	6857 — 6826	+5 57.98	−2 49.9

The following comparisons were made but not used:

Date	Cordoba A	$\Delta\alpha$	$\Delta\delta$
1914		m s	′ ″
May 19	13510—13542	−2 47.21	−4 26.7
May 20	13447—13370	+6 6.65	−5 15.5
May 21	13436—13375	+4 58.49	+4 22.0
June 29	13436—13375	+4 58.49	+4 22.0

REMARKS

1912. April 23. Satellite very faint. Seeing fair.
May 13. Satellite very faint. Interrupted by haze. Seeing poor.
May 17. Seeing poor.
May 20. Satellite very faint at times. Seeing poor.
June 8. Seeing poor.

1913. June 5. Satellite very faint. Observation unfinished on account of dawn. Seeing good but observation not satisfactory.
June 9. Seeing poor.
June 10. Seeing fair.
June 30. Seeing fair.
July 7. Satellite very faint. Seeing fair.

1914. Sept. 14. Seeing good.
Sept. 20. Satellite very faint. Seeing poor.

REMARKS (*Continued*)

1914. Sept. 22. Satellite very faint. Seeing poor.
1915. Oct. 27. Satellite very faint. Moonlight. Seeing poor.
1915. Oct. 28. Seeing fair.
Oct. 28. Seeing fair.
Oct. 29. Seeing poor.

ELEMENTS AND EPHEMERIS OF COMET *b* 1916 (WOLF).

BY F. E. SEAGRAVE.

The following orbit has been computed from observations take at different observatories, and an ephemeris formed.

The comet will make a near approach to the asteroid *Vesta* early in November. The nearest approach will occur on November 12, 1917 when the separation will amount to 0.126 of an astronomical unit. Figures showing the heliocentric position of both comet and *Vesta* on November 12, 1917 are appended.

ELEMENTS

$$T = \text{June } 16.51 \ 1917$$
$$\omega = 120^\circ \ 37' \ \ 5''.02$$
$$\pi = 303^\circ \ 54' \ \ 3''.12$$
$$\Omega = 183^\circ \ 16' \ 58''.10$$
$$i = 25^\circ \ 10' \ 12''.20$$
$$\text{Log } q = 0.226862$$

EPHEMERIS

1917	α	δ	Log r	Log Δ
June 2	21h 49m 39s	−20° 42′ 33″	0.22946	0.11396
6	21 58 37	−21 29 59	0.22820	0.10591
10	22 7 21	−22 13 5	0.22734	0.09798
14	22 15 53	−22 51 15	0.22692	0.09023
18	22 24 9	−23 24 35	0.22694	0.08248
22	22 32 7	−23 52 0	0.22736	0.07498
26	22 39 45	−24 13 37	0.22822	0.06754
30	22 46 41	−24 30 3	0.22947	0.05988

NOVEMBER 12, 1917

Comet *b* 1916	*Vesta*
$\lambda = 13^\circ \ 29' \ 16''$	$\lambda = 14^\circ \ 18' \ 21''$
$\beta = -4^\circ \ 51' \ 58''$	$\beta = -7^\circ \ \ 8' \ \ 2''$
$\text{Log } r = 0.40376$	$\text{Log } r = 0.39174$

$$\text{Log } \Delta = 9.09981$$
$$\text{Nearest approach} = \Delta = 0.1258$$

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MEASURES OF 100 DOUBLE STARS MADE WITH THE 26-INCH REFRACTOR OF THE LEANDER McCORMICK OBSERVATORY.

By CHARLES P. OLIVIER.

Cordoba [1] Δm 0.6
R.A. $0^h 19^m 42^s$ Decl. −33° 54′

16.826	121.9	3.29	+0.5	3	850
16.908	119.8	3.44	+0.2	3	600
16.867	120.8	3.36			

Distance increasing?

I 702 Δm 1.2
R.A. $0^h 22^m.9$ Decl. −25° 24′

16.722	84.2	1.03	+0.1	2	850
16.765	80.8	1.12	+0.4	2½	850
16.744	82.5	1.08			

λ 11 [**696**] Δm 0.3
R.A. $1^h 13^m 58^s$ Decl. −27° 2′

16.826	307.5	2.11	0.0	3	850
16.908	308.2	2.05	−0.3	2	600
16.867	307.8	2.08			

A 942 [**12853**] Δm 0.5
R.A. $1^h 25^m 47^s$ Decl. +57° 54′

16.842	31.4	1.44	−2.4	3	850
16.880	33.1	1.41	−0.9	3	560
16.861	32.2	1.42			

Σ 147 [**877**] Δm 0.9
R.A. $1^h 36^m 49^s$ Decl. −11° 49′

15.859	87.8	3.22	−0.6	1	560
16.735	89.3	3.15	−0.1	2	560
16.297	88.6	3.18			

OLIVIER 52 B.D. −1°.332 8.5 9.0
R.A. $2^h 0^m 46^s$ Decl. −4° 21′

16.908	126.2	4.80	−0.3	3	600
16.919	127.9	4.82	+1.4	2½	600
16.914	127.0	4.81			

Hu 16 [**1118**] Δm 1.5
R.A. $2^h 3^m 46^s$ Decl. −10° 34′

14.726	334.2	. . .	+0.2	2	850
15.788	338.8	0.99	+0.2	2½	850
16.836	335.2		+0.4	2	850
16.842	336.8	1.02	+0.7	3	850
16.048	336.2	1.00			

Σ 315 [**1154**] Δm 0.3
R.A. $2^h 44^m 29^s$ Decl. −10° 58′

16.932	156.5	2.50	+0.5	2	336
16.952	157.2	2.52	−0.9	2½	336
16.942	156.8	2.51			

J 303 9.2 — 10.4
R.A. $2^h 59^m 28^s$ Decl. +7° 41′

16.952	36.6	3.09	+1.6	3	336
16.958	39.2	2.90	−0.4	3½	560
17.003	37.2	2.91	+0.5	1½	560
16.971	37.7	2.97			

A 975 [**12925**] Δm 2.8

R.A. $3^h\ 0^m\ 58^s$ Decl. +56° 16′

16.842	199.8	1.19	−3.0	3	850
16.880	194.3	1.58	−2.2	3	560
16.888	199.9	1.50	−4	3	850
16.870	198.0	1.52			

J 304 9.5 — 10.8

R.A. $3^h\ 16^m\ 11^s$ Decl. +7° 43′

16.952	81.3	3.97	+0.5	2	336
16.958	82.4	3.99	−0.3	3	560
17.003	82.2	3.95	+0.4	1½	560
16.971	82.0	3.97			

J 305 Δm 0.2

R.A. $3^h\ 22^m\ 13^s$ Decl. +9° 31′

16.958	110.4	1.77	−0.2	3	560
17.003	110.6		+0.3	2	560
17.053	112.1	1.87	+1.8	2½	560
17.005	111.0	1.82			

Σ 408 [**1743**] Δm 0.2

R.A. $3^h\ 25^m\ 40^s$ Decl. −1° 37′

15.970	335.6	1.31	+0.1	2	850
16.735	335.9	1.42	+0.2	2½	600
16.352	335.8	1.36			

β 308 [**1795**] Δm 1.4

R.A. $3^h\ 33^m\ 2^s$ Decl. −7° 58′

16.932	325.7	1.70	+0.2	2	336
16.958	329.9	1.63	+0.6	2 —	560
17.053	327.2	1.90	+1.7	2	560
16.981	327.6	1.74			

A 987 [**12952**] Δm 0.4

R.A. $3^h\ 36^m\ 18^s$ Decl. +29° 26′

14.750	6.6	0.85	3	3	850
16.765	10.0	1.23	1.4	5	850
16.836	11.0	1.16	2.4	3	850
16.842	10.8	1.04	2.4	3	850
16.294	9.6	1.07			

β 880 [**1836**] Δm 0.4

R.A. $3^h\ 38^m\ 17^s$ Decl. +31° 51′

14.750	358.8	0.50	−3.	4	850
16.888	356.0	0.51	−1.1	3	850
15.819	357.4	0.52			

Ho 501 [**1835**] Δm 0.2

R.A. $3^h\ 38^m\ 8^s$ Decl. +35° 32′

14.750	188.8	1.20	−3.5	1	850
16.836	188.2	1.30	−2.0	3	850
15.793	188.5	1.25			

A 989 [**12954**] Δm 0.4

R.A. $3^h\ 37^m\ 18^s$ Decl. +29° 16′

16.842	1.4	3.00	−3.3	2½	850
16.880	1.7	3.02	−2.5	3	560
16.861	1.6	3.01			

No motion.

J 235 9.5 — 9.8

R.A. $3^h\ 40^m\ 23^s$ Decl. +0° 9′

16.919	260.6	2.46	+0.1	3	600
16.932	257.8	2.61	−0.3	2	336
16.952	260.7	2.22	+0.4	2	336
17.003	259.0	2.43	−0.4	2	560
16.952	259.5	2.43			

β 1184 [**1882**] Δm 0.3

R.A. $3^h\ 42^m\ 23^s$ Decl. +22° 4′

14.750	272.9	0.63	−3.8	4	850
16.836	275.1	0.70	−1.9	3	850
15.793	274.0	0.66			

Hu 1067 [**12958**] 9.0 — 9.8

R.A. $3^h\ 42^m\ 59^s$ Decl. +38° 11′

14.750	73.8	1.55	−4.1	4	850
16.756	72.2	1.33	−2.0	1	850
16.836	74.0	1.27	−2.9	3	850
16.117	73.3	1.38			

O Σ 66 [**1906**] Δm 0.3
R.A. 3h 45m 20s Decl. +40° 30′

14.750	142.9	0.85	−3.0	3	850
16.836	136.5	0.85	1.6	2½	850
16.888	138.2	0.94	−1.5	3	850
16.158	139.2	0.87			

β 401 [**1907**] Δm 4
R.A. 3h 45m 14s Decl. −1° 50′

15.010	250.9	4.39	+4.0	2	850
15.970	254.6	4.35	+0.4	3	850
16.836	251.8	4.20	−1.0	2	850
15.939	252.4	4.31			

Washburn 66 [**1925**] 8.7 − 11.9
R.A. 3h 47m 45s Decl. −8° 47′

15.970	27.9	2.21	+0.1	3	850
16.765	29.7	2.18	+0.2	3	850
16.368	28.8	2.20			

Apparently fixed.

J 236 Δm 0.4
R.A. 4h 3m 53s Decl. +1° 10′

16.952	221.1	4.32	−1.5	2½	336
17.053	220.4	4.36	+1.5	2	560
17.002	220.8	4.34			

Σ 518 [**2109**] 9.2 − 10.4
R.A. 4h 10m 40s Decl. −7° 48′

16.820	22.3	3.19	+0.3	3	850
16.826	19.5	3.32	−1.3	3	850
16.831	21.0	3.14	+0.2	4	850
16.826	20.9	3.22			

Σ 547 [**2198**] 9.3 − 14
R.A. 4h 20m 49s Decl. −1° 37′

16.831	71.7	2.21	+0.4	3	850
16.842	71.2	2.24	−1.3	4	850
16.836	71.4	2.22			

Hu 554 [**2212**] Δm 3.5
R.A. 4h 27m 0 Decl. +50° 48′

16.861	304.1	1.51	−2.7	2	850
16.908	306.9	1.60	−1.0	3	600
16.884	305.6	1.56			

Hu 1084 [**12999**] Δm 0.1
R.A. 4h 28m 54s Decl. +39° 32′

14.742	35.4	0.64	−3.	3	850
16.888	37.2	0.63	−2.3	3	850
15.815	36.3	0.64			

Possible motion?

A 836 [**13000**] 9.4 − 10.6
R.A. 4h 29m 15s Decl. −0° 58′

16.831	203.8	2.25	+0.6	3	850
16.842	203.2	2.37	−1.2	4	850
16.836	203.5	2.31			

J 238 Δm 0.2
R.A. 4h 32m 52s Decl. +17° 27′

16.958	66.0	2.79	−0.8	3	560
17.072	67.6	2.84	+2.1	3	560
17.015	66.8	2.82			

Hu 552 Δm 0.2
R.A. 4h 28m 22s Decl. +51° 56′

16.861	235.2	1.25	−2.6	3	850
16.908	231.7	1.30	−1.	2½	600
16.919	234.8	1.29	−1.7	2	600
16.896	233.9	1.28			

No motion.

A 1541 Δm 1.0
R.A. 4h 40m 22s Decl. +43° 13′

16.842	41.7	1.29	−2.9	3	850
16.908	42.7	1.46	−2.1	2½	600
16.875	42.2	1.38			

No motion.

Hu 551 Δm 0.9

R.A. $4^h\ 45^m\ 8^s$ Decl. $+19°\ 53'$

16.861	130.2	1.75	−2.3	3	850
16.908	130.1	1.99	−1.0	$2\frac{1}{2}$	600
16.919	127.8	1.91	−2.0	3	600
16.896	129.4	1.88			

A 179 [**2110**] Δm 2.2

R.A. $4^h\ 52^m\ 33^s$ Decl. $-6°\ 34'$

16.883	247.6	2.27	+0.8	$2\frac{1}{2}$	560
16.888	245.4	2.17	+1.1	2	560
16.886	246.5	2.22			

No motion.

A 1021 [**13027**] Δm 0.2

R.A. $4^h\ 57^m\ 9^s$ Decl. $-1°\ 2'$

16.831	333.6	0.76	−0.1	3	850
16.864	333.7	0.72	−0.1	3	850
16.848	333.6	0.74			

J 319 9.0 — 10.0

R.A. $4^h\ 59^m\ 28^s$ Decl. $-5°\ 20'$

16.919	230.5	3.06	−0.8	3	600
16.921	231.6	3.23	+0.5	5	600
16.932	234.1	3.33	−0.9	2	336
16.924	232.1	3.21			

A 482 [**2512**] 9 — 11

R.A. $5^h\ 0^m\ 13^s$ Decl. $-6°\ 40'$

16.883	172.7	4.21	+0.8	3	560
16.888	171.7	4.09	+1.3	2	560
16.886	172.2	4.15			

No motion.

A 485 9.0 — 11.6

R.A. $5^h\ 6^m\ 0^s$ Decl. $-9°\ 31'$

16.919	123.7	3.91	−0.8	3	600
16.921	122.7	4.37	+1.4	4	600
17.053	119.6	4.38	+0.9	3	560
16.964	122.0	4.22			

No motion.

J 326 Δm 0.5

R.A. $5^h\ 18^m\ 47^s$ Decl. $-1°\ 29'$

16.932	244.8	3.06	−0.9	3	336
16.932	245.2	3.09	−0.6	3	600
16.932	245.0	3.08			

J 676 = A 2649 8.8 — 9.0

R.A. $5^h\ 30^m\ 6^s$ Decl. $+7°\ 22'$

16.919	284.0	1.68	−0.5	3	600
16.921	282.1	1.79	+0.6	4	600
16.920	283.0	1.74			

A 320 [**2852**] Δm 1.2

R.A. $5^h\ 31^m\ 33^s$ Decl. $-2°\ 1'$

16.883	178.3	0.97	−0.2	2	560
16.902	179.5	0.97	−0.1	$2\frac{1}{2}$	850
16.921	182.2	0.96	+0.4	4	600
16.902	480.0	0.97			

No motion.

A 497 [**2914**] 9.0 — 11.3

R.A. $5^h\ 39^m\ 44^s$ Decl. $-7°\ 47'$

16.883	183.6	2.05	+0.7	3	560
16.888	182.9	2.21	+0.9	$2\frac{1}{2}$	560
16.886	183.2	2.13			

No motion.

J 682 Δm 0.6

R.A. $6^h\ 3^m\ 21^s$ Decl. $-7°\ 21'$

16.932	168.9	2.27	−0.5	2	336
16.998	170.1	2.50	+2.0	4	560
17.053	170.7	2.51	+1.3	3	560
17.994	169.9	2.43			

J 258 Δm 9.2 — 9.7

R.A. $6^h\ 16^m\ 28^s$ Decl. $+7°\ 42'$

16.919	257.4	2.56	−0.5	4	600
16.932	260.5	2.64	−1.2	3	600
16.995	259.6	2.73	+2.0	2	560
16.919	259.2	2.64			

J 687 Δm 0.1
R.A. 6h 15m 14s Decl. +7° 47′

17.072	352.2	2.13	+1.0	2	560
17.105	351.6	2.09	+0.2	4	560
17.088	351.9	2.11			

J 260 8.8 10.1
R.A. 6h 20m 20s Decl. +7° 46′

16.919	178.1	1.58	−0.7	2½	600
16.932	175.7	1.66	−1.4	2½	600
16.998	177.6	1.44	+1.4	3	560
16.950	177.1	1.56			

β 98 [3152] Δm 0.3
R.A. 6h 27m 45s Decl. −5° 16′

16.735	323.5	0.62	−1.0	2	600
16.921	323.1	0.62	+0.7	3	600
16.828	323.3	0.62			

Sirius **[3596]**
R.A. 6h 40m 44s Decl. −16° 35′

16.883	74.33	10.38	0.0	2½	560
16.888	73.80	...	+0.3	2	560
16.902	74.40	10.61	−0.5	2½	560
16.919	73.75	10.43	−0.3	2	600
16.921	73.76	10.58	+0.2	4	600
16.995	73.89	11.00	+1.0	3½	560
16.918	73.99	10.60			

The companion is now an easy object under fair conditions of seeing.

A 1057 [13110] Δm 0.1
R.A. 6h 43m 45s Decl. −8° 26′

16.820	98.7	1.10	−1.0	2	850
16.888	95.4	1.03	+0.1	3	560
16.854	97.0	1.06			

β 898 *b* 8.5 — 11.0
R.A. 6h 45m 54s Decl. −15° 55′

14.956	354.7	2.88	+0.7	2½	850
16.820	353.1	2.67	−0.1	2	850
16.831	353.5	3.32	−0.6	3	850
17.053	354.1	3.06	+0.4	3	850
16.415	353.8	2.98			

No motion.

Washburn 82 [3661] Δm 1.6
R.A. 6h 46m 12s Decl. −11° 10′

16.831	224.4	1.91	0.0	3	850
16.902	222.3	1.87	−0.9	2	850
16.866	223.4	1.89			

Apparently fixed.

A 514 [3731] Δm 0.2
R.A. 6h 52m 35s Decl. −9° 58′

16.883	95.4	1.46	+0.1	3	560
16.902	98.8	1.44	−0.5	2	560
16.921	97.0	1.52	+0.9	2½	600
16.902	97.0	1.47			

No motion.

A 515 [3719] Δm 0.7
R.A. 6h 53m 10s Decl. −10° 3′

16.883	305.2	1.72	+0.7	2	560
16.902	303.1	1.57	−0.3	3	560
16.995	303.7	1.56	+1.1	3	560
16.927	304.0	1.62			

No motion.

β 573 [3791] Δm 0.2
R.A. 6h 57m 7s Decl. −10° 44′

14.956	267.4	0.69	0.0	2½	850
16.902	268.3	0.89	−0.3	2½	560
16.921	268.4	0.85	+0.5	3	600
16.260	268.0	0.81			

H 48 [3835] Δm 0.0
R.A. 7h 1m 29s Decl. −12° 48′

14.868	328.6	2.52	−1.0	3	850
16.902	328.9	2.44	−0.0	3	560
15.885	328.8	2.48			

Σ 1104 [4098] Δm 1.1
R.A. 7h 24m 49s Decl. −14° 47′

A — C

14.956	336.8	2.14	+0.6	2	850
16.883	338.3	2.30	+0.5	2	560
16.902	337.6	2.13	−0.2	2	560
16.247	337.6	2.19			

A — D

16.902	7.7	42.35	+0.2	2	560

A 1071 **13138** Δm 0.4

R.A. 7h 32m 7s Decl. −8° 32′

16.820	175.5	1.89	−1.2	3	850
16.883	176.8	1.85	+0.6	3	560
16.852	176.2	1.87			

β 333 [**1103**] 8.5 10.4

R.A. 7h 56m 59s Decl. −22° 6′

15.911	47.7	1.42	−0.2	2½	850
16.902	43.7	1.91	−0.1	2	560
16.924	45.7	1.76		3½	600
16.578	45.7	1.70			

Evidently fixed.

Olivier 53 9.7 12

R.A. 7h 58m 38s Decl. −22° 28′

15.911	337.4	2.43	−0.6	2	850
16.902	337.9	2.75	+1.2	3	560
16.998	336.2	2.57	+1.5	3	560
16.604	337.4	2.58			

A 514 [**1113**] 9.5 11.5

R.A. 8h 0m 53s Decl. −2° 11′

16.883	75.3	2.25	+0.7	3	560
17.072	76.0	2.28	+0.4	2	560
16.978	75.6	2.26			

Olivier 54 9.4 10.9

R.A. 8h 0m 8s Decl. −22° 34′

16.902	74.8	3.87	+0.2	3	560
16.998	78.7	3.55	+1.6	3	560
17.072	74.9	3.68	+0.6	3	560
16.994	76.1	3.70			

Olivier 31 9.3 9.9

R.A. 8h 1m 56s Decl. −22° 34′

14.868	356.2			2	850
14.956	356.2			2	850
15.911	355.9	3.15	0.5	3	560
16.902	356.0	3.03	−0.2	3	560
15.659	356.1	3.09			

A 335 [**1172**] Δm 0.4

R.A. 8h 5m 57s Decl. −4° 37′

16.883	122.8	1.48	+0.9	2	560
16.921	121.8	1.48	+0.3	5	600
16.995	124.5	1.45	+0.7	2½	560
16.933	124.0	1.47			

Distance increased 0″.35 in 14 years.

β 206 [**1681**]

R.A. 8h 31m 9s Decl. −24° 46′

15.911	280.1	1.77	−0.5	2	560
16.998	280.9	1.81	+1.1	2	560
16.454	280.5	1.79			

Apparently fixed.

Hough 363 [**1998**]

R.A. 9h 10m 27s Decl. −19° 42′

14.956	177.0	2.24	0.2	2½	850
16.902	179.7	2.47	+0.3	2½	560
17.072	176.4	2.30	−0.3	2	560
16.310	177.7	2.34			

Distance 0″.78 greater than at discovery.

A 125 [**5026**] Δm 2.5

R.A. 9h 13m 55s Decl. −9° 59′

16.883	24.8	2.81	−0.1	3	560
16.902	22.6	3.04	+0.7	3	560
16.995	26.2	3.07	+0.2	4	560
16.927	24.5	2.97			

No motion.

A 126 [**5031**] Δm 0.2

R.A. 9h 14m 28s Decl. −9° 7′

16.883	144.0	1.36	0.0	2½	560
16.902	145.1*	1.22	+0.5	2½	560
16.892	144.6	1.28			

J 83 Δm 0.3

R.A. 9h 45m 19s Decl. +4° 5′

16.998	232.9*	1.89	+0.9	3	560
17.105	231.7	2.01	−1.1	2	560
17.052	232.3	1.95			

J 89 9.6 — 10.5
R.A. $10^h 40^m 21^s$ Decl. $-3° 21'$

	°	″			
16.998	87.5	3.90	+0.4	3	560
17.017	88.0	3.79	+0.8	1½	560
17.034	88.8	3.97	+0.7	2	560
17.015	88.1	3.89			

Σ 1500 Δm 0.2
R.A. $10^h 54^m 56^s$ Decl. $-2° 56'$

15.210	309.4	1.89	+0.2	2½	850
16.921	312.0	1.58	−0.4	3½	850
16.995	309.2	1.69	−0.4	3	560
16.375	310.2	1.72			

J 81 Δm 0.2
R.A. $11^h 4^m 55^s$ Decl. $+10° 14'$

17.031	140.2	1.87	+1.0	3	560
17.080	137.2	1.88	+1.2	3	560
17.056	138.7	1.88			

J 82 Δm 0.4
R.A. $11^h 6^m 56^s$ Decl. $+11° 26'$

17.031	112.8	1.96	+1.1	3	560
17.080	109.1	2.10	+1.3	3	560
17.056	111.0	2.03			

J 86 9.2 — 10.3
R.A. $11^h 31^m 17^s$ Decl. $+4° 41'$

16.998	93.5	2.35	0.0	2	560
17.031	97.1	2.38	+0.2	2	560
17.014	95.3	2.36			

Σ 1290 Δm 1.8
R.A. $12^h 5^m 15^s$ Decl. $-5° 19'$

15.287	148.1	5.88	+0.1	3	560
17.031	149.9	5.90	−0.1	3	560
16.159	149.0	5.89			

Σ 1619 Δm 0.1
R.A. $12^h 10^m 0^s$ Decl. $-6° 42'$

15.210	279.0	7.60	−0.1	2½	560
15.355	278.5	7.48	+1.3	1½	850
15.282	278.8	7.54			

Σ 1669 A — B **[6239]** Δm 0.1
R.A. $12^h 36^m 4^s$ Decl. $-12° 28'$

	°	″			
15.287	125.3	5.66	−0.3	2	560
16.355	126.6	5.52	+0.3	2	560
15.821	126.0	5.59			

A — C

16.355	227.8	57.96	+0.4	2	560

β 222 **[6413]** Δm 0.7
R.A. $13^h 12^m 0^s$ Decl. $-24° 1'$

15.287	13.4	1.78	0.0	3	560
15.317	14.8	1.32	0.0	2	600
17.031	11.1	1.60	+0.1	2	560
15.878	13.1	1.57			

Apparently fixed.

β 114 **[6528]** Δm 0.2
R.A. $13^h 29^m 3^s$ Decl. $-8° 6'$

15.287	144.4	1.53	−0.1	3½	560
15.317	145.0	1.38	+0.1	2	600
17.020	144.4	1.43	−0.4	3	560
15.875	144.6	1.45			

Probably in slow motion.

Σ 1788 **[6668]** Δm 0.3
R.A. $13^h 49^m 43^s$ Decl. $-7° 34'$

15.287	82.1	3.11	+0.3	3½	560
15.317	82.8	2.87	−0.2	2½	600
15.302	82.4	2.99			
16.355	81.2	2.82	+0.3	2	560
17.020	80.2	2.96	+0.1	3	560
16.688	80.7	2.89			

Σ 1802 **[6725]** Δm 0.7
R.A. $14^h 2^m 40^s$ Decl. $-12° 27'$

15.317	280.2	5.00	−0.2	3	600
17.020	280.2	5.23	−0.3	2½	560
17.031	280.7	5.01	−0.3	2½	560
16.456	280.4	5.09			

Distance increasing?

Hu 47 [**6740**] Δm 0.1
R.A. 14h 5m 58s Decl. −2° 12′

16.355	243.3	1.49	+0.6	3½	560
17.080	241.3	1.69	−0.2	3½	560
16.718	242.3	1.59			

Σ 1837 [**6851**] Δm 1.4
R.A. 14h 19m 48s Decl. −11° 13′

15.317	296.0	1.62	−0.2	2½	600
16.475	299.7	1.44	+1.2	3	560
17.020	295.1	1.52	−0.7	2	560
16.271	296.9	1.53			

A 2233 Δm 0.4
R.A. 16h 21m 47s Decl. +1° 14′

16.475	25.0	2.67	+0.9	1	560
17.080	25.1	2.65	−1.7	3	560
16.778	25.0	2.66			

A 2243 Δm 0.6
R.A. 17h 21m 50s Decl. +2° 25′

16.475	299.4	2.04	+0.3	3	850
17.080	299.1	1.97	−2.4	2	560
16.778	299.2	1.99			

Σ 2400 [**8830**] 8.5 — 11.0
R.A. 18h 14m 44s Decl. +16° 8′

14.457	177.8	3.86	−0.1	3	850
16.678	175.3	3.85	+2.0	3	850
15.568	176.6	3.86			

β 829 [**9633**]
R.A. 19h 14m 6s Decl. +5° 30′

12.557	311.6	0.83	+0.5	3	850
12.690	314.4	0.84	+0.4	1	1300
12.624	313.0	0.84			

β 678 [**10656**] 8.5 — 11.5
R.A. 20h 55m 25s Decl. −8° 44′

15.730	209.7	2.55	+1.7	3	850
16.678	208.7	2.47	+0.8	3	850
16.204	209.2	2.51			

[illegible] 22° motion in 38 years.

H I 47 [**10801**] Δm 0.1
R.A. 21h 6m 50s Decl. −15° 25′

15.788	136.8	3.33	+0.3	3	850
16.678	137.7	3.35	+1.0	3	850
16.233	137.2	3.34			

Σ 2862 [**11190**] Δm 0.2
R.A. 22h 1m 58s Decl. +0° 5′

15.700	101.7	2.39	−0.2	3	850
15.744	99.8	2.36	+0.1	1	850
15.722	100.8	2.38			

Σ 2909 [**11743**] Δm 0.1
R.A. 22h 23m 41s Decl. −0° 32′

14.750	127.7	2.96		3	850
14.761	127.6	2.75	−0.7	3	850
14.756	127.6	2.86			

β 477 [**12012**] Δm 0.3
R.A. 22h 47m 0s Decl. −22° 14′

14.742	276.9	2.67	−0.1	3	850
15.744	277.2	2.63	−0.4	3	850
15.243	277.0	2.65			

A 2296 9.6 — 11
R.A. 22h 53m 35s Decl. +3° 28′

16.683	48.5	0.63	+1.4	1	850
16.826	51.4	0.64	+0.4	3	850
16.754	50.0	0.64			

Howe 62 [**12146**] 9 — 11.5
R.A. 23h 0m 18s Decl. −4° 48′

14.780	211.1	4.27	+0.5	2½	850
16.744	214.1	4.44	+0.6	3	600
16.826	211.3	4.18	0.0	3	850
16.116	212.1	4.30			

Apparently fixed.

O. Stone 59 [12377] Δm 0.7

R.A. 23h 22m 27s Decl. −27° 44′

	°	″			
14.742	212.3	1.63	0.0	2½	850
16.908	213.0	1.56	+0.8	2	600
15.825	212.6	1.60			

I 691 9.8 10.2

R.A. 23h 41m.6 Decl. −20° 22′

15.859	163.5	2.70	+0.2	2	850
16.722	166.8	2.38	+0.4	2½	850
16.765	164.6	2.49	−0.4	4	850
16.826	166.6	2.61	+0.2	4	850
16.513	165.4	2.54			

I 696 Δm 2.0

R.A. 23h 42m.2 Decl. −22° 50′

15.859	198.0	0.86	+0.3	2	850
16.765	203.7	0.84	−0.3	2	850
16.312	200.8	0.85			

A 1245 [13658]

R.A. 23h 12m 15s Decl. +8° 55′

	°	″			
14.800	208.2	1.65	−0.4	2½	850
16.678	208.2	1.72	−0.5	3	850
15.739	208.2	1.68			

No motion.

Σ 3045 [12625] 8.3 9.7

R.A. 23h 49m 18s Decl. +1° 55′

14.780	264.4	1.70		2	850
16.683	264.1	1.50	−0.3	3	850
16.741	264.7	1.52	−0.5	2	600
16.068	264.3	1.57			

A 2200 Δm 0.2

R.A. 23h 49m 53s Decl. +3° 55′

16.683	35.8	0.88	0.0	3½	850
16.765	33.8	0.92	0.0	4	850
16.724	34.8	0.90			

Star	Jonckheere			Olivier			Ol. — J.	
	°	″		°	″		°	″
J 81	143.0	1.93	2n	138.7	1.88	2n	−4.3	−0.05
82	108.4	1.24	2n	111.0	2.03	2n	+2.6	+0.79
83	53.6	1.59	2n	232.3	1.95	2n	−1.3	+0.36
86	94.1	1.69	2n	95.3	2.36	2n	+1.2	+0.67
89	90 ±	1.8 ±	.	88.1	3.89	3n		. . .
235	80.6	1.07	1n	259.8	2.43	4n	−0.8	+1.36
236	218.0	3.02	1n	220.8	4.34	2n	+2.8	+1.32
238	70.3	1.80	1n	66.8	2.82	2n	−3.5	+1.02
258	258.2	1.73	2n	259.2	2.64	3n	+1.0	+0.91
260	172.8	1.53	1n	177.1	1.56	3n	+4.3	+0.03
303	36.8	2.40	1n	37.7	2.97	3n	+0.9	+0.57
304	79.2	2.90	1n	82.0	3.97	3n	+2.8	+1.07
305	110.6	1.30	1n	111.0	1.82	2n	+0.4	+0.52
319	235.4	2.73	1n	232.1	3.21	3n	−3.3	+0.48
326	245.8	2.76	2n	245.0	3.08	2n	−0.8	+0.32
676	100.4	1.74	1n	283.0	1.74	2n	+2.6	+0.00
682	173.4	2.19	1n	169.9	2.43	3n	−3.5	+0.24
687	171.5	2.07	1n	351.9	2.11	2n	+0.4	+0.04
				Mean:				+0.57

University of Virginia, Feb. 20, 1917.

THE YALE INDEX TO STAR CATALOGUES.

BY MARGARETTA PALMER.

In connection with the parallax determinations at the Yale Observatory and to avoid vain search through many catalogues for a star whose position might be found in only one or two, Dr. W. L. ELKIN began an index to the principal catalogues in use at the Observatory.

The first printed reference to this work is found in Dr. ELKIN'S report to the Board of Managers of the Observatory, June 23, 1897. He there states that "Miss NEWTON has been occupied in preparing a series of references to other catalogues in an interleaved copy of the *Bonner Durchmusterung*."

For various reasons it has never seemed advisable to push this work to completion but, during the stages of its progress, it has been of assistance in the practical work of the observatory. It is believed that, were it possible at the present time to publish a complete and thoroughly revised Index such as has been used at Yale in its unfinished state, it would possess practical advantages not only at the present time but even after the completion of the *Geschichte des Fixsternhimmels*, the stupendous task undertaken by the KÖNIGLICH PREUSSISCHE AKADEMIE. The *Geschichte*, when finished, will be invaluable as a universal compendium of the sidereal astronomy of position for the period 1750 to 1900 inclusive. The *Yale Index* is, for the same period, a handy reference book for the stars of the *Bonner Durchmusterung* only.

Owing to the present international situation, the unfinished results of the *Geschichte* which have been so generously placed at the disposal of those who needed them are not generally available. It, therefore, seems advisable to call the attention of those who use star catalogues of various periods to the incomplete *Yale Index*.

The Yale Observatory will furnish for any star contained in the *Bonner Durchmusterung* a list of catalogues in which that star is found and a statement as to the probability of finding it in other catalogues. As nearly all of the catalogues which are generally useful (1750 to 1900 inclusive) and many of the smaller and little used, have thus far been entered in the *Index*, the list furnished would be more complete than those which have generally resulted from individual efforts for the determination of proper-motion of comparison stars. If desired, instead of a list of references to catalogues containing the required positions, a transcript of the complete record of each catalogue will be furnished.

Obviously the possession of an interleaved copy of the *Durchmusterung*, showing at a glance the number and names of the catalogues in which any star is found, would be of greater practical advantage than any correspondence method of obtaining such information. In the absence of such a published work of reference, it is believed that the correspondence method may be of service. To avoid unnecessary delay in verifying unchecked entries from small catalogues, it is desirable that astronomers requesting references or transcriptions state clearly whether they desire a complete list of all catalogues where the required stars are to be found or merely a good working group for certain periods. Requests for references or star-positions or inquiries in regard to the same should be addressed to *Index Department, Yale Observatory, New Haven, Conn.*

ELEMENTS OF SCHAUMASSE COMET.

(T)	1917, May 18.89 G. M. T.	(i)	158° 35′
(ω)	119° 5′	(q)	0.762
(Ω)	8° 19′		

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MICROMETRIC MEASURES OF DOUBLE STARS,

By EDISON PETTIT.

The following list of one hundred and thirty-eight miscellaneous double stars, is selected from *B. G. C.* among objects apparently needing measures. Measures were made with the $11\frac{1}{2}''$ equatorial and $6''$ micrometer of the Washburn College Observatory, Topeka, Kansas, in the years 1915 and 1916. Each measure was made with four or more settings in both angle and distance.

During the prosecution of this program several new pairs were discovered, not being found in catalogs or published lists of measures. The number suggested that perhaps many of the fainter pairs were not recorded, and it was decided that a test would be made on +38° which goes through the zenith, to see in what measure the fainter pairs were not recorded. In this way, beginning September, 1916, during the three months following, approximately thirty pairs were discovered of which some nineteen were found by Prof. DOOLITTLE to be new pairs. Most of these objects however are fainter than the 8.7 magnitude. A list of these measures with positions for 1880 is appended to the present list of double stars.

During the prosecution of the work of measuring the catalog stars measures of several well known pairs were carried on as a check on the accuracy of the measures in general. Among such stars are 61 *Cygni* and some of the fixed Σ stars.

22 ESPIN 113

1916.586	124.8	6.41
.592	129.1	6.65
.597	126.7	6.24
1916.592	126.9	6.43

Possible change in angle measure in 1902.

29 ESPIN 114

1917.049	168.5	4.65
1916.586	164.1	4.60
1916.597	163.8	4.39
1916.744	165.5	4.55

82 Σ 11

1915.857	192.3	8.47
1915.890	192.4	8.49
1917.047	193.5	8.27
1916.238	192.7	8.41

109 KU 4

1916.591	131.9	1.85
.597	133.7	1.89
.603	133.5	1.80
1916.597	134.0	1.85

Possible change in distance since 1901.

110 Σ 18

1916.622	90.6	.77
.625	88.8	.85
.630	90.8	.70
1916.626	90.1	0.77

111 WEISSE 1

1916.589	107.2	5.42
.597	107.5	5.58
.602	107.3	5.73
1916.596	107.4	5.58

118 Σ 23

1916.591	322.8	4.46
.597	322.5	4.42
.603	322.0	4.54
1916.597	322.5	4.47

Change marked since 1903.

128 Σ 24

1916.589	248.9	5.24
.597	249.5	5.24
.603	247.7	5.15
1916.596	248.7	5.23

137 KR 4

1916.622	188.8	2.01
.625	189.8	2.07
.630	189.1	2.20
1916.626	189.2	2.09

149 H 1019

1916.641	95.3	1.40
.680	96.5	1.33
.721	95.0	1.39
1916.681	95.9	1.37

152 OΣ 6

AB — C

1916.575	113.3	13.49
.586	113.6	13.86
.597	113.6	13.34
1916.586	113.5	13.53

No change since 1898. Cannot separate *AB*.

173 H 1022

1916.641	30.7	5.56
.680	33.6	5.83
.688	32.0	5.59
1916.670	32.1	5.66

Orange-green.

190 Hu 2

1916.597	333.6	2.63
.603	331.3	2.83
.606	336.1	2.85
1916.602	334.	2.77

191 Σ 29

1916.589	167.5	5.53
.597	168.0	5.07
.603	168.5	5.29
1916.596	168.0	5.30

240 Kr 7

1916.589	11.2	4.07
.597	8.7	4.36
.602	10.0	3.98
1916.596	10.0	4.11

269 Espin

1916.603	113.4	5.97
.606	112.6	6.00
.611	113.6	6.13
1916.607	113.3	6.03

271 β 780

1916.603	138.5	2.32
.622	138.7	2.25
.625	136.1	2.25
1916.617	137.8	2.31

281 Hu 511

1916.603	177.4	4.36
.611	177.9	4.40
.611	178.0	4.52
1916.609	177.8	4.43

332 β 230

1916.611	322.3	3.95
.688	323.	3.96
.699	323.9	3.95
1916.676	323.2	3.95

310 Σ 44

1916.611	268.6	10.68
.622	268.0	10.04
.625	268.2	10.16
1916.619	268.3	10.28

316 Σ 45

1916.611	89.9	13.79
.613	88.1	13.73
.622	89.2	13.75
1916.615	89.1	13.76

Yellowish blue.

368 Σ 49

1916.688	320.5	6.82
.718	319.5	7.11
.721	322.2	7.23
1916.709	320.7	7.05

422 Σ 59

1916.622	148.0	2.17
.625	150.0	2.00
.630	145.4	2.29
.688	148.3	2.10
.707	148.7	2.21
.718	149.4	2.21
1916.665	148.6	2.17

426 η Cass Σ 60

1916.704	252.3	6.27
.707	252.8	6.44
1917.049	255.4	6.18
1916.820	253.5	6.30

445 β 781

1916.625	32.5	.91
.630	29.7	.89
.680	29.2	.99
1916.645	30.5	.94

473 Espin 118

1916.688	62.4	2.21
.707	62.1	2.26
.718	61.0	2.27
1916.704	61.8	2.25

P. A. reversed in B's general catalogue.

1223 A. G. 37

1916.772	290.3	5.24
.797	291.7	5.08
.849	290.4	5.63
1916.806	290.8	5.32

1572 Σ 355

1916.589	146.1	2.38
.591	146.3	2.65
.606	145.0	2.60
1916.595	145.8	2.54

1591 Σ 360

1915.890	131.1	2.94
1916.586	131.8	2.13
1916.589	132.7	2.20
1916.355	131.9	2.42

Change evident in P.A. Decreasing since 1892.

2027 OΣ 531

1917.063	121.8	1.54
1916.882	122.1	1.29
1917.003	120.4	1.33
1916.983	121.4	1.39

1122 Castor Σ 1110

1915.279	219.9	5.59
.281	219.4	5.53
.287	219.7	5.53
.290	219.2	5.58
1915.285	219.5	5.56

1760 Σ 1270

1915.213	262.8	1.46
.227	262.3	1.64
.238	263.3	1.76
1915.226	262.8	1.62

1920 Σ 1308

1915.208	84.7	10.74
.213	84.8	10.67
.227	85.3	10.32
1915.216	84.9	10.57

1926 H 1161

AB

1915.243	183.3	7.12
.293	181.4	7.17
.386	181.0	7.27
1915.307	181.9	7.18

AC

	272.0	32.37
	271.4	31.90
	271.2	32.07
	271.5	32.11

No motion since 1900 unless *AC* distance.

1931 H 4172

1915.254	218.2	7.41
.257	219.9	6.99
.284	218.9	7.39
1915.265	219.0	7.16

1939 Σ 1312

1915.279	148.7	4.95
.282	150.0	4.90
.295	149.7	4.82
1915.286	149.4	4.89

1916 H 119

1915.254	328.4	54.09
.257	329.5	53.84
.279	328.9	53.19
1915.263	328.9	53.71

B at least one magnitude too great in the catalog. Fixed 1905.

1915 Σ 1317

1917.074	62.7	7.88
1915.246	62.7	7.96
.257	61.4	8.10
1915.892	62.3	7.98

1950 Σ 1304

1915.238	318.5	24.48
.241	319.3	24.25
.249	318.9	24.35
1915.243	318.9	24.36

1957 H 805

1915.238	71.0	9.30
.246	71.3	8.85
.386	71.3	9.40
1915.290	71.2	9.18

1961 Σ 1319

1915.293	51.9	13.68
1916.403	50.4	13.55
1917.047	51.3	13.48
1916.248	51.2	13.57

1972 Σ 1321

1915.254	69.7	19.47
.279	69.2	19.41
.295	68.	19.03
1915.276	69.1	19.31

1987 Σ 1327

AB

1915.238	71.0	10.72
.243	73.0	10.37
.257	73.0	10.52
1915.246	72.2	10.53

AC

	22.5	26.73
	22.5	26.75
	23.9	26.94
	23.0	26.80

1998 Ho 363

1915.254	178.4	3.47
1915.279	176.8	3.21
1916.386	178.8	3.23
1915.640	177.9	3.34

Seems to be motion in distance since 1905.

1999 Σ 1329

1915.254	250.7	18.67
.279	251.9	18.80
.295	250.3	18.65
1915.276	250.9	18.70

5002 H 808

1915.295	222.3	21.97
1916.403	221.1	21.85
1916.808	220.9	21.92
1916.169	221.4	21.91

Change in angle evident since 1895.

5003 Σ 1332

1915.227	24.7	5.95
.238	20.9	6.07
.243	23.1	6.24
1915.236	22.9	6.09

Change in angle since 1893.

5006 OΣ (App) 98

1915.200	169.5	110.84
.213	169.6	111.73
.221	169.8	110.86
1915.213	169.7	111.14

A = Yellowish; *B* = Blueish.

5020 Ho 364

1915.284	333.3	4.09
.386	334.6	4.19
.395	334.7	4.59
1915.355	334.2	4.32

Some change in distance since 1903.

5021 H 126

1915.284	132.5	93.92
.293	132.7	93.88
.295	132.5	93.96
1915.291	132.6	93.92

Hard to identify with *H* star 1820. P.A. too small, distance too great.

5025 S 595

1915.284	284.3	67.59
1916.332	284.1	67.47
1916.403	284.2	67.57
1916.006	284.2	67.54

Change due to P.M.?

5017 Σ 1344

1915.279	104.2	3.94
.386	106.5	3.92
.395	105.1	4.35
1915.353	105.2	4.07

Change in distance questionable, since 1867. Magnitude closer than catalog shows.

5093 Σ 1349

1915.279	165.3	19.17
1915.324	166.1	19.55
1915.389	165.6	19.57
1917.019	165.0	19.23
1915.759	165.5	19.45

5094 Σ 1355

1915.284	338.5	3.04
.386	338.1	3.19
.397	337.8	2.98
1915.356	338.1	3.07

Evident change in angle since 1893.

5116 Σ 1360

1915.284	243.5	14.55
.287	243.7	14.64
.295	243.7	14.28
1915.289	243.6	14.49

5120 Σ 1350

1915.243	67.0	10.80
.246	67.0	10.64
.279	67.2	10.87
1915.256	67.1	10.77

5119 OΣ (App) 102

1915.284	43.2	48.86
.287	43.5	49.07
.295	43.3	49.00
1915.289	43.3	48.98

5152 Σ 1371

1915.284	279.3	7.86
.287	279.4	7.86
.295	279.4	8.09
1915.289	279.4	7.93

5156 S 601

1915.284	91.4	51.83
.287	91.1	51.55
.389	90.7	51.91
1915.320	91.1	51.76

5188 Σ 1376

1915.284	313.6	5.61
.287	312.8	5.57
.295	312.6	5.38
1915.289	313.1	5.52

Possible change in distance since 1897 measure.

5200 Σ 1378

1915.249	00.1	5.07
.254	00.5	4.97
.279	359.9	5.34
1915.261	00.2	5.12

5212 OΣ 521

1915.254	291.9	11.62
.287	290.8	11.29
.395	290.7	11.40
1915.312	291.1	11.43

Change indicated since 1898 measure.

5217 H 1172

1915.284	272.0	13.87
.287	273.2	11.10
.386	272.1	11.34
1915.319	272.5	11.10

5227 OΣ 522

1915.257	125.1	14.13
.287	125.2	14.24
.389	122.2	14.29
1915.311	124.1	14.25

Some change likely since 1898 measure.

5233 Σ 1389

1915.284	308.7	2.53
.389	309.2	2.75
.397	308.2	2.57
1915.357	308.7	2.61

Change in both angle and distance since 1903.

5284 Hu 124

1915.238	9.2	13.18
1916.332	7.8	13.02
1916.403	9.1	13.52
1915.991	8.7	13.36

Blue-white.

5289 H 4277

1915.238	32.4	21.91
1916.403	32.6	21.85
1915.820	32.5	21.88

5302 Σ 1404

1915.238	294.6	6.64
.389	293.6	7.01
.397	295.9	6.61
1915.341	294.7	6.67

5315 Σ 1407

1915.389	52.9	5.28
.397	52.6	5.08
.413	53.1	5.13
1915.399	52.8	5.16

5322 Σ 1408

1915.394	13.1	3.65
.397	12.7	3.66
.411	13.3	3.52
1915.400	13.0	3.61

5336 Σ 1409

1915.397	188.9	13.71
.414	189.0	13.07
.416	189.0	13.22
1915.409	188.7	13.33

5356 Σ 1415

1915.397	166.7	17.07
1915.416	166.8	17.11
1916.403	166.4	16.95
1915.739	166.6	17.05

5360 H 1176

1916.477	318.1	8.99
.485	317.4	9.06
.490	317.4	9.33
1916.484	317.6	9.13

5376 H 3324

1915.397	198.7	18.84
1915.416	198.6	18.74
1916.402	199.2	18.78
1915.738	198.8	18.78

5398 Σ 1410

AB

1915.397	342.3	14.40
.416	341.2	14.42
1916.402	339.3	14.27
1915.738	340.9	14.36

AC — Mag. 10.7

	358.4	41.38
	358.8	41.10
	358.6	41.24

Σ gives 151°.7–42″ for *C*.

5403 Kr 36

1915.397	242.3	6.58
1915.416	243.0	5.62
1916.433	244.5	6.13
1915.882	243.2	6.11

Measure discordant. Pair faint.

5422 Σ 1428

1915.397	84.8	3.48
1915.416	84.5	3.40
1916.433	83.3	3.82
1915.882	84.2	3.56

5451 H 5480

1916.477	80.7	5.90
.603	80.3	6.06
.617	81.7	5.53
1916.566	80.9	5.83

5455 Σ 1437

1915.379	289.1	23.63
1915.416	289.8	23.63
1916.477	290.3	23.63
1915.757	289.7	23.63

6243 γ *Virginis* Σ 1670

1916.474	325.0	5.88
.485	325.2	5.98
.490	325.3	6.03
1916.483	325.2	5.96

6390 Σ 1719

1915.243	1.2	7.45
1915.455	1.6	8.08
1916.458	2.2	7.34
1915.719	1.7	7.62

6397 Σ 1721

1915.293	359.6	6.90
1916.476	359.1	6.36
1916.575	355.7	6.42
1916.115	358.1	6.56

6398 Σ 1722

1915.293	339.4	3.39
1916.455	337.8	3.29
1916.460	338.4	3.40
1915.069	338.5	3.36

6530 Σ 1757

1916.455	86.5	2.83
.460	85.9	2.82
.474	85.2	2.94
1916.463	85.9	2.86

Motion is in B's plot since 1903.

7487 ξ *Scorpii* Σ 1998

*AB

1916.497	172.2	1.06
.562	169.7	1.25
.567	172.9	1.15
1916.542	171.6	1.15

AC

	60.4	7.57
	56.9	7.85
	57.0	7.63
	58.1	7.68

*The quadrant is given as defined in Aitken's orbit. (Pub. L. O. XII p. 103.)

7878 μ *Draconis* Σ 2130

1916.630	131.5	2.71
.641	129.0	2.64
.680	128.5	2.32
1916.650	129.7	2.56

Both yellowish.

8302 95 *Herculis* Σ 2264

1916.562	259.5	6.20
.570	259.1	6.29
.586	260.1	6.34
1916.573	259.6	6.28

8310 70 *Ophiuchi* Σ 2272

1916.575	139.8	5.02
.586	141.8	4.97
.589	140.2	5.07
1916.583	140.6	5.03

8311 OΣ 534

1916.530	284.4	2.46
.567	278.7	2.90
.570	277.1	2.63
1916.556	280.1	2.66

8356 β 244

1916.550	260.1	2.39
.562	263.2	2.32
.570	261.3	2.19
1916.561	261.5	2.30

8357 Σ 2284

1916.564	191.5	3.63
1916.575	191.2	3.46
1916.586	191.6	3.66
1916.597	191.3	3.52
1916.603	190.4	3.84
1916.606	192.6	3.52
1916.588	191.4	3.60

8362 *A.G.* 216

1916.564	93.2	2.52
.570	88.9	2.35
.592	90.8	2.78
1916.575	91.0	2.55

8381 Σ 2283

1916.567	82.3	1.39
.570	86.2	.87
.586	83.4	1.00
1916.574	84.0	1.09

Change questionable since 1902 measure.

8385 Σ 2285

1916.567	333.2	3.65
.575	335.4	3.61
.592	334.8	3.73
1916.578	334.5	3.66

9930 Hu 80

1916.575	4.7	2.41
.591	4.8	2.07
.597	3.6	2.12
1916.588	4.4	2.20

9935 Σ 2640

1916.575	20.4	5.40
.586	20.8	5.48
.591	20.6	5.33
1916.584	20.6	5.41

9939 β 470

1916.575	214.3	2.70
.586	215.6	2.63
.589	217.6	2.28
1916.583	215.8	2.54

9943 A 382

1916.630	88.6	1.69
.633	90.3	1.56
.641	89.1	1.95
1916.635	89.3	1.74

9944 Σ 2642

1916.575	176.9	2.18
.586	176.6	2.31
.591	176.8	2.07
1916.584	176.8	2.15

Change in angle only since 1899.

9949 Σ 2634

1914.865	12.5	6.13
1916.567	12.1	5.49
1916.575	11.6	5.40
1916.002	13.1	5.67

9951 Ho 119

1916.575	199.6	3.27
.597	198.8	3.49
.602	201.3	3.29
1916.591	199.9	3.35

9982 Σ 2644

1916.575	210.4	2.91
.586	209.5	2.98
.591	209.3	3.23
1916.584	209.7	3.04

9986 H 907

1916.617	133.6	6.01
.622	133.9	6.50
.625	135.0	6.04
1916.621	134.3	6.18

9992 H 1491

1916.589	118.2	4.99
.591	118.5	4.85
.597	119.7	4.77
1916.592	118.8	4.87

10015 H 2941

1916.622	112.1	6.30
.625	111.2	6.30
.627	110.8	6.10
1916.625	111.4	6.23

10017 Σ 2653

1916.624	268.8	2.50
.627	266.2	2.70
.633	265.1	2.38
1916.628	266.7	2.53

10030 A 385

1916.625	259.2	2.95
.627	259.7	2.85
.630	261.5	2.73
1916.627	260.1	2.84

10045 Espin 27

1916.625	310.1	4.16
.630	310.5	4.08
.633	310.0	4.10
1916.629	310.2	4.11

10050 Ho 123

1916.625	226.4	3.37
.630	227.0	2.96
1916.627	226.7	3.17

10072 β 981

1916.641	216.9	0.90
.688	216.8	0.86
.707	219.5	0.84
1916.679	217.7	0.87

10097 Σ 2664

1914.863	310.6	25.10
.866	311.5	25.02
.869	310.1	25.02
1914.866	310.7	25.04

10732 61 *Cygni* Σ 2758

1916.332	130.7	23.54
.485	131.0	23.47
.567	131.3	23.66
1916.461	131.0	23.55

11131 29 *Aquarii* S 802

1915.827	244.3	3.88
.832	243.4	4.01
.857	242.4	4.15
1915.839	243.5	4.01

11183 Σ 2863

1915.712	279.0	7.51
.715	281.5	7.46
.723	282.7	7.17
1915.716	281.0	7.38

11186 Σ 2864

1915.712	222.7	7.47
.715	221.2	7.30
.720	222.4	7.32
1915.715	222.1	7.36

11190 Σ 2862

1915.712	100.1	2.90
.715	100.2	2.50
.737	99.6	2.94
1915.721	100.0	2.78

Probably no change since 1902.

11191 Espin 103

1915.723	208.1	1.69
1915.737	210.7	2.13
1916.591	210.1	1.56
1916.017	209.6	1.79

11512 β 170

1915.712	59.6	1.98
.715	57.7	1.69
.720	56.4	1.98
1915.715	57.9	1.88

11528 Σ 2870

1915.712	272.2	5.57
.737	269.5	5.65
.832	270.9	5.57
1915.760	270.8	5.60

11635 Kr 57

1916.630	228.2	1.52
.641	228.7	1.48
.680	225.9	1.61
1916.650	227.6	1.44

11636 β 378

AB

1916.630	90.3	3.38
.641	91.5	2.94
.680	90.6	3.14
1916.650	90.8	3.15
	35.6	6.28
	35.0	7.23
	27.5	6.90
	32.7	6.80

C very faint. Mag. 13.

11657 Σ 2895

1916.630	37.2	9.19
.641	37.4	8.74
.688	38.5	8.49
1916.653	37.7	8.81

11702 Σ 2902

1916.603	89.1	6.52
.611	89.4	7.02
.614	89.3	6.93
1916.609	89.3	6.82

11715 *ν Aquarii* Sh 345

1916.721	312.5	6.35
.745	310.6	5.94
.770	311.1	6.24
1916.745	311.4	6.18

Yellow.

11767 Σ 2913

1916.748	327.9	8.39
.770	329.6	8.12
.775	329.5	8.14
1916.764	329.0	8.22

White blue not red or yellow as given in catalog.

11997 Σ 2947

1916.641	64.4	3.83
.688	64.2	3.55
.699	66.1	4.06
1916.676	65.2	3.85

12707 Arg. 47

1916.721	291.0	10.43
.759	291.8	10.45
.770	291.2	10.14
1916.750	291.3	10.34

Yellow blue.

DM(66°)6 Hu 1003–12758
R.A. 3m 52s.5
Dec. 66° 37′.5
Mags. 10–11

1916.575	33.6	2.43
1916.589	32.7	2.04
1916.592	32.6	2.14
1916.585	32.9	2.20

This star is not No. 22 in *B. G. C.* No. 22 is (66°)8.

D.M. (50°) **18** Espin 749
R.A. 0h 11m 47s.9
Dec. 50° 38′.0
Mags. 10–11

1916.688	163.5	8.86
1916.710	163.5	9.49
1916.721	165.8	8.97
1916.706	163.9	9.11

DM (59°) **13** Fox
R.A. $0^h16^m33^s.1$
Dec. 59°38′.2
Mags. 9.2–10.6

1916.688	154.3	6.33
1916.719	153.7	6.34
1916.703	154.0	6.34

DM (37°) **606** Espin 231
R.A. $2^h35^m20^s.4$
Dec. 37°55′.4
Mags. 8–10

1916.808	82.2	4.26
1916.813	80.4	4.58
1916.849	82.1	4.09
1916.823	81.6	4.31

DM (37°) **3199** Olivier 14
R.A. $18^h35^m47^s.9$
Dec. 37°59′.9
Mags. 10–10

1916.833	165.8	1.85
.874	165.7	1.38
1916.854	165.7	1.62

DM (37°) **3335** *A.G.* 227 9037
R.A. $19^h1^m14^s.1$
Dec. 37°52′.9
Mags. 9–9

1917.066	10.4	5.83
1916.833	10.5	6.03
.874	11.0	6.24
1916.924	10.6	6.03

DM (38°) **3913** Aitken 1416
R.A. $20^h3^m13^s.7$
Dec. 38°16′.8
Mags. 8–9.7

1916.778	42.0	4.80
.797	41.2	4.99
.813	43.6	4.90
1916.796	42.3	4.90

DM (25°) **1130** *A.G.*
R.A. $20^h5^m22^s.8$
Dec. 25°10′.0
Mags. 8.5–9.8

1916.625	10.0	5.19
.630	10.9	5.76
.633	12.3	5.61
1916.629	11.1	5.52

DM (38°) **3968** Aitken 1421
R.A. $20^h10^m26^s.2$
Dec. 38°5′.3
Mags. 9.5–10

1916.762	198.9	2.82
.795	195.2	2.29
.874	195.5	2.73
1916.810	196.5	2.61

DM (37°) **1208** Espin 254
R.A. $21^h6^m1^s.4$
Dec. 38°3′.5
Mags. 9.5–9.5

1916.764	330.1	1.82
.797	327.0	2.17
.849	331.4	2.09
1916.803	329.5	2.03

DM (61°) **2268** Fox
R.A. $22^h10^m21^s.5$
Dec. 61°19′.1
Mags. 8.5–10

1916.680	176.9	5.53
.688	177.6	5.49
.701	175.8	5.49
1916.673	176.8	5.50

(1)
R.A. $0^h3^m47^s.3$
Dec. 37°43′.6
Mags. 9.5–9.5

1916.839	112.6	10.06
1916.871	110.5	9.99
1916.975	113.0	9.83
1916.895	112.0	9.96

(2)
R.A. $0^h25^m17^s.5$
Dec. 59°59′.1
Mags. 10–10

1916.871	198.4	3.26
1916.877	196.5	2.79
1917.003	198.1	3.27
1916.917	197.5	3.08

(3)
DM (62°) **316**
R.A. $1^h44^m59^s$
Dec. 62°18′.3
Mags. 9.2–9.6

1916.885	136.5	3.35
1916.891	136.8	3.66
1916.995	134.8	3.78
1916.924	136.0	3.60

(4)
DM (34°) **115**
R.A. $2^h14^m32^s.2$
Dec. 34°8′.5
Mags. 9–10–10 Triple

A and *BC*

1916.863	292.8	13.25
1916.756	292.5	13.31
1916.822	292.6	13.69
1916.814	292.6	13.42

B and *C*

1916.863	150.5	1.25
1916.882	148.8	1.38
1917.003	152.7	1.50
1916.916	150.3	1.38

(5)
DM (34°) **439**
R.A. $2^h20^m1^s.6$
Dec. 34°17′.7
Mags. 9.5–10.2

1916.874	320.4	4.44
1916.877	324.6	4.65
1916.882	321.6	5.10
1916.877	322.2	4.73

(6)
DM (31°) **152**
R.A. $2^h21^m13^s.3$
Dec. 31°19′.5
Mags. 9.6–9.9

1916.756	231.7	6.06
1916.822	231.3	5.82
1916.877	231.0	5.91
1916.818	234.3	5.93

(7)
DM (63°) **371**
R.A. $2^h48^m5^s.6$
Dec. 63°30′.0
Mags. 8.7–9.2

1916.885	208.7	9.63
1916.891	207.8	9.39
1916.995	209.2	8.79
1916.924	208.6	9.27

(8)
DM (37°) **902**
R.A. $4^h7^m22^s.9$
Dec. 37°58′.1
Mags. 9.7–10.2

1916.770	97.5	3.19
1916.899	99.3	2.67
1916.995	99.3	2.57
1916.888	98.7	2.79

(9)
DM (38°) **1123**
R.A. $5^h12^m15^s.7$
Dec. 38°20′.2
Mags. 9–9

1916.847	170.5	2.64
1916.882	166.7	2.48
1916.995	170.9	2.67
1916.908	169.4	2.60

(10)
R.A. $5^h19^m58^s.7$
Dec. 37°53′.4
Mags. 10.2–10.2

1916.847	152.1	4.28
1916.882	153.0	4.38
1916.899	151.3	4.04
1917.049	152.0	4.26
1916.919	152.8	4.24

(11)

R.A. $7^h38^m58^s.2$

Dec. 38°49′.8

Mags. 9.1–10.4

1916.770	6.5	1.44
1916.882	6.5	1.30
1916.899	6.9	3.98
1916.850	6.6	4.24

(12)

R.A. $9^h3^m13^s.12$

Dec. −15° 6′.1

Mags. 10–11

1915.208	135.6	15.36
1915.227	134.5	15.65
1915.238	1 6.0	15.40
1915.221	135.3	15.47

(13)

R.A. $9^h1^m49^s.1$

Dec. −15°18′.8

Mags. 9.1–12

1915.238	248.7	4.11
1915.279	250.2	3.67
1915.284	244.4	4.44
1915.268	247.7	4.05

(14)

DM (−8°) **2207**

R.A. $9^h14^m17^s.9$

Dec. 8°10′.4

Mags. 8–9.2

1915.279	236.5	29.01
1915.385	236.2	28.67
1915.394	236.0	29.23
1915.383	236.2	28.97

(15)

R.A. $9^h10^m2^s.5$

Dec. −20°1′.3

Mags. 10.5–11

1915.295	258.8	13.31
1915.808	259.0	12.78
1917.049	259.2	12.36
1916.381	259.0	12.82

(16)

R.A. $17^h2^m37^s.7$

Dec. 38°48′.8

Mags. 8.8–13.5

1916.874	104.55	12.73
1916.882	107.35	10.72
1917.066	103.6	12.42
1916.941	105.1	11.96

Companion star seen with difficulty in $11\frac{1}{2}''$.

(17)

R.A. $17^h12^m37^s.0$

Dec. 38°0′.2

Mags. 9.5–10.0

1916.800	17.8	8.68
1916.808	18.8	9.20
1916.813	18.4	8.40
1916.807	18.3	8.93

(18)

R.A. $21^h15^m24^s.1$

Dec. 37°48′.6

Mags. 10–10.2

1916.833	124.7	6.37
1916.871	124.5	6.95
1916.882	126.1	6.63
1916.862	125.1	6.65

(19)

DM (59°) **2823**

R.A. $23^h57^m43^s$

Dec. 59°59′.7

Mags. 9–10

1916.772	7.8	7.96
1916.797	5.7	7.54
1916.813	8.3	8.05
1916.794	7.3	7.85

Washburn College Observatory, Topeka, Kansas, 1917 February 2.

NEW ASTRONOMICAL WORK.

Total Eclipse of the Sun, June 8, 1918, Published by the Nautical Almanac Office, Washington, D.C., 1917.

Annuario, Published by Observatorio Nacional do Rio de Janeiro, 1917.

CONTENTS.

Editor, Benjamin Boss, Albany, N.Y., Associate Editors: E. E. Barnard, Ernest W. Brown, F. R. Moulton and R. S. Woodward.
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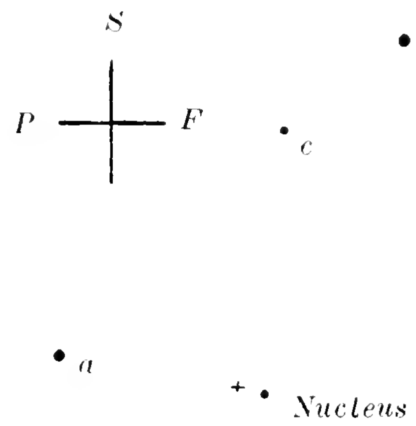
THE PROPER-MOTION OF THE GREAT NEBULA OF *ANDROMEDA* (M 31).

By E. E. BARNARD.

The large radial velocity obtained spectroscopically by SLIPHER and others for the Great Nebula of *Andromeda* (M 31) would suggest that this object might show a visual displacement across the sky in a reasonable period of time. In the hope of ultimately detecting motion in this nebula I began a series of measures of the nucleus in 1898 with respect to three small stars near it. These observations were repeated in 1909 without showing any definite motion. In 1915-16 I again repeated the measures. The various mean results are given here. The individual measures are reserved for later publication with observations of other nebulæ.



NUCLEUS AND *a*

	°	°	"	"	
1898.709	261.28	±0.018	124.67	±0.044	(10 *n.*)
1909.883	261.31	±0.035	124.67	±0.049	(4 *n.*)
1916.339	261.29	±0.022	124.63	±0.032	(15 *n.*)

NUCLEUS AND *b*

1898.723	160.51	±0.028	228.14	±0.028	(10 *n.*)
1909.888	160.57	±0.059	228.27	±0.085	(3 *n.*)
1916.395	160.73	±0.017	228.44	±0.016	(14 *n.*)

NUCLEUS AND *c*

1898.512	177.22	. . .	157.72	.	(1 *n.*)
1909.888	177.16	±0.029	157.73	±0.071	(3 *n.*)
1916.121	177.22	±0.024	157.70	±0.024	(9 *n.*)

The means of the probable errors for a single night's measures for these stars are: $\pm 0^\circ.074$ and $\pm 0''.107$.

These results do not indicate any visual displacement of the nebula in the interval of some eighteen years. The individual measures show that the parallax must be beyond the reach of ordinary micrometer work. The differences in the measures with *b* can be attributed to a small motion in that star, for the measures with *c* negative any motion of the nebula in that direction. Unfortunately there was only one measure of *c* obtained in 1898. The estimated magnitudes of these stars are: a = 12.3 (5), b = 12.3 (4), c = 13.8 (5).

The nucleus of the nebula is about 2″ to 3″ in diameter, but it is so strongly condensed that under good conditions it can be bisected with almost the same accuracy as the comparison stars. From its nature, the brightness of the nucleus varies greatly with the size of the telescope and the magnifying power used. With the 40-inch telescope it is of about 13 to 14 magnitude, or about the same brightness as star *c*.

In these remarks it is assumed that the comparison stars are not connected with the nebula, which is a reasonable assumption.

Throughout these measures, and indeed for many years before, I have always, when observing M 31, carefully examined the place of the nova of 1885, which is 16″ from the nucleus toward the star *a*. Nothing, however, has been seen in its position.

There are some early measures of the nucleus and the star a. In *A. N.* Bd. 112, p. 286 the following observations are quoted and seem to have been made prior to 1867, but I have not the exact dates:

STRUVE	$\Delta\alpha$	$+10^s.81$	$\Delta\delta$	$+18''.7$
VOGEL		$+10\ .79$		$+19\ .1$
Mean =	$\Delta\alpha$	$+10^s.81$	$\Delta\delta$	$+19''.0$

These give: P. A. 261°.28. Dist. 124″.75.

RAYET in 1898 obtained (*Comptes Rendus* for Sept. 26, 1898):

$$\Delta\alpha\ +10^s.91\quad \Delta\delta\ +18''.48$$

which give: P. A. 261°.28 Dist. 125″.12.

There is a larger value attributed to D'ARREST of $\Delta\alpha$ $+11^s.26$, $\Delta\delta$ $+19''.5$. I do not know the date of this observation.

In *A. N.* Bd. 112, p. 246, E. LAMP gives two measures of the nucleus and the star a made by LAMONT at Munich in 1836. After adding 180° to the angles the measures are:

1836	Oct. 13	261°.01	124″.7
	Oct. 14	261 .19	126 .4
Mean =		261°.11	125″.5

Though the distances are discordant, they show that no considerable relative motion has occurred in the past eighty years.

Yerkes Observatory, Williams Bay, Wisconsin.
1917 May 22.

TWO NEW VARIABLE STARS.

By E. E. BARNARD.

On my photographs with the BRUCE telescope of this observatory I have found the following two variable stars. Professor E. C. PICKERING has kindly verified these and informed me that they are new so far as shown by the records of the Harvard College Observatory.

VARIABLE *A*

$$1900.0\quad \alpha\ 18^h\ 25^m\ 14^s.3\quad \delta + 13°\ 21'.6$$

It was measured in the sky on 1917 June 2 with respect to what is supposed to be BD + 13°3659.

1917.420 P.A. 314°.6 Dist. 101″.9

From this the above position was obtained. On this date the variable was estimated to be 12.8 magnitude. The star varies through several magnitudes with a maximum of perhaps 12th magnitude.

VARIABLE *B*

$$1900.0\quad \alpha\ 18^h\ 25^m\ 14^s.6\quad \delta + 9°\ 46'.7$$

This star is close north preceding the 8.4 magnitude star BD + 9°3740. On June 2 the variable was measured with respect to the BD star.

1917.420 P.A. 307°.5 Dist. 55″.6

This gives the position:

$$1917.0\quad \alpha\ 18^h\ 25^m\ 59^s.84\quad \delta + 9°\ 47'\ 16''.8$$

It was estimated to be 14.7 magnitude. It perhaps gets as bright as the 12th magnitude.

Yerkes Observatory, Williams Bay, Wisconsin.
1917 June 4.

THE *MOON'S* PLACE AT THE SOLAR ECLIPSE OF 1918 JUNE 8.

By ERNEST W. BROWN.

Two trial months of the *Moon's* place have been computed from the new tables. The first of these, in 1912, was undertaken in order to prepare and test forms for the computation of the annual ephemeris. The second, from May 25, to June 21, 1918, was partly a test of the tables and partly a test of the forms after the latter had been printed. This later month contains the solar eclipse which is visible next year on this continent. It was computed by Dr. H. B. Hedrick and the work checked by the writer. It is of interest to note the differences from the published ephemerides and to find what corrections must be made so that the predicted data concerning the eclipse may be as near the actual phenomena as possible.

The following table gives the differences from the values of the longitude, latitude and parallax as published in the *American Ephemeris* for 1918. The corrections here given are to be added to the published positions in order to obtain the positions computed from the new tables.

Table of Corrections to the *Moon* Ephemeris for 1918. G. M. T.

Date		Long.	Lat.	Par.	Date		Long.	Lat.	Par.
		″	″	″			″	″	″
May	25.5	+5.3	+0.3	+0.45	June	9.5	+6.1	+1.0	+.50
	26.0	5.1	.3	.44		10.0	6.6	1.0	.51
	26.5	4.9	.1	.44		10.5	6.8	1.0	.51
	27.0	4.5	.2	.44		11.0	6.9	1.0	.49
	27.5	4.0	.1	.44		11.5	7.0	1.1	.49
	28.0	3.7	.2	.45		12.0	7.2	1.2	.48
	28.5	3.5	.2	.46		12.5	7.3	1.1	.47
	29.0	3.2	.3	.45		13.0	7.5	1.1	.46
	29.5	3.0	.3	.45		13.5	7.7	1.2	.45
	30.0	2.8	.4	.47		14.0	7.7	1.3	.45
	30.5	2.6	.4	.47		14.5	7.8	1.2	.45
	31.0	2.5	.5	.47		15.0	7.8	1.3	.44
	31.5	2.4	.5	.46		15.5	7.9	1.2	.44
June	1.0	2.4	.6	.46		16.0	7.9	1.2	.45
	1.5	2.4	.7	.47		16.5	7.7	1.2	.43
	2.0	2.4	.8	.47		17.0	7.6	1.1	.42
	2.5	2.5	.9	.47		17.5	7.6	1.0	.42
	3.0	2.5	1.0	.48		18.0	7.6	1.0	.42
	3.5	2.6	1.1	.49		18.5	7.6	1.0	.42
	4.0	2.8	1.2	.50		19.0	7.5	.8	.43
	4.5	2.9	1.3	.50		19.5	7.4	.7	.43
	5.0	3.2	1.2	.51		20.0	7.2	.6	.43
	5.5	3.4	1.2	.53		20.5	7.0	.5	.43
	6.0	3.7	1.2	.52		21.0	6.8	.3	.44
	6.5	4.1	1.2	.52		21.5	6.6	.2	.45
	7.0	4.4	1.2	.53		22.0	6.4	.2	.45
	7.5	4.8	1.1	.54		22.5	6.0	.1	.45
	8.0	5.2	1.2	.53		23.0	5.8	+ .1	.45
	8.5	5.6	1.1	.52		23.5	+5.6	.0	+.46
	9.0	+6.1	+1.1	+ .51					

In Table II of a paper on "The Longitude of the *Moon* from 1750 to 1910,"* I obtained the average annual corrections which should be made to results obtained from Hansen's tables with Newcomb's corrections, in order to compare the observations with my theory. The average correction for the year 1918 has been found in continuation of the column "Cor." of that table, and is $+5''.5$. The mean correction of the longitude for the month given above is $+5''.3$, a closeness of agreement which is accidental in view of the presence of corrected terms which have periods of the order of a year. It is nevertheless satisfactory.

The chief interest, at the present moment, is the solar eclipse which is central on June 8.42, G. M. T. I shall give the theoretical corrections to the position of the *Moon* at this time and join thereto an estimate for empirical and other changes which are required to make the predicted and observed places agree.

Longitude. From the above table, the theoretical addition is $5''.6$. This includes the great empirical term which has been added to the Almanac position since 1883 but takes no account of the minor fluctuations. Since attempts to represent these by a formula which will serve for prediction have had poor success, the best we can do is to make an attempt to predict by means of the values during the past few years. In the following table are given, the annual mean corrections found at Greenwich to Hansen's longitude with Newcomb's corrections, the portion of these due to the difference between Hansen's and my theory, and finally the minor fluctuations as compared with my theory. It is assumed that the latter are of long period so that these corrections may be taken to be applicable to the *mean* longitude.

*M. N. R. A. S., Vol. LXXIII, p. 705.

Table of Corrections to the Mean Longitude

Year	Theor. Cor.	Obs. Cor.	Minor Fluc.	Year	Theor. Cor.	Obs. Cor.	Minor Fluc.
1906.5	+3.13	+5.91	+2.78	1914.5	+5.87	+12.86	+6.99
1907.5	2.80	5.96	3.16	1915.5	5.72	+12.5	+6.8
1908.5	3.08	5.97	2.89	1916.5	5.90		
1909.5	3.46	6.41	2.95	1917.5	5.80		
1910.5	3.70	7.85	4.15	1918.5	5.45		
1911.5	4.07	8.31	4.27	1919.5	5.15		
1912.5	4.88	9.79	4.91	1920.5	+5.10		
1913.5	+5.77	+11.93	+6.16				

I shall assume a correction of 7″.0 for 1918.5 (which can be changed later if necessary when the observations for 1916, 1917 are available) and give the changes due to this assumption.

The change in the true longitude is sufficiently accounted for by adding to it,

$$7'' \{1 + .14 \cos l + .02 \cos 2D + .02 \cos (2D - l)\}$$

where l is the mean anomaly, and $2D$ the argument of the variation. At the given date, $l = 68°$, $2D = 351°$. Hence the correction is $+7''.5$. Adding to this the theoretical correction, $+5''.6$, we obtain the total correction $+13''.1$.

In the Introduction to the *American Ephemeris* for 1918, it is stated that 6″.2 has been added to the ephemeris longitude in computing the longitude for the eclipse. A further addition of $+6''.9$ is therefore indicated.

Latitude. The theoretical correction is 1″.4. The addition of 7″.5 to the true longitude adds to the latitude $-7''.5 \times .090 = -0''.67$. No correction has been applied in the new tables for the difference between the center of mass and center of figure of the *Moon*, in the direction of its N — S axis. My analysis and that of Newcomb indicated that a correction of about $-0''.5$ should be applied to account for this.* The sum of the three corrections is nearly zero. Hence, no correction is needed to Hansen's latitude at the time of the eclipse. The correction applied in the *American Ephemeris* is $+1''.2$. Hence a further correction of $-1''.2$ is indicated.

Parallax. The theoretical correction is $+0''.52$. The change due to the empirical addition to the longitude is $+0''.005$ and may be neglected. The correction applied in the *American Ephemeris* is $+0''.47$. Hence the correction to the Hansenian parallax is $+0''.52$ and that to the eclipse value in the *American Ephemeris* is $+0''.05$.

* Hansen applied $-1''.00$

Yale University, 1917 May 5.

OBSERVATIONS OF ASTEROIDS,

MADE WITH THE PHOTOGRAPHIC TELESCOPE OF THE U. S. NAVAL OBSERVATORY,

By GEORGE H. PETERS

[Communicated by Rear Admiral T. B. Howard, U. S. N., Superintendent.]

Name	Mag.	Date 1916	G. M. T.	α 1916.0	δ 1916.0
			h m	h m s	° ′ ″
306 *Unitas*	11.5	Feb. 7	14 41.5	8 23 39.5	+16 49 19
440 *Theodora*	12.4	Feb. 7	15 49.8	8 43 23.6	+16 59 44
479 *Caprera*	12.6	Feb. 7	15 49.8	8 52 27.7	+16 46 17
322 *Phaeo*	13.0	Feb. 7	16 58.3	9 21 41.2	+ 4 17 52
306 *Unitas*	11.5	Feb. 9	14 58.3	8 21 16.2	+17 01 11
376 *Geometria*	12.1	Mar. 4	13 47.8	8 44 19.2	+14 54 03
96 *Aegle*	10.7	Mar. 4	14 30.9	9 13 36.4	+14 10 55
490 *Veritas*	12.6	Mar. 4	15 35.8	8 50 42.5	+ 9 42 44

OBSERVATIONS OF ASTEROIDS (*Continued*)

Name	Mag.	Date 1916	G. M. T.	α 1916.0	δ 1916.0
			h m	h m s	
(51) *Nemausa* . .	9.5	Mar. 4	15 35.8	8 52 38.3	+ 8 54 27
(122) *Gerda*	11.2	Mar. 8	14 04.3	9 53 09.3	+11 17 18
(797) *Montana*	12.5	Mar. 9	13 06.3	9 13 08.6	+ 8 17 52
(797) *Montana*	12.5	Mar. 9	14 13.3	9 13 09.3	+ 8 17 59
(236) *Honoria*	12.2	Mar. 9	15 49.2	9 39 49.0	+ 7 34 29
(352) *Gisela* .	12.5	Mar. 10	15 27.4	10 11 53.6	+ 5 05 06
(236) *Honoria*	12.2	Mar. 11	15 27.0	9 38 37.0	+ 7 15 42
(551) *Ortrud* . .	12.8	Mar. 11	16 21.0	10 36 35.9	+ 9 01 56
(339) *Dorothea* .	13.2	Mar. 11	17 30.0	11 35 53.4	+ 2 13 02
(122) *Gerda*	11.2	Mar. 23	13 21.5	9 45 25.8	+12 36 25
(352) *Gisela* .	12.5	Mar. 23	14 27.0	10 02 28.6	+ 6 17 28
(126) *Velleda* .	12.4	Mar. 23	15 33.0	10 53 11.9	+ 9 52 46
(250) *Bettina* . .	11.3	Mar. 23	16 39.5	12 03 16.2	+ 8 46 47
(250) *Bettina*	11.3	Mar. 30	15 57.0	11 57 51.3	+ 8 55 14
(363) *Padua* .	12.0	Mar. 30	15 57.0	12 07 36.6	+ 7 51 34
(551) *Ortrud* .	12.8	Mar. 31	14 20.5	10 24 42.0	+10 11 03
(59) *Elpis* . .	11.4	Mar. 31	14 20.5	10 27 01.9	+ 8 51 56
(39) *Lætitia* . .	10.0	Mar. 31	15 20.0	11 25 54.5	+ 8 54 04
(97) *Klotho* . .	10.8	Mar. 31	16 10.5	11 38 46.9	+ 8 32 43
(446) *Aeternitas* .	11.7	Mar. 31	17 19.5	13 08 13.0	+ 1 10 38
(121) *Hermione*	11.9	Mar. 31	17 19.5	13 15 48.1	+ 1 29 14
(250) *Bettina* .	11.3	Apr. 9	15 17.0	11 50 11.7	+ 8 57 30
(363) *Padua* . .	12.0	Apr. 9	15 17.0	11 59 48.7	+ 8 23 22
(536) *Merapi* .	12.2	Apr. 9	16 07.5	12 33 26.5	+19 48 33
(39) *Lætitia* .	10.0	Apr. 10	13 29.2	11 20 12.0	+ 9 47 41
(536) *Merapi* .	12.2	Apr. 10	14 49.2	12 32 46.0	+19 48 49
(41) *Daphne* .	8.8	Apr. 10	16 04.0	12 45 41.7	+ 4 23 37
(601) *Nerthus* .	13.0	Apr. 10	16 04.0	12 32 12.4	+ 5 01 17
(446) *Aeternitas*	11.7	Apr. 10	17 09.5	12 59 16.3	+ 1 37 21
Asteroid .	12.5	Apr. 10	17 09.5	13 08 58.4	+ 0 16 10
(121) *Hermione*	11.9	Apr. 10	17 09.5	13 09 04.1	+ 2 04 57
(250) *Bettina* .	11.3	Apr. 12	15 24.0	11 48 09.9	+ 8 55 42
(363) *Padua* . .	12.0	Apr. 12	15 24.0	11 57 42.1	+ 8 30 09
(121) *Hermione*	11.9	Apr. 29	14 44.3	12 57 05.5	+ 2 53 12
(276) *Adelheid* .	12.0	May 31	16 34.3	16 40 30.6	− 2 20 31
(276) *Adelheid*	12.0	June 3	15 35.0	16 38 15.0	− 2 04 40
(234) *Barbara* .	10.7	June 22	16 36.5	18 15 39.8	+ 0 33 49
(234) *Barbara*	10.7	June 25	16 02.9	18 12 53.2	+ 0 18 10
(148) *Gallia* .	11.3	June 26	17 00.0	18 49 31.0	+ 4 32 54

Prof. Comstock has kindly looked over this list, and states that the object of 1916, April 10, designated "asteroid," is considered new. It was found near the southern edge of both plates several days after exposure. It trailed slightly in declination and is not quite as sharply defined as an asteroid usually is, though not diffuse. Clouds interrupted exposures to find this object again on April 25 and 28. It was not found on the plates for (121) of April 29, but the night was hazy, causing (121) to be rather faint and the motion of the unidentified object in declination, which cannot be determined, may have been south. Should plates of this region near this date be available at other observatories an examination of them is desirable.

OBSERVATIONS OF COMET *1917 b* (SCHAUMASSE)

MADE WITH THE 26-INCH EQUATORIAL OF THE U. S. NAVAL OBSERVATORY,

By H. E. BURTON.

[Communicated by Rear Admiral T. B. HOWARD, U. S. Navy, Superintendent.]

Date Wash. M. T.	✱	Comp.	Δα	Δδ	App. α	App. δ	log pΔ α	log pΔ δ	Red. to App. Pl. α	Red. to App. Pl. δ
1917 h m s			m s	′ ″	h m s	° ′ ″			s	″
May 9 15 53 59	1	14 , 3	+3 12.96	+0 44.2	23 30 58.50	+23 11 41.7	9.660n	0.600	+1.20	+1.2
May 18 15 29 57	2	20 , 4	+5 43.97	+3 19.1	1 4 51.21	+47 15 7.6	9.828n	0.543	+0.84	+0.4

Mean Places of Comparison Stars for 1917.0

✱	α	δ	Authority	
1	23h 27m 44s.34	+23° 10′ 56″.3	*A. G. Berlin B*	9010
2	0 59 6 .40	+47 11 48 .1	*A. G. Bonn*	866

NOTES

1917. May 9. Comet faint on account of dawn. Seeing poor.

1917. May 18. Seeing poor.

OBSERVATIONS OF COMETS,

MADE WITH THE 26-INCH EQUATORIAL OF THE U. S. NAVAL OBSERVATORY,

By H. E. BURTON.

[Communicated by Rear Admiral T. B. HOWARD, U. S. Navy, Superintendent.]

Date Wash. M. T.	✱	Comp.	Δα	Δδ	App. α	App. δ	log pΔ α	log pΔ δ	Red. to App. Pl. α	Red. to App. Pl. δ
					Comet 1916 *b* (WOLF)					
1917 h m s			m s	′ ″	h m s	° ′ ″			s	″
Jan. 25 17 33 27	1	25 , 5	+2 14.83	+ 6 39.9	16 52 32.74	− 5 28 3.2	9.509n	0.774	+0.50	− 8.6
Feb. 24 17 5 3	2	30 , 6	+1 40.19	+ 3 27.9	17 57 3.49	− 2 32 17.0	9.453n	0.758	+1.03	−10.1
Mar. 6 17 26 56	3	10 , 2	−1 36.60	− 5 29.0	18 19 51.13	− 0 57 20.7	9.341n	0.748	+1.18	−10.0
Apr. 3 15 3 13	4	30 , 6	+3 38.50	+ 2 21.9	19 25 56.11	+ 5 4 7.0	9.567n	0.709	+1.59	− 8.4
Apr. 9 15 29 33	5	30 , 6	−3 33.79	− 5 31.0	19 40 33.01	+ 6 37 30.5	9.510n	0.690	+1.60	− 7.3
May 1 14 8 0	6	25 , 5	−3 45.87	+ 0 47.1	20 34 14.31	+12 41 22.0	9.594n	0.657	+1.90	− 4.0
					Comet 1917 *a* (MELLISH)					
Mar. 22 8 29 52	7	3 , 1	+0 10.46	+12 59.1	2 8 42.25	+15 6 34.1	9.674	0.738	+0.63	+ 5.6
Mar. 24 7 10 30	8	25 , 5	−3 39.58	− 0 37.0	2 7 23.35	+15 48 2.0	9.675	0.710	+0.62	+ 5.6
Mar. 30 7 31 17	9	20 , 4	−4 1.57	+ 7 45.7	1 59 11.27	+17 45 54.6	9.681	0.722	+0.58	+ 5.7
					Comet 1917 *b* (SCHAUMASSE)					
M[illegible] 1 15 53 37	10	25 , 5	[illegible]1 13.11	− 1 16.0	23 11 58.53	+14 13 14.8	9.646n	0.677	+1.43	+ 2.2

Mean Places of Comparison Stars for 1917.0

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	16 50 17.11	− 5 31 34.5	*A. G. Straszburg* 5782	6	20 37 58.28	+12 41 8.9	*A. G. Leipzig I* 8088
2	17 55 22.27	− 2 36 4.8	*A. G. Straszburg* 6029	7	2 8 31.16	+14 53 29.1	*A. G. Leipzig I* 654
3	18 21 26.55	− 0 51 41.7	*A. G. Nicolajew* 4577	8	2 11 2.31	+15 48 33.4	*A. G. Berlin A* 630
4	19 22 16.02	+ 5 1 53.5	*A. G. Albany* 6684	9	2 3 12.26	+17 38 3.2	*A. G. Berlin A* 602
5	19 44 5.20	+ 6 43 8.8	*A. G. Leipzig II* 9560	10	23 13 10.54	+11 14 28.6	*A. G. Leipzig I* 9268

NOTES

1916 *b*

1917 Jan. 25. Very faint nucleus. Seeing poor.

Feb. 24. Comet faint. Haze. Seeing fair.

Mar. 6. Comet faint. Thin clouds. Seeing fair.

Apr. 3. Comet faint. Not visible in 5-inch finder. Seeing fair.

Apr. 9. Comet faint. Moonlight. Seeing fair.

May 1. Seeing poor.

1917 *a*

1917 Mar. 22. Haze. Seeing poor. This observation was made with the 12-inch equatorial.

24. Comet visible in 2-inch finder. Bright nucleus. Has tail 5′ or more in length. Estimated position angle of tail with respect to nucleus is 70°. Seeing fair.

30. Seeing poor.

1917 *b*

May 1. Visible in 5-inch finder. Nebulous. Nucleus not well defined. Seeing poor.

NEW VARIABLE STAR IN *CYGNUS*,

By CAROLINE E. FURNESS.

During the past two months, Dr. J. VAN DER BILT, of the Observatory of Utrecht, while detained from sailing for Holland, has been a guest at this Observatory. He has been using the stereo-comparator for the purpose of determining the proper-motions of stars in the KAPTEYN areas on plates taken by DONNER at Helsingfors. While making a rapid survey of the plates to see if there were any large proper-motions he discovered a variable star concerning which he has left the following note:

"DONNER's Region 894 shows a star on the plate taken 18 October 1908 which is missing on that taken 14 October 1896.

"The position referred to *BD* +41° 4094 = *A. G.* Bonn 15350, is −47ˢ.5 −2′ 51″, which gives

1855.0	21ʰ 13ᵐ 25ˢ	+41° 16′ 55″
1900.0	21 15 9	+41 28 14

"In the following table the distances of the comparison stars and the variable from *BD* +41° 4094 are given and their magnitudes on the International scale.

Star	Mag	Δα	Δδ
a	11.6	−19ˢ	−1′ 0″
b		−24	+0 34
c		−24	+0 53
d		−31	+0 31
e	12.9	−40	−3 20
v		−47	−2 51
f		−49	−1 9
g	12.7	−56	−2 23

"On the plate taken 18 October 1908 the variable has the brightness of 12.8 mag."

When the discovery was reported to Prof. PICKERING, the variation was confirmed by Miss LEAVITT by the examination of sixteen Harvard plates, and the magnitudes of the comparison stars given in the table were communicated.

The results of the proper-motion investigation are not yet ready for publication.

Vassar College Observatory, May 4, 1917.

THE COMPANION OF *SIRIUS*.

By E. E. BARNARD.

The following observations of the companion of *Sirius* are a continuation of those printed in *A. J.* *30*, 21. On 1916 November 2 with seeing about 4 on a scale of 5, using an occulter and hexagonal diaphragm, the companion was noted as "Very bright. It is perhaps as bright as 7th magnitude; certainly as bright as the 8th magnitude."

Date		P. A.	Dist.	H. A.	Remarks
				h m	
1916.776	Oct. 10	73.21	10.83	E 1 40	Companion easy.
.790	15	74.06	10.86	E 1 20	Faint in clouds.
.806	21	73.90		E 0 30	Lost in bad seeing.
.806	21	70.58	10.66	W 0 50	Very difficult. Seeing very bad.
.820	26	73.61	10.15	0 00	Very difficult. Seeing very bad.
.826	28	75.47	10.90	E 0 50	Seeing bad in angle measures. Well seen for distances.
.834	31	73.89	10.88	E 1 20	Faint and difficult. Seeing very poor.
.839	Nov. 2	73.03	10.84	E 0 20	Seeing = 4.
.850	6	76.17	10.70	E 0 50	Seeing very bad.
.872	14	73.30		E 0 50	Lost in clouds.
.941	Dec. 9	74.78	10.89	W 0 20	Seeing very bad.
.960	16	74.02	10.71	E 0 45	Blurring. Seeing = 2.
.990	27	73.63	10.77	E 0 30	Faint. Seeing very bad.
.998	30	73.52	10.67	E 0 10	Seeing very bad.
1917.064	Jan. 23	75.10	10.72	W 2 30	Difficult from poor seeing.
.113	Feb. 10	73.64	10.87	W 1 20	Seeing 1-2. Very heavy wind.
.121	13	74.09	10.81	E 0 10	Well seen. Seeing 2-3.
.140	20	72.97	10.89	W 0 10	Seeing = 2.
.160	27	73.40	10.92	W 0 20	Seeing = 3, but variable.
.217	Mar. 20	73.05	11.05	W 0 20	
1916.946		73.77	10.80		

Yerkes Observatory, Williams Bay, Wisconsin, 1917 May 23.

CONTENTS.

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ON THE ERRORS IN THE SUM OF A NUMBER OF TABULAR QUANTITIES,

By FRANK SCHLESINGER.

Most printed tables have been computed with such care that it can be assumed that the maximum errors are $+0.5$ and -0.5 in units of the last tabular place. We may also assume that any error between these limits is as likely to occur as any other. Under these circumstances it is required to compute the errors that we should expect to find in the sum (or difference) of any number of tabular quantities. This problem has been proposed for solution by Professor E. W. BROWN in a recent number of the *Annals of Mathematics;* he points out its importance in connection with the use of his new lunar tables.

We begin with the simplest case, namely that in which the quantities are taken directly from the tables without interpolation. Let us denote by e an error in the sum of n items and by $f(e)$ the frequency with which that error occurs; by E an error in the sum of $n+1$ items and by $F(E)$ its frequency. The number of errors in the sum n items that fall in the immediate vicinity of e_1 is proportional to $f(e_1)$; if we combine each of these with an additional item they become dispersed throughout the interval $e_1 - 0.5$ to $e_1 + 0.5$; and they are uniformly dispersed, since within these limits any error in the additional item is as likely to occur as any other. Conversely, the total number of errors that occur in the immediate vicinity of E_1 in the sum of $n+1$ items is made up of contributions from all the errors between $E - 0.5$ and $E + 0.5$ in the sum of n items, and each of the latter contributes in proportion to $f(e)$. We have therefore

$$(1) \qquad F(E) = \int_{E-\frac{1}{2}}^{E+\frac{1}{2}} f(e)\, de$$

This theorem enables us to derive the law of distribution of errors in the sum of $n+1$ items if we have the law for n items. Starting therefore with $n = 1$ the repeated application of the theorem will furnish a complete solution of the problem.

For $n = 1$ the frequency is equal to a constant. This constant must be unity since the condition

$$\int_{-\frac{1}{2}}^{+\frac{1}{2}} de = 1$$

must be fulfilled in order that the integral between any two limits of e may express the fraction of the total number of errors that will fall between these limits. We have therefore for the sum of two items

$$F(E) = \int_{E-\frac{1}{2}}^{E+\frac{1}{2}} de$$

But as there are no errors in the single items that numerically exceed 0.5, the limits of this integration are in general greater than the conditions of the problem permit. For positive values of e the integration should be performed from $E - 0.5$ to $+0.5$, and for negative values from -0.5 to $E + 0.5$. We have accordingly the frequency function for the sum of two items

$$\left.\begin{array}{lll} 1 + E \ldots\text{from} & E = -1 & \text{to } E = 0 \\ 1 - E \ldots\text{from} & E = 0 & \text{to } E = +1 \end{array}\right\} \qquad (2)$$

Beyond these limits the frequency is obviously zero since the sum of two items cannot be in error by more than one unit. This function can be represented by two straight lines intersecting the axis of frequencies at $+1$ and the axis of errors at $+1$ and -1 respectively.

For the sum of three items we have

$$F(E) = \int_{E-\frac{1}{2}}^{E-\frac{1}{2}} (1 \pm e)\, de$$

We must now distinguish three cases from one another: (1) e may be less than -0.5; the proper integral is then

$$\int_{-1}^{E+\frac{1}{2}} (1+e)\,de\ ;$$

(2) e may lie between -0.5 and $+0.5$; then the integral becomes

$$\int_{E-\frac{1}{2}}^{0} (1+e)\,de \quad + \quad \int_{0}^{E+\frac{1}{2}} (1-e)\,de\ ;$$

(3) e may be greater than $+0.5$:

$$\int_{E-\frac{1}{2}}^{+1} (1-e)\,de$$

The frequency function for the error of the sum of three items is therefore

$$(3)\qquad \left\{ \begin{array}{lll} \frac{1}{2}(\frac{3}{2}+E)^2 & \text{from } -\frac{3}{2} & \text{to } -\frac{1}{2} \\ \frac{3}{4}-E^2 & \text{from } -\frac{1}{2} & \text{to } +\frac{1}{2} \\ \frac{1}{2}(\frac{3}{2}-E)^2 & \text{from } +\frac{1}{2} & \text{to } +\frac{3}{2} \end{array} \right.$$

This function can be represented by three vertical parabolas, the middle one of which has its concavity toward negative frequencies and intersects the axis of frequencies at $+0.75$. The two others have their concavities toward positive frequencies; they are tangent to the middle parabola, and to the axis of errors at $+1.5$ and -1.5 respectively.

I have continued these integrations as far as $n = 6$; as the resulting functions are symmetrical with respect to the axis of errors, it is necessary to set down only the formulae for positive errors. For $n = 4$,

$$(4)\qquad \left\{ \begin{array}{ll} +\frac{1}{2}E^3 - E^2 + \frac{2}{3} & \text{from } \ \ 0 \text{ to } +1 \\ \frac{1}{6}(2-E)^3 & \text{from } +1 \text{ to } +2 \end{array} \right.$$

For $n = 5$,

$$(5)\qquad \left\{ \begin{array}{ll} +\frac{1}{4}E^4 - \frac{5}{8}E^2 + \frac{115}{192} & \text{from } -\frac{1}{2} \text{ to } +\frac{1}{2} \\ -\frac{1}{6}E^4 + \frac{5}{6}E^3 - \frac{5}{4}E^2 + \frac{5}{24}E + \frac{55}{96} & \text{from } +\frac{1}{2} \text{ to } +\frac{3}{2} \\ \frac{1}{24}(\frac{5}{2}-E)^4 & \text{from } +\frac{3}{2} \text{ to } +\frac{5}{2} \end{array} \right.$$

For $n = 6$,

$$(6)\qquad \left\{ \begin{array}{ll} -\frac{1}{12}E^5 + \frac{1}{4}E^4 - \frac{1}{2}E^2 + \frac{11}{20} & \text{from } \ \ 0 \text{ to } +1 \\ +\frac{1}{24}E^5 - \frac{3}{8}E^4 + \frac{5}{4}E^3 - \frac{7}{4}E^2 + \frac{5}{8}E + \frac{17}{40} & \text{from } +1 \text{ to } +2 \\ \frac{1}{120}(3-E)^5 & \text{from } +2 \text{ to } +3 \end{array} \right.$$

In general, the frequency function for n items is made up of n algebraic expressions each of which is of the degree $n-1$, and each applies to a field of one unit; the aggregate of these expressions forms a continuous function (beyond $n = 1$) and one that has a continuous derivative (beyond $n = 2$). The first and the last of these expressions are of the form

$$\frac{1}{(n-1)!}\left(\frac{n}{2} \pm E\right)^{n-1}$$

The aggregate of expressions fulfills the condition, as should obviously be the case,

$$\int_{-\frac{n}{2}}^{+\frac{n}{2}} F(E)\,dE = 1\ ,$$

These properties furnish convenient controls on the correctness of the integrations.

We have therefore solved this case of our problem as far as the sum of six items. Except for the labor involved in the derivation of these expressions and in their numerical application, there would be no difficulty in extending these results to greater values of n. There exists, however, a form of solution that is very convenient, that is nearly exact even for small values of n, and that increases in accuracy as n becomes greater. This solution is based upon the fact that the exact frequency functions derived above approach more and more closely to the Gaussian or exponential error function

$$\frac{h}{\sqrt{\pi}}\, e^{-h^2E^2}\ .$$

Here e is the base of Napierian logarithms and h is the measure of precision. To prove this let us consider the sum of a large number of items (n) of which p are positive and $n-p$ are negative. Since positive and negative errors are equally likely to occur, the chance that exactly p of them should be positive is

$$\frac{n!}{p!\,(n-p)!}$$

Since any error between -0.5 and $+0.5$ is as likely to occur as any other the average of the positive errors will be $+0.25$ and of the negative -0.25. Consequently on the average the error of the sum of these n items is

$$+0.25\ (2p-n)$$

We have now two expressions that connect the frequency of an error with the size of that error. By passing to the limit in the same way as that pointed out many years ago by HAGEN*, we conclude similarly that the errors in the sum of a large number of tabular quantities follow the exponential law. This proof applies not only to such a frequency law for the single items as we have here, but more generally to any frequency function that is symmetrical with respect to the axis of errors, and still more generally to all frequency functions that satisfy the two conditions

$$\int_{-\infty}^{0} f(e)\,de = \int_{0}^{+\infty} f(e)\,de$$

$$\int_{-\infty}^{0} e\,f(e)\,de = \int_{0}^{+\infty} e\,f(e)\,de$$

We next inquire as to the lower limit of n to which it is permissible to apply the exponential law. A practical method for testing this is to compute the number of errors that fall within specified limits by the rigorous formulæ and to compare these with the number that would occur in the exponential distribution. For example, formula (5) yields the numbers that occur in the second column of Table 1, computed by integrating these expressions between the limits 0 to +0.5, +0.5 to +1.0, etc., up to +2.5. It is assumed that there are 1000 sums. The probable error for this case comes out 0.447, since this satisfies the equation

$$\int_{-0.447}^{+0.447} \left(\tfrac{1}{4}E^4 - \tfrac{5}{8}E^2 + \tfrac{115}{192}\right) de = \tfrac{1}{2}$$

I next compute the number of errors between the same limits in the exponential distribution (column three), assuming that the probable error is again 0.447 and making use of the very convenient table that is printed at the end of the second volume of CHAUVENET's Astronomy as well as in many other places.

The agreement between these two sets of numbers is very good and shows that even for so small a number of items as 5, the distribution of errors is practically the same as that which follows from the exponential law.

In any set of errors the mean error is defined as that

* See page 17 of MERRIMAN's Method of Least Squares.

TABLE 1, $n = 5$

Limits of Error		Number of Errors from the Exact Formulæ	Number of Errors from the Exponential Formula
−2.5 to	−2.0	0	1
−2.0	−1.5	8	11
−1.5	−1.0	51	54
−1.0	−0.5	163	159
−0.5	0.0	275	275
0.0	+0.5	275	275
+0.5	+1.0	163	159
+1.0	+1.5	54	54
+1.5	+2.0	8	11
+2.0	+2.5	0	1

one whose square is the mean of the squares of all the errors. In the present case the square of the mean error of a single item is 1/12, since this is the value of the integral

$$\int_{-1/2}^{+1/2} E^2\,dE$$

Similarly we may find the square of the mean error for the sum of 2, 3, .. 6 items from the expressions

$$\int_{-\frac{n}{2}}^{+\frac{n}{2}} E^2 F(E)\,dE\ ;$$

but it is at once obvious that the square of the mean error for the sum of n items must be exactly equal to n times the square of the mean error for one item. In Table 2 the mean error is given in the second column for all values of n up to 6.

The probable error r in any set of errors is the one that occupies the middle place if the errors are ar-

TABLE 2

n	The Mean Error	The Probable Error	Ratio of the Prob. Error to the Mean Error
1	0.289	0.250 = 0.250 $\sqrt{n}$	0.866
2	.408	.293 = .207 $\sqrt{n}$	.717
3	.500	.353 = .204 $\sqrt{n}$	.705
4	.577	.403 = .201 $\sqrt{n}$	.697
5	.645	.447 = .200 $\sqrt{n}$	.692
6	.707	.488 = .199 $\sqrt{n}$	.689
large	.289 $\sqrt{n}$	.195 $\sqrt{n}$	.675

ranged in the order of their size; or, it is the one defined by the equation

$$\int F(E)\,dE = \tfrac{1}{2}$$

In the third column of Table 2 are the values of the probable errors computed from formulæ (2) to (6) above.

If the number of items is large and we may regard as complete the coincidence of the true error curve with the exponential curve, then the probable error of the sum of n items may at once be computed from the mean error by multiplying it by the well-known ratio 0.6745; that is, the probable error is then equal to

$$0.6745\sqrt{\frac{n}{12}} = 0.195\sqrt{n}$$

as indicated in the last line of the table. With this value of the probable error we may not enter such a table as that numbered IXA in Volume 2 of CHAUVENET's Astronomy and thus compute the number of errors that we should expect to fall between any pair of specified limits.

It is worthy of remark that the coefficients of $\sqrt{n}$ in the third column of Table 2 follow very closely the empirical expression

$$0.195 + \frac{1}{40n}\,.$$

Except for $n = 1$ the error is always in the fourth decimal place.

We now proceed to the case in which the items to be added together are the results of linear interpolations between tabular numbers. In carrying out the interpolations the computer sometimes retains additional decimal places beyond the last tabular place; or more usually he rounds off the interpolated number to the last tabular place. We will consider both kinds of interpolations.

Let A_1 and A_2 be two successive arguments in the table, and T_1 and T_2 the corresponding tabular numbers, whose true values, if carried out to a sufficient number of places would be $T_1 - \Delta_1$ and $T_2 - \Delta_2$. Δ_1 and Δ_2 are the errors of T_1 and T_2 and they may have any values between -0.5 and $+0.5$. Let $A_1 + a$ be the number for which we wish to interpolate, where a may have any value between zero and unity. If the computer retains additional places* in the interpolations, then the error incurred will be $\Delta_1 + a(\Delta_2 - \Delta_1)$, which we shall call e. It is required to find the frequency function for e.

A special phase of this problem has been treated by Dr. R. S. WOODWARD in his chapter on Probabilities in MERRIMAN and WOODWARD's Higher Mathematics. He has shown how to derive the frequency function for any particular value of the interpolating fraction. This result does not help us in the present problem since we require a more general frequency function. That is, if we are going to add together a number of these unrounded interpolations, the sum will involve in general about as many interpolating fractions as there are times, and we must not use the frequency function that corresponds to any particular one of them. In other words, in establishing our frequency function we should regard a as a third independent variable (any of whose values between 0 and 1 is as likely to occur as any other) the two other variables being of course Δ_1 and Δ_2.

The problem can be simplified by the following considerations. No matter what the value of a, e must lie between Δ_1 and Δ_2. For fixed values of Δ_1 and Δ_2 any error between them is as likely to occur as any other, since any value of a is as likely to occur as any other within the limits 0 and 1. Therefore the number of errors at any particular value of e will be augmented only if this e lies between Δ_1 and Δ_2, and this number will share equally with that contributed to any other error between these values of Δ_1 and Δ_2. If these two are close together and e lies between them, then the number of errors occurring at e will be relatively large, since there are fewer errors with which to share. Accordingly for fixed values of Δ_1 and e, the former being greater than the latter, we have for the frequency of e,

$$\int_{\Delta_2 = -\frac{1}{2}}^{\Delta_2 = e} \frac{d\Delta_2}{\Delta_1 - \Delta_2} = \log\left(\Delta_1 + \tfrac{1}{2}\right) - \log\left(\Delta_1 - \Delta_2\right)$$

Similarly, for fixed values of Δ_1 and e, but the former smaller than the latter, the frequency is

$$\int_{\Delta_2 = e}^{\Delta_2 = +\frac{1}{2}} \frac{d\Delta_1}{\Delta_2 - \Delta_1} = \log\left(\tfrac{1}{2} - \Delta_1\right) - \log\left(e - \Delta_1\right)$$

* It makes no practical difference whether several or only one additional place is retained. The formulæ derived below apply [illegible] to the case where several additional decimals are retained, but it would be easy to modify them for the case of a single additional decimal. This would merely increase the probable and mean errors by negligibly small quantities.

Now considering Δ_1 to be variable, the total frequency of e is

$$\int_{\Delta_1=-\frac{1}{2}}^{\Delta_1=e} \{\log(\tfrac{1}{2}-\Delta_1) - \log(e-\Delta_1)\}\, d\Delta_1$$

$$-\int_{\Delta_1=e}^{\Delta_1=+\frac{1}{2}} \{\log(\Delta_1+\tfrac{1}{2}) - \log(\Delta_1-e)\}\, d\Delta_1$$

$$= -2(\tfrac{1}{2}-e)\log(\tfrac{1}{2}-e) - 2(\tfrac{1}{2}+e)\log(\tfrac{1}{2}+e) \equiv \varphi(e)$$

This is the frequency function for the errors of an unrounded interpolation; for brevity we denote it by $\phi\ (e)$. It satisfies the condition, as it should,

$$\int_{-\frac{1}{2}}^{+\frac{1}{2}} \phi(e)\, de = 1$$

The frequency becomes zero for $e = \pm 0.5$ and has a maximum value of 1.386 for $e = 0$. The probable error (r) of an unrounded interpolation must satisfy the equation

$$\int_{-r}^{+r} \phi(e)\, de = \tfrac{1}{2}$$

In this case r comes out equal to 0.187, which is considerably less than 0.250, the probable error of a quantity that is taken directly from the table without the necessity for interpolating. The square of the mean error (m) of an unrounded interpolation is equal to

$$2\int_0^{\frac{1}{2}} e^2\phi(e)\, de$$

$$= -4\int_0^{\frac{1}{2}} e^2(\tfrac{1}{2}-e)\log(\tfrac{1}{2}-e)de$$

$$-4\int_0^{\frac{1}{2}} e^2(\tfrac{1}{2}+e)\log(\tfrac{1}{2}+e)\, de$$

$$= +4\int_0^{\frac{1}{2}} \{(\tfrac{1}{2}-e)^3 - (\tfrac{1}{2}-e)^2 + \tfrac{1}{4}(\tfrac{1}{2}-e)\}\log(\tfrac{1}{2}-e)de$$

$$-4\int_0^{\frac{1}{2}} \{(\tfrac{1}{2}+e)^3 - (\tfrac{1}{2}+e)^2 + \tfrac{1}{4}(\tfrac{1}{2}+e)\}\log(\tfrac{1}{2}+e)de$$

$$= \tfrac{1}{18} \text{ or } m = \sqrt{\tfrac{1}{18}} = 0.236$$

This again is considerably less than 0.289, the mean error of a direct tabular quantity.

We next inquire into the errors of the sum of two unrounded interpolations. Such an error E can be considered as made up of two errors e_1 and $E - e_1$ in the single items. The frequency with which E will occur is therefore proportional to the product of $\varphi\ (e_1)$ and $\phi\ (E - e_1)$, and is to be derived by performing the integration

$$\int_{e=E-\frac{1}{2}}^{e=E+\frac{1}{2}} \varphi(e)\,\varphi(E-e)\, de \tag{7}$$

For positive values of E the upper limit of integration should be replaced by $+0.5$, and for negative values the lower limit should be replaced by -0.5.

If we substitute the values of $\varphi\ (e)$ and $\phi\ (E - e)$ in this integration we get an expression that is complicated and difficult to handle. I have therefore sought to replace ϕ by an algebraic function that would represent the frequencies with sufficient approximation. All the necessary conditions for such a function are fulfilled by this substitute for ϕ:

$$\phi_1(e) = \tfrac{4}{3}(1 - 8e^3), \text{ for positive values of } e.$$
$$= \tfrac{4}{3}(1 + 8e^3), \text{ for negative values of } e.$$

Below is given a comparison of this approximation with the rigorous values.

Table 3

	$\phi\ (e)$ Rigorous	$\phi_1(e)$ Approximate
$e =\ \ 0.0$	1.386	1.333
$e = \pm 0.1$	1.346	1.323
$e = \pm 0.2$	1.222	1.248
$e = \pm 0.3$	1.001	1.045
$e = \pm 0.4$	0.655	0.651
$e = \pm 0.5$	0.000	0.000

The agreement of the two is sufficiently close for our purposes; ϕ_1, like ϕ, satisfies the condition

$$\int_{-\frac{1}{2}}^{+\frac{1}{2}} \phi_1(e)\, de = 1$$

The probable error that follows from ϕ_1, is 0.190 as against the rigorous value 0.187; and the mean error derived from the approximate expression is exactly equal to the rigorous value. We may therefore replace ϕ by ϕ_1, with little error.

Resuming on this basis the consideration of the errors of the sum of two unrounded interpolations, (7) becomes, for positive values of E not greater than 0.5,

$$\frac{16}{9}\int_{e=-E}^{e=0}(1+8e^3)(1-8\overline{E-e}^3)\,de$$

$$+\frac{16}{9}\int_{e=0}^{e=E}(1-8e^3)(1-8\overline{E-e}^3)\,de$$

$$+\frac{16}{9}\int_{e=E}^{e=\frac{1}{2}}(1-8e^3)(1+8\overline{E-e}^3)\,de$$

And for positive values of E greater than 0.5,

$$\frac{16}{9}\int_{e=E-\frac{1}{2}}^{e=\frac{1}{2}}(1-8e^3)(1-8\overline{E-e}^3)\,de$$

After suitable reductions these become

$$(8)\quad\left\{\begin{array}{l}\frac{8}{7}-\frac{32}{5}E^2+\frac{32}{3}E^3-\frac{64}{9}E^4+\frac{256}{105}E^7,\ \text{for } 0 < E < \frac{1}{2}\\[2mm] \frac{32}{21}-\frac{32}{9}E+\frac{32}{5}E^2-\frac{32}{3}E^3+\frac{64}{9}E^4-\frac{256}{315}E^7,\\ \hfill\text{for } \frac{1}{2} < E < 1\end{array}\right.$$

These two expressions, together with two symmetrical ones for negative values of E, form the approximate frequency function for the errors in the sum of two unrounded interpolations. The probable error comes out equal to 0.237 and the mean error is exactly equal to

$$\sqrt{\frac{2}{18}}\quad\text{or}\quad 0.333\,.$$

It can readily be shown that if we add together a large number of unrounded interpolations we obtain a frequency function that approaches the exponential form. It does this very rapidly, so that the sum of even two interpolations affords a good approximation. In Table 4 are compared the number of errors that fall within successive limits according to the expressions (8) and according to the exponential distribution. For the latter the probable error is assumed to be 0.237. The total number of errors is taken to be 1000. Similar numbers apply to negative errors.

Table 4

Limits of Error		Number of Errors from Expressions (8)	Number of Errors from the Exponential Law
0.0 to	0.1	112	112
0.1	0.2	103	103
0.2	0.3	88	88
0.3	0.4	71	69
0.4	0.5	54	50
0.5	0.6	37	34
0.6	0.7	22	21
0.7	0.8	10	12
0.8	0.9	3	6
0.9	1.0	0	3
Beyond	1.0	0	2

This agreement is so good that for the sum of more than two unrounded interpolations we may proceed as follows. The mean error of any number of such items is exactly equal to

$$\sqrt{\frac{n}{18}}\quad\text{or}\quad 0.236\sqrt{n}$$

From this we compute the probable error by multiplying by 0.6745 and we obtain $0.159\sqrt{n}$. With this we enter the usual tables to get the number of errors between any specified limits. It may be worth while to remark that the only inaccuracy in this process arises from the fact that the true law of distribution is not exactly exponential and that for small values of n the probable error thus computed will be two small; but even for n so small as 2, the inaccuracy thus incurred is less than six per cent, and this decreases very rapidly as n increases.

We come now to the last case, namely that in which the computer rounds off his interpolations to the last tabular place. Here again we may distinguish several sub cases, depending upon the nature of the interpolating fraction and the number of significant places in the tabular differences. Dr. Woodward has also treated special phases of this problem in the place already cited. He has shown how to derive the frequency function when the interpolating fraction has the form m/n, where m and n are any whole numbers. This again will not help us much in the present problem since the sum of a number of these rounded interpolations will in general involve about as many different interpolating fractions as there are items in the sum.

In what follows I assume that the interpolating fraction is expressed as a decimal, which is almost invariably the case in practice. Furthermore, to

begin with, I assume that the number of decimals in this fraction is large and that the number of significant figures in the tabular differences is also large. The additional error committed by rounding off the interpolations has maximum values equal to -0.5 and $+0.5$, and under the assumptions that we have just made (but not otherwise) any error between these limits is as likely to occur as any other. To find the frequency function for the combination of the error of rounding with that of an unrounded interpolation, it is evident from what has gone before that we should form the integral

$$\int_{e=E-\frac{1}{2}}^{e=E+\frac{1}{2}} \phi(e)\, de$$

where ϕ has the same definition as before. Remembering that for positive values of e the upper limit should be replaced by $+0.5$, and performing the integration we get

$$(9) \qquad 1 - E - (1-E)^2 \log(1-E) + E^2 \log E$$

This expression, together with a symmetrical one for negative values of E, forms the frequency function for a rounded interpolation. The probable error, computed in the usual way, is 0.270. The mean error comes out equal to

$$\sqrt{\frac{5}{36}} = 0.373$$

This is as it should be, since a moment's reflection will show that the square of the mean error of a rounded interpolation should equal the sum of the squares of that of a tabular place and of an unrounded interpolation.

The next step is to deduce the frequency curve for the sum of two rounded interpolations; but this proves to be unnecessary, for the reason that the law of frequency for the sum of several items approaches the exponential law so rapidly that the sum of only two items conforms to it within narrow limits. In fact the frequency law for a single item is not a bad approximation to the exponential law. In Table 5 are compared the frequencies that follow from (9), for a single rounded interpolation, with those that follow from the exponential expression that corresponds to a probable error equal to 0.270. The total number of errors is supposed to be 1000. Similar numbers apply to negative errors.

TABLE 5: A Rounded Interpolation

Limits of Error	Number of Errors from Expression (9)	Number of Errors from the Exponential Law
0.0 to 0.1	99	99
0.1 0.2	93	93
0.2 0.3	81	82
0.3 0.4	71	68
0.4 0.5	57	53
0.5 0.6	43	39
0.6 0.7	29	27
0.7 0.8	19	17
0.8 0.9	7	10
0.9 1.0	1	6
Beyond 1.0	0	6

Except for the disparity in the number of large errors the agreement is very good.

For any number of items (n) in each sum the mean error of the sum is exactly equal to

$$\tfrac{1}{6}\sqrt{5n} \quad \text{or} \quad 0.373\sqrt{n}$$

The probable error accordingly, if we assume that the true error function agrees with the exponential law within negligible limits, is $0.251\sqrt{n}$. With this value we can now enter the usual tables and we thus calculate the number of errors that we should expect to fall within specified limits. The inaccuracy that this process entails is due to the fact that the true error function does not follow the exponential law exactly, and that the probable error thus computed is somewhat smaller than the truth; but even for a single item in each sum the inaccuracy thus incurred is only about seven per cent. of the probable error, and for the sum of two items only about three per cent. and this percentage rapidly decreases as n increases.

These results for a rounded interpolation apply strictly only when the number of decimals in the interpolating fraction is large. If, for example, there is only one decimal in this fraction, then it cannot be assumed that all errors introduced by the rounding are equally likely to occur; a tabulation for every possible digit in the last place of the tabular difference and in the interpolating fraction shows that then 27 per cent. of the rounding errors are zero; 8 per cent. are ± 0.1; 24 per cent. ± 0.2; 8 per cent. ± 0.3; 24 per cent. ± 0.4; and 9 per cent. ± 0.5. Consequently the mean error (due to rounding only) is 0.280, as against 0.289 if all errors were equally likely to occur. Similarly, assuming that there are two decimals in the interpolating fraction (and not less than three significant places in the tabular differences)

I have made a tabulation of the errors that would follow from every possible combination of the four digits involved; that is, the final two in the tabular difference and the two in the interpolating fraction. This shows a much more nearly uniform distribution than in the case of a single decimal. The mean error is now 0.283. It appears then that in either of these cases not much error would have been incurred had we used the formulae already deduced. If, however, the computer wishes to take account of this small correction he may do so by entering the exponential tables with a probable error of $0.217\sqrt{n}$ if there is one decimal in the interpolating fraction, and $0.218\sqrt{n}$ if there are two, numbers that are less than two per cent. smaller than the corresponding ones deduced on the assumption of a large number of decimals.

The chief results of this investigation may be summarized as follows. The frequency of errors in the sum of a number of tabular quantities follows closely the exponential law unless the number of items in each sum is very small. If these quantities are taken directly from tables without the necessity for interpolating, then the exponential expression that should be used is the one corresponding to a probable error of $0.195\sqrt{n}$; but if the number of items in each sum is less than 5 or 6, then we should make use of the exact formulae derived above. If the quantities are interpolated from the tables, and if the computer retains an additional decimal beyond the last tabular place, then the proper exponential expression to use is the one that corresponds to a probable error of $0.159\sqrt{n}$; this is a sufficiently close approximation unless n is 1 or 2, in which cases we should use the more precise formulae given above. Finally, if the computer rounds off his interpolations to the last tabular place, he should use the exponential expression that corresponds to a probable error of $0.25\sqrt{n}$, unless n is 1, for which case the rigorous formula has been deduced.

The slowness with which these errors accumulate will, I think, surprise most computers. Suppose that we have a large number of sums for each of the three cases, and suppose that the number of tabular items in each sum is 26 for direct tabular numbers, 40 for the unrounded interpolations, and 16 for the rounded interpolations; then in each case half the sums will have actual errors that are less than one unit in the last tabular place.

Allegheny Observatory, University of Pittsburgh,
1 July 1917.

NOTE

TRAITÉ DES FONCTIONS ELLIPTIQUES (LEGENDRE)

Errors in Tome II.

By C. J. MERFIELD.

When using the above memoir recently, several typographical errors were noted.

These errors are given below, together with the necessary corrections.

Table IX $F\,(\theta.\varphi)$

φ	θ	Erroneous	Correct	Page
5°	7°	0.0872981052	0.0872681052	297
20	9	0.3429353662	0.3492353662	297
6	14	0.1147309355	0.1047309355	301
42	27	0.7475401816	0.7457401816	313

Table IX $E\,[\theta.\varphi.]$

φ	θ	Erroneous	Correct	Page
5	10°	0.0023731277	0.0872631277	296
78	22	1.3107285082	1.3197285082	310
6	86	0.1016293396	0.1045293396	360

Table VI

$\theta = 5°.1$ Log: $c = 5.94887..$ should be $8.94887..$

$9\ .1$ Log $b°$ 9.9999996 . should be $9.9999896..$

$34\ .1$ Log c 9.94756 . should be $9.74756..$

Log $E'c$ at 37°, the first difference should be 238151846 and not 238181846 as given.

The Observatory, South Yarra, Melbourne, 1917 April 16.

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Editor, Benjamin Boss, Albany, N. Y.; Associate Editors: E. E. Barnard, Ernest W. Brown, F. R. Moulton and R. S. Woodward
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THE PRINCIPAL AXES OF DISTRIBUTION OF STELLAR MOTION,

By H. RAYMOND.

1. *The velocity figure.* The following method of discussing the preferential motion of the stars was suggested by NEWCOMB's paper on the position of the Galactic plane (*Contributions to Stellar Statistics.* On the Position of the Galactic and Other Principal Planes toward which the Stars tend to Crowd. Carnegie Inst. of Wash. Pub. No. 10) and is a generalization of the process there described.

Suppose all the stars with whic we have to deal, including the *Sun*, transferred to a single point O, fixed relatively to the geometric center of gravity of the stars, and allowed to move with their present velocities for, say, a century. They will have expanded into a roughly globular cluster, and we wish to find any systematic deviation of this "velocity-figure" from the spherical form which can be described as an extension along any line, or a compression toward any plane.

Pass a line through the origin O and let the projection upon it of a star in the velocity-figure be P. Find $p = OP$ for each star, and form the mean of the squares $\overline{pp}$. The line for which $\overline{pp}$ is a maximum will be called the "axis of preference," and that for which $\overline{pp}$ is a minimum the "axis of avoidance." The axis of preference corresponds to the line of relative motion of KAPTEYN's two star streams, or to that determined by the vertex and antivertex: while an axis of avoidance would be to the motions of the stars what the Galactic axis is to their positions in space.

Let the coördinates of a star in the velocity-figure, referred to suitable axes through O, be ξ, η, and ζ, and the direction cosines of an arbitrary line through O be l, m and n. Then for that line $p = l\xi + m\eta + n\zeta$.

$$\overline{pp} = \text{mean of } (l^2\xi^2 + m^2\eta^2 + n^2\zeta^2 + 2lm\xi\eta + 2ln\xi\zeta + 2mn\eta\zeta)$$

$$(1) \qquad = l^2\overline{\xi\xi} + m^2\overline{\eta\eta} + n^2\overline{\zeta\zeta} + 2lm\overline{\xi\eta} + 2ln\overline{\xi\zeta} + 2mn\overline{\eta\zeta}$$

The condition, that for a maximum or a minimum the differential of $\overline{pp}$ is zero, gives

$$(\overline{\xi\xi}l + \overline{\xi\eta}m + \overline{\xi\zeta}n)\,dl + (\overline{\zeta\eta}l + \overline{\eta\eta}m + \overline{\eta\zeta}n)dm + (\overline{\xi\zeta}l + \overline{\eta\zeta}m + \overline{\zeta\zeta}n)dn = 0 \qquad (2)$$

Also, since $l^2 + m^2 + n^2 = 1$,

$$l\,dl + m\,dm + n\,dn = 0 \qquad (3)$$

These equations may be treated by LAGRANGE's method of indeterminate multipliers. Multiply (3) by the indeterminate factor λ, substract from (2), and equate the coefficients of dl, dm, and dn separately to zero. We thus get

$$\left.\begin{aligned}(\overline{\xi\xi} - \lambda)\,l + \overline{\xi\eta}m + \overline{\xi\zeta}n &= 0\\ \overline{\xi\eta} + (\overline{\eta\eta} - \lambda)m + \overline{\eta\zeta}n &= 0\\ \overline{\zeta\xi}l + \overline{\eta\zeta}m + (\overline{\zeta\zeta} - \lambda)n &= 0\end{aligned}\right\} \qquad (4)$$

The four equations (3) and (4) give the values of λ, l, m, and n for which $\overline{pp}$ is a maximum or a minimum.

Eliminating l, m, and n from (3) and (4) we get the cubic in λ,

$$\begin{vmatrix}\overline{\xi\xi} - \lambda & \overline{\xi\eta} & \overline{\xi\zeta}\\ \overline{\xi\eta} & \overline{\eta\eta} - \lambda & \overline{\eta\zeta}\\ \overline{\xi\zeta} & \overline{\eta\zeta} & \overline{\zeta\zeta} - \lambda\end{vmatrix} = 0 \qquad (5)$$

If for brevity we put $A = \overline{\xi\xi}$, $B = \overline{\eta\eta}$, etc., we have

$$\begin{vmatrix}A - \lambda & D & E\\ D & B - \lambda & F\\ E & F & C - \lambda\end{vmatrix} = 0 \qquad (5)$$

This has three real and positive roots, which, substituted in (4), give three sets of values of l, m, n. Those corresponding to the greatest root λ_1 of (5) are the direction cosines of the axis of preference; those corresponding to the least root λ_3 give the axis of avoidance; while to λ_2 corresponds a line for which pp is a minimax. The three lines are mutually orthogonal, and (as also the roots $\lambda_1, \lambda_2, \lambda_3$) are not changed by a different choice of coordinate axes. In fact, they will be recognized as the three principal axes of inertia of the velocity-figure, considered as a rigidly connected system of equally weighted points. (See PLUMMER's comments on NEWCOMB's paper, *Monthly Notices R. A. S.*, LXV, 565.)

It is easy to see that $\lambda_1, \lambda_2, \lambda_3$, are the mean square components of stellar motion in the direction of the three principal axes of the velocity-figure. For if we had at the first chosen u, v, w, coinciding in order with the three principal axes of stellar motion, we should have had $\overline{uv} = \overline{uw} = \overline{vw} = 0$, whence, (putting u for ξ in (5), etc.),

$$\lambda_1 = \overline{uu} \qquad \lambda_2 = \overline{vv} \qquad \lambda_3 = \overline{ww} \tag{6}$$

2. *Effect of Solar Motion.* Thus far I have followed NEWCOMB, the only real difference being that his equations relate to the positions of the stars on the sky, and mine to their positions in the velocity-figure. Now the solar motion introduces a difficulty from which NEWCOMB's problem, because of the practically central position of the *Sun*, is free. But suppose, in NEWCOMB's problem, the *Sun* removed toward one pole of the galaxy so far that none of the stars used appear farther than 45° from the other pole. If ρ be the distance of any star, it is evident that pp would be always greater than $\tfrac{1}{2}\rho^2$ for the plane perpendicular to the Galactic axis, and less than $\tfrac{1}{2}\rho^2$ for any plane through that axis. The method would therefore lead to the erroneous result that the stellar system is elongated in the direction of the Galactic poles. In the same manner the solar motion will introduce a fictitious elongation of the velocity-figure in the direction of the apex and antapex, because the coordinates are necessarily referred in the first instance to the *Sun*, which being in motion, occupies an eccentric position in the velocity-figure.

The difficulty may be avoided as follows: Transform to a new set of axes parallel to the old, but passing through the *Sun* (in the velocity-figure) as origin. Let the new coordinates be ξ, η, ζ, and the coordinates of the *Sun* (or of the solar motion) referred to the old axes X, Y, Z. We have then $\xi = x + X$, $\eta = y + Y$, $\zeta = z + Z$. It follows that $\xi^2 = x^2 + Xx + X(x + X)$, etc. Taking the mean we have, since $\bar{x} = -X$, a constant,

$$\left.\begin{aligned} \xi\xi &= A = \overline{xx} - \bar{x}\,\bar{x} & \xi\eta &= D = \overline{xy} - \bar{x}\,\bar{y} \\ \eta\eta &= B = \overline{yy} - \bar{y}\,\bar{y} & \xi\zeta &= E = \overline{xz} - \bar{x}\,\bar{z} \\ \zeta\zeta &= C = \overline{zz} - \bar{z}\,\bar{z} & \eta\zeta &= F = \overline{yz} - \bar{y}\,\bar{z} \end{aligned}\right\} \tag{7}$$

The expressions $X = -\bar{x}$, $Y = -\bar{y}$, $Z = -\bar{z}$, give the solar motion by the method of BRAVAIS (*Journal de Math. de Liouville*, t. VIII. For a full discussion see WEERSMA, *Groningen Publications* No. 21). The spherical coördinates are given by

$$\left.\begin{aligned} M \cos A \cos D &= -\bar{x} \\ M \sin A \cos D &= -\bar{y} \\ M \sin D &= -\bar{z} \end{aligned}\right\} \tag{8}$$

3. *Expressions for the coördinates x, y, z.* Suppose the x axis directed toward the First Point of Aries, the y axis toward the point 6^h, $0°$, and the z axis toward the North Pole. Then

$$\left.\begin{aligned} x &= h\rho \cos\alpha \cos\delta - kR\mu \sin\alpha - kR\mu' \cos\alpha \sin\delta \\ y &= h\rho \sin\alpha \cos\delta + kR\mu \cos\alpha - kR\mu' \sin\alpha \sin\delta \\ z &= h\rho \sin\delta \qquad\qquad\qquad\qquad + kR\mu' \cos\delta \end{aligned}\right\} \tag{9}$$

Here R is the distance of a star, ρ its radial velocity, μ and μ' its proper-motion in right-ascension and declination respectively (in arc of a great circle), h and k the factors required to reduce ρ and μ, μ' to the units employed.

The radial velocities cannot yet be combined with the proper-motions on a large scale, because of their small number, but especially because of our too scanty knowledge of the individual distances R. We are compelled, therefore, to separate the tangential and radial parts of the motions, and write for the former

$$\left.\begin{aligned} x_1 &= -R\mu \sin\alpha - R\mu' \cos\alpha \sin\delta \\ y_1 &= +R\mu \cos\alpha - R\mu' \sin\alpha \sin\delta \\ z_1 &= \qquad\qquad\qquad + R\mu' \cos\delta \end{aligned}\right\} \tag{10}$$

and for the radial part

$$\left.\begin{aligned} x_2 &= \rho \cos\alpha \cos\delta \\ y_2 &= \rho \sin\alpha \cos\delta \\ z_2 &= \rho \sin\delta \end{aligned}\right\} \tag{10'}$$

The distances R, being unknown, must eventually be dropped, but are retained for the present.

Since $R\mu = -x \sin\alpha + y \cos\alpha$ and $R\mu' = x \cos\alpha \sin\delta - y \sin\alpha \sin\delta + z \cos\delta$, we have

$$(11)\quad \begin{cases} x_1 = x(1 - \cos^2\alpha \cos^2\delta) - y \sin\alpha \cos\alpha \cos^2\delta - z \cos\alpha \sin\delta \cos\delta \\ y_1 = -x \sin\alpha \cos\alpha \cos^2\delta + y\,(1 - \sin^2\alpha \cos^2\delta) - z \sin\alpha \sin\delta \cos\delta \\ z_1 = -x \cos\alpha \sin\delta \cos\delta - y \sin\alpha \sin\delta \cos\delta + z \cos^2\delta \end{cases}$$

The values of the squares and products of x_1, y_1, z_1, in terms of those of x, y, x, can be found from (11) by multiplication.

4. *Effect of dropping the radial motions.* Thus far we have made no hypothesis as to the distribution of motions. It is not necessary that the velocity-figure should have any sort of symmetry, or that the motions of the stars in one part of space should be distributed in the same way* as those in any other part. For example, the plane bisecting at right angles the line of separation of the two streams in KAPTEYN's theory is a plane of avoidance, but not necessarily a plane of symmetry. But when we drop the radial motions we are no longer thus free. We are compelled to introduce hypotheses, and the most simple and obvious which seem to be sufficient are the following:

I. The stars are equally distributed over the sky.

II. Their distribution in distance is the same in all parts of the sky.

III. The motions of the stars are distributed in the same way in all accessible parts of space.

These hypotheses are not rigorously true, but are subject to systematic as well as chance deviations. These must be eliminated or minimized by special devices. I is obviously not true, but this can be allowed for by a simple choice of weights. The correction of errors due to the inexactness of II is treated in §6. III seems to be justified in a broad way by the results of former investigations in this field. There is, indeed, the difficulty presented by the unequal distribution of spectral classes, each having its own typical velocity-figure, which calls for their individual treatment.

These hypotheses are contained in the single one: If we divide the space occupied by the stellar system into pyramids having their common vertex at the *Sun* and enclosing equal solid angles, and form the velocity-figure for the stars within each pyramid, these figures will be alike in form, real and apparent size, orientation, and number of stars.

If this were rigidly true, then for each motion (x, y, z) in one region there would be an identical motion in every other region. We then have the proportion: the parts of the sums Σx_1, Σy_1, Σz_1, Σx_1x_1, etc., contributed by these identical motions, one in each area, is to the sums Σx, Σy, Σz, Σxx, etc., contributed by the same motions, as $\int\int x_1 \cos\delta\, d\alpha d\delta$, etc., $\int\int x_1x_1 \cos\delta\, d\alpha d\delta$, etc., is to $\int\int x \cos\delta\, d\alpha\, d\delta$, etc., $\int\int xx \cos\delta\, d\alpha\, d\delta$, etc., integrated over the sphere. The latter integrals are simply $4\pi x$, since x, etc., are here constant. Thus, for example,

$$\Sigma x_1 / \Sigma x = \int\int x_1 \cos\delta\, d\alpha\, d\delta \,/\, 4\pi x$$

The values of x_1, which varies over the sphere, may be supplied from (11) in terms of the constants x, y, z. Treating all other possible sets of identical motions in the same way, performing the integrations, and remembering that the ratio of the sums equals the ratio of the means, we have

$$(12)\quad \begin{cases} \overline{x}_1 = \frac{2}{3}\,\overline{x} & \overline{x_1x_1} = \frac{8}{15}\,\overline{xx} + \frac{1}{15}\,\overline{yy} + \frac{1}{15}\,\overline{zz} & \overline{x_1y_1} = \frac{7}{15}\,\overline{xy} \\ \overline{y}_1 = \frac{2}{3}\,\overline{y} & \overline{y_1y_1} = \frac{1}{15}\,\overline{xx} + \frac{8}{15}\,\overline{yy} + \frac{1}{15}\,\overline{zz} & \overline{x_1z_1} = \frac{7}{15}\,\overline{xz} \\ \overline{z}_1 = \frac{2}{3}\,\overline{z} & \overline{z_1z_1} = \frac{1}{15}\,\overline{xx} + \frac{1}{15}\,\overline{yy} + \frac{8}{15}\,\overline{zz} & \overline{y_1z_1} = \frac{7}{15}\,\overline{yz} \end{cases}$$

* Not necessary, that is, for a solution. If the result is to have any value, of course the distribution in different parts of the sky must be approximately the same.

These equations will give the values of $\bar{x}$, $\overline{xx}$, etc., to be used in (7). I will refer to the process summarized in (12) as the "correction for perspective."

If (10′) be used in the same way as (10), employing the relation $\rho = x \cos\alpha \cos\delta + y \sin\alpha \cos\delta + z \sin\delta$, we find in like manner:

$$(12')\quad \begin{cases} \bar{x}_2 = \frac{1}{3}\bar{x} & \overline{x_2x_2} = \frac{1}{5}\overline{xx} + \frac{1}{15}\overline{yy} + \frac{1}{15}\overline{zz} & \overline{x_2y_2} = \frac{2}{15}\overline{xy} \\ \bar{y}_2 = \frac{1}{3}\bar{y} & \overline{y_2y_2} = \frac{1}{15}\overline{xx} + \frac{1}{5}\overline{yy} + \frac{1}{15}\overline{zz} & \overline{z_2x_2} = \frac{2}{15}\overline{xz} \\ \bar{z}_2 = \frac{1}{3}\bar{z} & \overline{z_2z_2} = \frac{1}{15}\overline{xx} + \frac{1}{15}\overline{yy} + \frac{1}{5}\overline{zz} & \overline{y_2z_2} = \frac{2}{15}\overline{yz} \end{cases}$$

5. *Effect of the Galactic condensation.* If we had the total motions of the stars, their greater numbers in the galactic region could make no difference. The case is otherwise when we have either the tangential or the radial components only. For suppose an extreme case, — all the stars concentrated in the galactic plane. It is clear that all the radial components would lie in that plane, and consequently the tangential components would show a preponderance normal to it.

The difficulty can be avoided by weighting each region in proportion to its area. In the present investigation the means of the components x_1, y_1, z_1, of tangential motion, their squares and products, were taken in 108 nearly equal trapeziums, the means of these means being then taken with weights proportional to their areas. The components of real motion were then formed by (12), and these employed to find A, B, etc., by (7). An alternative but more laborious procedure would be to find $\overline{\xi_1\xi_1} = \overline{x_1x_1} - \bar{x}_1\bar{x}_1$, etc., for each area, and take their means by weights, from which $\overline{\xi\xi}$, $\overline{\xi\eta}$, etc., would be found by the last six equations of (12), and these used in (7).

The question of distribution does not arise in most other methods of investigating systematic motion, because the form of the velocity-figure (projected) is completely found for each area, the results being integrated to give the general velocity-figure by a second process. Such a modification of the present method is possible. If in a limited area $A = \overline{\mu\mu} - \bar{\mu}\bar{\mu}$, $B = \overline{\mu'\mu'} - \bar{\mu}'\bar{\mu}'$, $C = \overline{\mu\mu'} - \bar{\mu}\bar{\mu}'$, the maximum and minimum values of a mean square component are given by the two roots of the quadratic

$$\begin{vmatrix} A - \lambda & C \\ C & B - \lambda \end{vmatrix} = 0$$

or $\lambda = \frac{1}{2}(A + B) \pm \frac{1}{2}[(A - B)^2 + 4C^2]^{\frac{1}{2}}$. The position angle of the longer axis is given by

$$\tan\theta = \frac{2C}{B - A + [(A - B)^2 + 4C^2]^{\frac{1}{2}}} = \frac{A - B + [(A - B)^2 + 4C^2]^{\frac{1}{2}}}{2C}$$

or that of either axis by $\tan 2\theta = 2C/(B - A)$. This is essentially CHARLIER'S Method. (Meddelanden fran Lunds Ast. Obs., Ser. II, No. 9.)

6. *Effect of the greater distances of the Galactic stars.* The variation of the mean distance of the stars from one point of the sky to another would be of no importance if we had the total motion, or if we were using the radial velocities. In the case of the tangential motions it changes the size of the velocity-figure for such areas as depart from the mean. We therefore multiply the partial means by areas of the components by factors inversely proportional to the effective mean parallaxes for those areas, and the means of the squares and products by factors proportional to the effective mean squares of the parallaxes. As the only variation of a systematic character that need be feared is that with galactic latitude, this operation can be performed by zones parallel to the Galaxy. It is clear that the factors for $\bar{x}_1$, etc., and those for $\overline{x_1y_1}$, etc., must be mutually consistent, unless the "alternative procedure" of the last section be followed.

The effective mean parallax can be found by the ratio of observed solar motion for an area, $(\bar{x}_1 \cdot \bar{x}_1 + \bar{y}_1 \cdot \bar{y}_1 + \bar{z}_1 \cdot \bar{z}_1)^{\frac{1}{2}}$, to that predicted from $(\bar{x} \cdot \bar{x} + \bar{y} \cdot \bar{y} + \bar{z} \cdot \bar{z})^{\frac{1}{2}} \sin\Delta$, Δ being the distance from the apex. This is not quite correct for the second order quantities, $\overline{x_1x_1}$, $\overline{x_1y_1}$, etc., which play the chief part in this method; moreover, near the apex and antapex the factor depends on the ratio of two small quantities. This was therefore rejected.

It is not difficult to show that, for any given area, the mean square parallax being the same all over the sky,

$$\overline{\xi_1\xi_1} + \overline{\eta_1\eta_1} + \overline{\zeta_1\zeta_1} = \overline{x_1x_1} + \overline{y_1y_1} + \overline{z_1z_1} - \bar{x}_1\,\bar{x}_1 - \bar{y}_1\,\bar{y}_1 - \bar{z}_1\,\bar{z}_1$$
$$= \lambda_1 l^2 + \lambda_2 m^2 + \lambda_3 n^2$$

where l, m, n are the direction cosines of the area, referred to the three principal axes of motion, and the λ's are the three roots of the cubic (5). Or if solar motion be not eliminated, we have the form

$$(13)\qquad \text{Predicted}\quad (\overline{x_1x_1} + \overline{y_1y_1} + \overline{z_1z_1}) = \lambda_1 l^2 + \lambda_2 m^2 + \lambda_3 n^2 + M^2 \sin^2 \Delta$$

For l, m, n, Δ, we may substitute their values in terms of the longitudes and latitudes λ, β, of an arbitrarily chosen point, the longitudes L_1, L_2, L_3, L, and latitudes B_1, B_2, B_3, B, of the three axes of motion and the apex. If the resulting expression be multiplied by the differential area $\cos\beta\, d\lambda d\beta$, and integrated with regard to λ from 0° to 360° and with regard to β between the limiting latitudes β_1 and β_2 of the zone, we have, for the predicted mean over a zone

$$\text{Predicted}\ (x_1x_1 + y_1y_1 + z_1z_1) = H + K\ f \qquad (14)$$

Where

$$(15)\qquad \begin{cases} H = H_1 + H_2 \qquad\qquad K = K_1 + K_2 \\ H_1 = \tfrac{1}{2}(\lambda_1 \cos^2 B_1 + \lambda_2 \cos^2 B_2 + \lambda_3 \cos^2 B_3) + (\lambda_1 \sin^2 B_1 + \lambda_2 \sin^2 B_2 + \lambda_3 \sin^2 B_3) \\ K_1 = \tfrac{1}{2}(\lambda_1 \cos^2 B_1 + \lambda_2 \cos^2 B_2 + \lambda_3 \cos^2 B_3) - (\lambda_1 \sin^2 B_1 + \lambda_2 \sin^2 B_2 + \lambda_3 \sin^2 B_3) \\ H_2 = \tfrac{1}{2} M^2 \cos^2 B + M^2 \sin^2 B \qquad\qquad K_2 = \tfrac{1}{2} M^2 \cos^2 B - M^2 \sin^2 B \\ f = (\sin^3 \beta_2 - \sin^3 \beta_1)/3(\sin \beta_2 - \sin \beta_1) \end{cases}$$

If the pole of the zones coincides with one of the three poles, say (L_3, B_3),

$$(16)\qquad H_1 = \tfrac{1}{2}(\lambda_1 + \lambda_2) + \lambda_3 \qquad K_1 = \tfrac{1}{2}(\lambda_1 + \lambda_2) - \lambda_3$$

If the solar motion is eliminated area by area, as in the "alternative procedure" of §5 (second paragraph) H and K in (14) should of course be replaced by H_1 and K_1.

The correction factors for squares and products are, then,

$$\frac{\text{Predicted}\ (x_1x_1 + \overline{y_1y_1} + \overline{z_1z_1})}{\text{Observed}\ (x_1x_1 + y_1y_1 + z_1z_1)}\,.$$

As these factors do not ordinarily differ greatly from unity, their square roots may be used with $\bar{x}_1$, $\bar{y}_1$, $\bar{z}_1$, without serious errors. This procedure is not entirely free from objection, but it can be improved only after investigating the laws of distribution of the stars and their velocities.

Because of the unequal distribution of the material, the parallax factors will, in general, slightly change the size of the velocity-figure and solar motion. As this seems to introduce an unnecessarily arbitrary element, it was allowed for by multiplying λ_1, λ_2, λ_3, by such a factor as would make their sum, the total mean square motion, the same as before; and similarly with M. This did not change the form or orientation of the velocity-figure nor the direction of solar motion.

7. *Solution by zones.* If for any reason it seems desirable to make a solution for a zone, formulas connecting $\bar{x}$, $\overline{xx}$, etc., with $\bar{x}_1$, $\overline{x_1x_1}$, etc., must be found, valid for the limited surface used. In order to integrate over a zone the coördinates must be transformed to x', y', z', where z' is the direction of the common pole of the zones.

Performing the integrations of §4 over a zone of limiting latitudes β_1, β_2, we get, instead of (12)—dropping the primes:

$$(17)\qquad \begin{cases} \bar{x}_1 = \tfrac{1}{2}(1+f)\,\bar{x} \qquad \overline{x_1x_1} = a\,\overline{xx} + b\,\overline{yy} + c\,\overline{zz} \qquad \overline{x_1y_1} = e\,\overline{xy} \\ \bar{y}_1 = \tfrac{1}{2}(1+f)\,\bar{y} \qquad \overline{y_1y_1} = b\,\overline{xx} + a\,\overline{yy} + c\,\overline{zz} \qquad \overline{x_1z_1} = g\,\overline{xz} \\ \bar{z}_1 = \quad (1-f)\,\bar{z} \qquad \overline{z_1z_1} = c\,\overline{xx} + c\,\overline{yy} + d\,\overline{zz} \qquad \overline{y_2z_1} = g\,\overline{yz} \end{cases}$$

$$(18)\qquad \begin{cases} a = \tfrac{3}{8} + \tfrac{1}{4}f + \tfrac{3}{8}\varphi = e + \tfrac{1}{8}d \qquad b = \tfrac{1}{8} - \tfrac{1}{4}f + \tfrac{1}{8}\varphi = \tfrac{1}{8}d \qquad c = \tfrac{1}{2}(f - \varphi) \\ d = 1 - 2f + \varphi \qquad e = \tfrac{1}{4} + \tfrac{1}{2}f + \tfrac{1}{4}\varphi = f + \tfrac{1}{4}d \qquad g = \tfrac{1}{2} + \tfrac{1}{2}f - \varphi \\ f = (\sin^3 \beta_2 - \sin^3 \beta_1)/3(\sin \beta_2 - \sin \beta_1) \qquad \varphi = (\sin^5 \beta_2 - \sin^5 \beta_1)/5(\sin \beta_2 - \sin \beta_1) \end{cases}$$

It will be noticed that, if the velocity-figure be the same everywhere, $\overline{xx} + \overline{yy} + \overline{zz}$ should be equal for all zones; so that this process affords a means of equalizing the effect of distance. The relative weights of the various quantities are very unequal; $\overline{zz}$, and in general, all quantities involving z being weak in the polar zones.

8. *The equivalent ellipsoid.* It remains to point out the relation between the parameters found by this method and those resulting from some other methods. If the velocities in each of the three principal directions be distributed according to the error-law, as supposed in SCHWARZSCHILD's hypothesis somewhat generalized, the number of points of the velocity-figure in a differential volume du, dv, dw at the point u, v, w (origin at the center of gravity)

$$(19)\qquad \begin{gathered} Ke^{-F(u,v,w)}\,du\,dv\,dw \\ F(u,v,w) = u^2/\alpha^2 + v^2/\beta^2 + w^2/\gamma^2 \end{gathered}$$

In this the mean square components λ and the mean components $\bar{u}$, $\bar{v}$, $\bar{w}$ (without regard to sign) are readily found to be

$$(20)\qquad \begin{gathered} \lambda_1 = \tfrac{1}{2}\alpha^2 \qquad \lambda_2 = \tfrac{1}{2}\beta^2 \qquad \lambda_3 = \tfrac{1}{2}\gamma^2 \\ \bar{u} = \sqrt{\frac{2\lambda_1}{\pi}} \qquad \bar{v} = \sqrt{\frac{2\lambda_2}{\pi}} \qquad \bar{w} = \sqrt{\frac{2\lambda_3}{\pi}} \end{gathered}$$

On the two stream theory, the same simple relation holds between $\bar{v}$ and λ_2, $\bar{w}$ and λ_3. In the preferred direction we would have, in EDDINGTON's notation,

$$(21)\qquad \begin{gathered} \lambda_1 = \frac{1}{2h^2} + n_1V_1^2 + n_2V_2^2 = \frac{1}{2h^2} - V_1V_2 \\ n_1 + n_2 = 1 \qquad n_1V_1 + n_2V_2 = 0 \end{gathered}$$

where n_1 and n_2 are the proportional numbers of stars in the two streams, V_1 and V_2 their stream-velocities relative to their common center of gravity, while h is the reciprocal of the radius of either sphere of the "dumb-bell," and corresponds in a way to $1/\beta$ of the ellipsoidal theory. $\bar{u}$ comes out in exponential form, and can be readily evaluated in any particular case. However, as this requires that $\lambda_2 = \lambda_3$, and in the present paper these are found to be unequal, it is hardly worth while to elaborate the comparison.

9. *Material and order of computation.* The following results are based upon 5943 proper-motions of less than 80″ per century from the *Preliminary General Catalogue* of Prof. LEWIS BOSS. They were divided into two groups, "S" of less than 20″, "L" of more than 20″ per century. S was subdivided according to type in 7 classes. The sky had been divided into 108 nearly equal areas in 9 zones parallel to the galaxy, and the proper-motions, with the coefficients for computing solar motion by AIRY's method tabulated, in the course of the researches by the author of the catalogue. As the AIRY coefficients are the negative of those of equations (10), these tabulations were directly available.

x_1, y_1, z_1, were computed by (10), their squares and products formed, and the means taken in each of the 108 trapeziums, both for the type-groups separately and for the whole. This is by far the most laborious part of the work, but it is a matter of mere routine. It would have been possible to avoid a considerable part of this work by forming the means of μ, μ', $\mu\mu$, $\mu'\mu'$, and $\mu\mu'$ by areas, and taking the coefficients in (10) for the center of each area. (10) would then be modified by affecting, x_1, y_1, z_1, μ, μ', by the sign for the mean (dropping R) while the forms for the squares and products are obvious. But this makes no allowance for curvature or unequal distribution within an area, and makes a large saving of labor only when there are a good number of stars in a trapezium. I therefore thought it unwise to use it in this first trial of the method. The means of the means-by-areas were then taken for the whole sky, weighting each trapezium in proportion to its area. The second column of Table I gives the result for group "S."

The means were then corrected for perspective by (12), as shown in the fourth column of Table I. The corrections for solar motion were applied by (7) and the solar motion found by (8). The results appear in the sixth column.

The determinant (5′) in expanded form may be put

$$\lambda^3 + a\lambda^2 + b\lambda + c = 0$$

where

$$\begin{aligned} a &= -(A + B + C) \\ b &= AB + AC + BC - D^2 - E^2 - F^2 \\ c &= -(ABC + 2DEF) + AF^2 + BE^2 + CD^2 \end{aligned}$$

It will be noticed, however, that if any constant be added to A, B, and C, the same constant will be added to each of the roots. In particular, if we add $-B$,

the formation of the cubic will be simplified, and one of the roots made small. The reduced cubic was

$$x^3 - 13.640\,x^2 - 49.658\,x + 63.077 = 0$$

Its roots were $+16.429$, $+1.010$, -3.800; and the corresponding roots of (5′)

$$\lambda_1 = 29.813 \qquad \lambda_2 = 14.394 \qquad \lambda_3 = 9.584$$

The direction cosines of the three principal axes could now be found by (4); but their spherical co-ordinates may be found directly, thus: Put $A - \lambda_1 = A_1$, $B - \lambda_1 = B_1$, $C - \lambda_1 = C_1$; $A_1B_1 - D^2 = a_1$, $A_1C_1 - E^2 = b_1$, $B_1C_1 - F^2 = c_1$, $A_1F - DE = -d_1$, $B_1E - DF = -e_1$, $C_1D - EF = -f_1$; then

$$\left.\begin{aligned} \tan A_1 &= b_1/f_1 = d_1/e_1 = f_1/c_1 \\ \tan D_1/\cos A_1 &= a_1/e_1 = d_1/f_1 = e_1/c_1 \\ \tan D_1/\sin A_1 &= a_1/d_1 = d_1/b_1 = e_1/f_1 \end{aligned}\right\} \tag{22}$$

Similarly for the other two axes. The agreement of the several values of A and D checks the solution of the cubic (5′). The three poles are given in the eighth column of Table I.

TABLE I — 5384 Stars, p.m. < 20″. First Approx.

$\bar{x}_1$	$-\ .0030$	$\bar{x}$	$-\ .0045$	M	3″.83	λ_1	29.81
$\bar{y}_1$	$+\ 2.425$	$\bar{y}$	$+\ 3.187$	A	270°.4	λ_2	14.39
$\bar{z}_1$	$-\ 1.413$	$\bar{z}$	$-\ 2.119$	D	+33 .6	λ_3	9.58
$\overline{x_1x_1}$	9.592	$\overline{xx}$	10.779	A	10 .779	A_1	92°.4
$\overline{y_1y_1}$	23.129	$\overline{yy}$	39.786	B	29 .629	D_1	+ 5 .2
$\overline{z_1z_1}$	12.903	$\overline{zz}$	17.874	C	13 .884	A_2	353 .2
$\overline{x_1y_1}$	$-\ 0.464$	$\overline{xy}$	$-\ 0.994$	D	− 0 .980	D_2	+ 60 .4
$\overline{x_1z_1}$	$+\ 0.927$	$\overline{xz}$	$+\ 1.986$	E	+ 1 .976	A_3	185 .3
$\overline{y_1z_1}$	$-\ 2.418$	$\overline{yz}$	$-\ 5.180$	F	+ 1 .573	D_3	+ 29 .1

A_3, D_3 lies so near the galactic pole (190°.33, + 27°.35) that the difference may be neglected, and the correction for parallax was accordingly made by (14), (15), and (16). The means were then taken as before, the second approximation following the same order as the first. The results appear in Tables I and III. A third approximation seemed unnecessary.

TABLE II — SOLAR MOTION

Group	No. Stars	A	D	M
B $(Oe_5 - B_5)$	492	274°.2	+39°.3	2″.33
A $(B_8 - A_4)$	1647	266 .7	+27 .9	4 .03
F $(A_5 - F_*)$	656	262 .9	+27 .0	6 .36
G	446	256 .6	+45 .8	2 .60
K	1227	272 .5	+38 .4	4 .02
M	223	272 .2	+45 .7	3 .45
X	693	271 .4	+42 .1	3 .06
S(p.m. < 20″)	5384	269 .0	+32 .5	3 .83
L(20″ − 80″)	559	269 .3	+31 .5	22 .17
All (p.m. < 80″)	5943	269 .1	+32 .3	5 .42

10. *Numerical results.* The apices of solar motion as found for the different groups are given in Table II. The seventh group, here called "X," consists of a few stars of types N and O, and those faint stars whose type is unknown. The latter are generally believed to belong largely to "late" types, and their solar and systematic motions resemble those of "late" rather than "early" type.

Table III gives the elements of systematic motion for the same groups. The R.A. and Decl. of the three principal axes are followed by the mean square motions in the three directions. The last three columns give the mean component in each direction, as found by equations (20), divided through by the solar motion of the group in order as far as possible to eliminate the mean parallax of the group. These equations are valid for the velocities only on the ellipsoidal hypothesis, but are approximately true for the proper-motions.

Table IV gives the result of special solutions. In the first two lines are shown the result of dividing the material of the 5384 stars along the galaxy. The differences are rather smaller than might have been

expected. The other pairs of solutions result from a division of these, and of Types *A* and *K*, into Galactic and Non-Galactic stars, the former lying between Galactic latitudes +30° and −30°. The solutions had to be made by the method of § 7. The other groups are rather small to divide. No very certain trend can be found in these results. Those for the non-galactic regions are especially liable to error due to defects in the fundamental hypotheses. A solution of the group "*S*" by the "alternative procedure" of § 5 is also given for comparison with Table I.

TABLE III SYSTEMATIC MOTION

Group	Pole of Preference		Intermediate Pole		Pole of Avoidance		Mean Square Motions			Mean Motions in Terms of M		
	A_1	D_1	A_2	D_2	A_3	D_3	λ_1	λ_2	λ_3	u/M	v/M	w/M
B	100.6	−38.1	75.6	+48.7	180.4	+12.6	4.58	1.94	1.33	0.73	0.47	0.40
A	92.4	− 1.8	355.7	+74.7	182.9	+15.2	25.65	9.15	2.57	1.00	0.60	0.32
F	93.1	− 2.1	1.0	+44.0	185.2	+45.9	58.05	28.78	22.66	0.96	0.67	0.60
G	93.8	− 3.1	356.3	+65.4	185.4	+24.4	32.26	15.81	9.96	1.74	1.22	0.97
K	93.1	+11.3	348.1	+52.4	191.2	+35.3	29.30	18.71	11.46	1.08	0.86	0.67
M	94.0	+25.3	338.6	+42.2	205.1	+37.2	28.14	12.58	3.48	1.23	0.82	0.43
N	83.9	+30.2	317.2	+15.8	192.6	+29.0	21.08	12.72	5.33	1.20	0.93	0.60
S	91.9	− 4.2	352.9	+64.7	183.8	+24.9	28.91	14.99	9.89	1.12	0.81	0.66
L	92.5	+35.3	331.6	+36.0	211.7	+34.5	745.6	261.4	194.7	1.01	0.61	0.52
All	92.3	+16.9	299.4	+71.2	184.7	+ 8.1	107.3	58.7	34.8	1.53	1.13	0.83

TABLE IV SPECIAL SOLUTIONS

Group	Region	A	D	M	A_1	D_1	A_2	D_2	A_3	D_3	λ_1	λ_2	λ_3
20″	North	270°	+33	3″.8	93°	+ 6	348°	+67°	185°	+22	26.7	13.7	9.4
	South	268	+32	3.6	94	− 1	359	+64	182	+28	27.5	14.5	9.0
20″	Gal.	270	+31	4.0	93	− 1	356	+59	189	+28	27.9	15.2	9.1
	Non-g.	270	+34	3.1	83	2	2	+75	173	−15	28.3	13.3	8.2
A	Gal.	270	+28	4.0	94	6	25	+71	182	+16	23.7	9.7	3.5
	Non-g.	262	+29	3.9	94	− 6	348	+68	186	+21	22.8	8.1	0.2
K	Gal.	272	+30	3.8	95	− 2	2	+46	187	+41	31.2	17.2	10.7
	Non-g.	284	+36	3.7	90	+15	297	+72	182	+ 8	26.2	17.6	11.4
S	Alternative Procedure				93	− 1	354	+61	185	+26	29.0	14.2	9.2

11. *The Apices of solar motion* for the separate types show the division into two groups already found by several investigators; *A* and *F* have apices in smaller R.A. and Decl. (smaller galactic longitude) than *K*, *M*, and *N*. *B* shows more affinity to the latter group than to the former in this regard. *G* is anomalous.

The *M*'s vary considerably more than would correspond to the differences in the positions of the apex, but this seems to be for the most part the effect of unequal mean parallax. *G*, however, appears to have a small *M* (see last three columns of Table III) perhaps due to the presence of a "solar" group whose motions agree with that of the *Sun*. This type is certainly less homogeneous than the generality of the others in several ways. Although it has a large number of near stars it has a small mean parallax, indicating great range in distance.

If Table II be compared with L. Boss's determinations by Airy's method in *A. J.* 614, 623-4, and with those by Schwarzschild's method in *A. J.* 676, all three being based on practically the same material, the agreement will be found close. The separation into two groups, *B* agreeing more nearly with the "late" types, and the anomalous position of the *G* apex, are common to all.

12. *Systematic Motion.* All groups show unequal motions in the three principal directions. The poles of avoidance cluster about the Galactic pole, mostly south preceding and north following; the remotest is about 20° away. The axes of preference, except for *B* type, lie near the Galactic plane and not far from its nodes on the equator.

The separation of the vertices or poles of preference into two groups pointed out in *A. J.* 676, the "late" types having their 6-hour vertex decidedly to the north of "early" types, is confirmed. There seems to be no good reason to doubt the reality of this phenomenon. The "early" type vertices are compactly grouped, those of "late" type show progression; but the weakness of some of the groups precludes emphasis on this point. Similar relations appear in the positions of the other two poles, but are less definite.

The mean square motions and mean motions shown in the last six columns of Table III show some interesting trends. The velocity-figure for *A* is both elongated and compressed. With advance of type the preferential motion shows comparatively little increase*, most of the increase of proper-motion being due to the other two components. The intermediate axis seems to increase rather steadily. The motion along the least axis increases abruptly from *A* to *F* and then remains practically constant; indeed, there is some indication of a maximum in the region *F* to *K*, but this depends chiefly on the drop in the weak group *M*. In the article in *A. J.* 676 already cited, the ratios *A*/*V*, *B*/*V*, of Table I correspond to π' times u/M, $\bar{v}/M$ (or $\bar{w}/M$) and show the same general trends.

With these results may be compared those of Gyllenberg (Medd. fran Lunds Ast. Obs., Nr. 59, and Serie II, Nr. 13), who found from radial velocities of 1526 stars the principal poles (84°.4, +5°.2), (177°.1, +29°.9), (336°.5, +59°.5), with similar results from sub-divisions of the material, including even the *B* and *A* types. These are nearly the same as those of Table III, but in the order 1, 3, 2. One would expect any changes in the *v* and *w* components to result in shifting the corresponding axes through some other angle than 90°; that the poles should differ so much in the two investigations, and that the difference should be so approximately of the nature of an interchange of axes, is remarkable.

Eddington and Hartley (*Monthly Notices* R.A.S., LXXV, 521) obtained results from radial velocities more like mine for "early" types and like Gyllenberg's for "late" types.

13. *The large proper-motion stars.* Insofar as the stars of large proper-motion are such merely because of large parallax, their $\bar{u}/M$, etc., should be the same for other stars; but insofar as they are really moving at high speeds, these quantities should be large. These quantities are small for the group "*L*," but large for the group "*All*," which is the combination of the two preceding groups in the table. This seems to call for explanation.

If we plot the projected velocity-figure for each area we get diagrams similar to those published by B. Boss in *A. J.* 635-636. Now the 5384 stars of the group "*S*" are included in a circle of radius 20″ and eccentricity 3″.3 sin Δ. (See *A. J.* 612, p. 98). This approximately allows for solar motion within the circle, but not for stars outside it, whose parallactic center comes, for Δ = 90°, close to the edge of the circle. The circle is therefore, for these large motions, too close to that end of the velocity-figure which is nearest the direction of the apex. In some cases the grouping has very few stars on that end or at the sides of the figure. The effect tends toward that of excluding all stars lying apexward from a line through the parallactic center at right angles to the

* *i. e.*, relative to *M*. If *M* increases, as shown by Campbell, *Lick Obs. Bull.* 196, then the velocity-figure increases more with advance of type than is indicated in Table III.

preferential motion. If this tendency were complete λ would be one-fourth and α one-half their proper values. Of course it is not complete anywhere, and is *nil* about the apex and antapex; but it suffices to reduce the axes, and especially the greater one, by considerable fractions. The effect on the stars *p.m.* < 20″ is imperceptible.

11. *The B type stars* show such anomalies of distribution that they were divided into galactic and non-galactic stars, corrected for perspective according to §7, the two parts being then combined by weights according to the number of stars (respectively 148 and 114) in each. Adjoining areas were combined freely to give enough stars to work with, but not opposite areas, except in the extra-galactic zones, where it could not perceptibly affect the result. The solar motion was eliminated at the beginning by the "alternative procedure" of §5. In view of the smallness of the peculiar motions relative to solar motion, this seemed less likely to introduce anomalies through lack of consistency between the parallax factors for the first and second order terms.

The vertex obtained lies near the antapex. This result persists in spite of rearrangement of the areas and other changes in the procedure, and seems to be inherent in the material. The displacement of the vertex is in the nature of an extrapolation of the difference already found between A, F, G and "later" types, but is much larger than would naturally be expected.

It will not do to say, in the language of the two-stream hypothesis, that the B stars belong to stream I only. This, if true, would result in a spherical velocity-figure and move the apex toward the anti-vertex of stream I. Such is not the case. The vertex is moved, not the apex.

The tendency may be due in part to the small size of the peculiar motions relative to solar motion, together with great range in distance, producing a false elongation of the velocity-figure. The first of these causes is present, the second may be. The magnitude of such an effect might be estimated, but to remove it we need to know the individual parallaxes.

If this were the whole explanation there should be signs of a similar effect among the other types; and the question arises, — ignoring for the moment the results found by SCHWARZSCHILD's method, — whether the separation of the vertices into two groups may not be due to such a cause, the "early" types having smaller motions than the "late". If so, there should be a correlation between the last three columns of Table III and D_1, small values of $\bar{u}$ M, etc., going with southerly values of D_1. No such relation appears. Further, such an effect depends also on range in distance. There are good reasons for believing that G has great range in distance, and the group "All" certainly has, yet they show no certain trace of such an effect. Finally, the effectiveness of the suggested causes in shifting the vertex is greater the more nearly spherical the velocity-figure. The "late" types have velocity-figures more nearly spherical than "early" types, yet their vertices are farthest from that of B. Table III seems, then, except for type B, to be essentially free from systematic error of this sort.

ENCKE'S COMET.

A cablegram received from Copenhagen states that ENCKE's Comet was observed by WOLF, at Heidelberg, September 11.5531 G. M. T., in R. A. $20^h\ 5^m\ 36^s$, Dec. $+13°\ 16'$. The comet was of 12th magnitude.

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VOL. XXX ALBANY, N.Y., 1917, SEPTEMBER 29 NO. 23

OBSERVATIONS OF ASTEROIDS,

MADE WITH THE PHOTOGRAPHIC TELESCOPE OF THE U. S. NAVAL OBSERVATORY,

By GEORGE H. PETERS.

[Communicated by Rear Admiral T. B. Howard, U. S. N., Superintendent.]

Name	Mag.	Date 1916	G. M. T.	α 1916.0	δ 1916.0
			h m	h m s	° ′ ″
(714) *[1911 L W]*	11.5	Aug. 2	15 49.0	20 12 23.3	+ 5 10 05
(572) *Rebekka* . . .	12.1	Sept. 17	16 17.5	22 03 59.3	− 1 01 09
(572) *Rebekka* . . .	12.4	Sept. 19	13 49.8	22 03 03.5	− 1 22 49
(441) *Bathilde* . . .	12.6	Sept. 19	14 45.5	22 41 48.2	+ 5 04 38
(219) *Thusnelda* . .	9.7	Sept. 19	15 36.5	23 00 06.1	+ 5 38 55
(374) *Burgundia* .	11.8	Sept. 19	15 36.5	23 03 34.8	+ 5 05 52
(371) *Bohemia* .	11.8	Sept. 19	16 28.5	23 11 07.1	+ 6 57 02
(407) *Arachne* . . .	11.5	Sept. 19	16 28.5	23 20 19.8	+ 7 43 57
(441) *Bathilde* . . .	12.6	Sept. 20	13 36.0	22 41 07.7	+ 4 58 54
(219) *Thusnelda* .	9.7	Sept. 20	14 32.5	22 59 39.1	+ 5 24 35
(374) *Burgundia* .	11.8	Sept. 20	14 32.5	23 02 54.0	+ 4 58 40
(371) *Bohemia* .	11.8	Sept. 20	15 23.0	23 13 20.4	+ 6 52 19
(407) *Arachne* . .	11.5	Sept. 20	15 23.0	23 19 29.7	+ 7 39 54
(133) *Cyrene*	11.4	Sept. 20	16 27.5	23 22 45.5	+ 2 15 07
(133) *Cyrene* . .	11.4	Sept. 21	16 01.5	23 22 00.1	+ 2 11 32
(32) *Pomona* .	11.0	Sept. 23	16 11.6	23 49 58.3	+ 5 00 31
(78) *Diana*	11.1	Sept. 23	16 11.6	23 52 56.0	+ 5 52 55
(241) *Germania* .	10.6	Sept. 23	16 57.7	0 05 22.5	+ 9 59 48
(32) *Pomona* . . .	11.1	Sept. 25	15 29.0	23 48 22.2	+ 4 16 41
(78) *Diana*	11.0	Sept. 25	15 29.0	23 51 04.3	+ 5 46 07
(303) *Josephina* .	11.7	Sept. 25	16 27.5	0 11 12.2	+ 4 52 58
(241) *Germania* . .	10.6	Sept. 26	15 58.1	0 03 07.1	+ 9 15 05
(303) *Josephina* . .	11.7	Sept. 26	16 53.0	0 10 22.9	+ 4 49 54
(305) *Gordonia* . . .	12.2	Oct. 17	15 03.5	0 27 38.9	+ 5 07 31
(268) *Adorea*	13.2	Oct. 17	16 27.0	1 15 41.1	+ 4 24 17
(401) *Ottilia*	12.8	Oct. 17	16 27.0	1 16 44.3	+ 5 16 30
(723) *Hammonia*	13.0	Oct. 17	16 27.0	1 22 52.2	+ 3 39 24
(305) *Gordonia* . . .	12.2	Oct. 21	15 05.8	0 25 91.6	+ 4 44 00
(268) *Adorea* . . .	13.2	Oct. 22	16 16.6	1 12 04.6	+ 4 02 46
(401) *Ottilia* . .	12.8	Oct. 22	16 16.6	1 13 95.7	+ 5 02 01
(723) *Hammonia*	13.0	Oct. 22	16 16.6	1 19 09.4	+ 3 11 15
(334) *Chicago* . . .	12.0	Oct. 24	15 43.4	1 53 57.1	+ 5 19 01

No.		Mag.	Date 1916	G. M. T.	α 1916.0	δ 1916.0
				h m	h m s	° ′ ″
245	*Vera*	11.3	Oct. 24	15 43.1	1 57 00.3	+ 7 03 41
742	*1913 QU*	12.0	Oct. 24	16 12.3	2 31 24.0	+ 5 39 29
334	*Chicago*	12.0	Oct. 26	16 01.1	1 52 41.1	+ 5 11 55
245	*Vera*	11.3	Oct. 26	16 01.1	1 55 19.9	+ 6 59 34
742	*1913 QU*	12.0	Oct. 26	17 12.9	2 29 35.0	+ 5 37 39
334	*Chicago*	12.0	Oct. 27	15 11.4	1 52 05.5	+ 5 08 31
245	*Vera*	11.3	Oct. 27	15 11.1	1 54 32.0	+ 6 57 37
257	*Silesia*	12.1	Oct. 27	16 28.1	2 39 25.2	+15 57 04
257	*Silesia*	12.1	Oct. 28	15 43.9	2 38 36.5	+15 54 43
334	*Chicago*	12.0	Nov. 1	14 43.9	1 49 02.2	+ 4 52 01
245	*Vera*	11.3	Nov. 1	14 43.9	1 50 29.7	+ 6 48 55
145	*Adeona*	11.0	Nov. 1	17 14.1	3 04 05.4	+ 4 56 41
538	*Friederike*	12.1	Nov. 1	17 14.1	3 09 29.5	+ 7 11 27
145	*Adeona*	11.0	Nov. 2	16 44.0	3 03 07.6	+ 4 56 11
538	*Friederike*	12.1	Nov. 2	16 44.0	3 08 43.6	+ 7 07 29
334	*Chicago*	12.0	Nov. 3	14 59.0	1 47 50.2	+ 4 45 44
245	*Vera*	11.3	Nov. 3	14 59.0	1 48 55.7	+ 6 46 03
785	*1914 UN*	13.6	Nov. 3	16 11.0	2 43 13.9	+ 5 26 22
245	*Vera*	11.3	Nov. 15	14 54.5	1 40 53.1	+ 6 38 12
245	*Vera*	11.3	Nov. 16	14 13.0	1 40 21.1	+ 6 38 20
785	*1914 UN*	13.6	Nov. 16	16 34.0	2 31 19.2	+ 5 11 36
378	*Holmia*	11.9	Nov. 18	14 37.1	3 05 32.3	+17 40 21
131	*Vala*	12.6	Nov. 18	15 42.2	2 53 29.7	+14 16 50
245	*Vera*	11.3	Nov. 25	13 17.3	1 36 31.7	+ 6 46 07
378	*Holmia*	11.9	Nov. 25	14 45.8	3 00 00	+16 52 33
245	*Vera*	11.3	Nov. 27	13 14.5	1 35 57.6	+ 6 49 25
308	*Polyxo*	11.1	Nov. 27	14 37.5	3 53 05.4	+14 23 06
60	*Echo*	10.1	Nov. 27	14 37.5	3 54 51.7	+14 48 42
102	*Miriam*	11.7	Nov. 27	15 18.5	4 19 30.2	+15 40 48
308	*Polyxo*	11.1	Dec. 1	14 37.5	3 49 32.7	+14 10 46
60	*Echo*	10.1	Dec. 1	14 37.5	3 51 01.2	+14 31 17
102	*Miriam*	11.7	Dec. 1	15 23.6	4 16 05.6	+15 40 50
17	*Thetis*	10.8	Dec. 2	17 02.0	4 38 08.8	+14 58 07
335	*Roberta*	12.3	Dec. 2	17 02.0	4 42 23.4	+14 31 45
245	*Vera*	11.3	Dec. 7	11 18.3	1 34 45.0	+ 7 14 22
245	*Vera*	11.3	Dec. 8	11 18.3	1 34 46.7	+ 7 17 41
17	*Thetis*	10.8	Dec. 22	14 40.0	4 19 11.5	+14 51 09
335	*Roberta*	12.3	Dec. 22	14 40.0	4 23 47.5	+14 15 20
532	*Herculina*	9.7	Dec. 22	15 28.0	5 28 29.5	+13 17 07
532	*Herculina*	9.7	Dec. 28	15 57.4	5 22 33.8	+13 47 31
532	*Herculina*	9.7	Dec. 29	14 24.5	5 21 41.4	+13 52 24
409	*Aspasia*	10.9	Dec. 29	16 06.5	6 41 53.1	+12 18 29
409	*Aspasia*	10.9	Dec. 30	16 18.1	6 40 51.5	+12 16 10

245 *Vera* was so far off the B. J. ephemeris that an extended series of observations was secured for the computation of new elements. 538 *Friederike* is probably correctly identified; the daily motion is right though it was considerably off the B. J. ephemeris. It was not on the ephemeris place on plates subsequently taken as a check. As previously stated in this *Journal*, photographic observations of asteroids at the Naval Observatory are made in the following zones:

+ 4° to + 8°, March, April, September, October;
0° to + 4°, May to August;
+12° to +16°, November to February.

SUNSPOT OBSERVATIONS,

MADE AT BERWYN, PENN., WITH A $4\frac{1}{2}$-INCH REFRACTOR,

By A. W. QUIMBY.

1917	Time	New Grs.	Total Grs.	Total Spots	Fac. Grs.	Def.
Jan. 1	8	–	7	20	1	fair
2	2	–	5	27	1	fair
3	4	–	3	20		poor
4	8	–	3	20	–	poor
6	8	1	3	12	2	poor
7	8	–	3	10	1	poor
8	3	3	6	22	2	poor
9	8	–	5	27	2	poor
10	9	–	3	20	1	poor
11	8	2	7	23	1	poor
12	8	–	6	27	3	fair
13	10	–	2	6	–	poor
14	8	–	2	6	–	poor
15	12	–	4	10	–	poor
16	4	–	4	10	3	fair
17	8	–	4	8	2	poor
18	12	2	5	19	5	fair
19	8	–	5	9	3	fair
20	8	–	3	4	2	poor
22	10	–	3	6		poor
23	9	–	3	11	–	poor
24	11	–	3	10	1	poor
25	3	3	6	16	2	fair
26	4	2	7	17	2	fair
27	8	–	4	7		poor
28	9	3	8	16	4	fair
29	8	–	2	2	–	poor
30	4	2	6	10	4	fair
31	8	–	3	3	–	poor
Feb. 1	11	–	3	5	3	fair
2	9	–	3	4	3	fair
3	4	1	5	13	2	fair
4	8	–	3	8	1	poor
5	9	–	2	4	–	poor
6	11	2	6	26	3	fair
7	9	1	7	50	3	fair
8	8	–	4	45	2	poor
9	2	1	7	60	3	fair
10	8	–	7	60	4	fair
11	8	–	7	58	3	fair
12	8	–	7	61	3	fair
13	8	–	7	42	3	fair
14	8	–	5	38	4	fair
15	8	–	4	32	3	fair
16	10	–	3	30	3	fair
Feb. 17	8		3	30	2	fair
18	2	1	4	31	1	fair
19	8	–	4	22	2	fair
20	4	–	3	13	2	fair
21	8	–	2	7	1	poor
22	10	–	2	1	1	fair
23	9	–	1	1	–	fair
24	4	1	1	1	3	fair
25	8	–	1	2	2	fair
26	8	–	1	1	–	poor
27	4	3	4	6	3	fair
Mar. 2	8	1	4	12	1	fair
5	3	3	7	44	2	fair
6	8	–	6	51	2	fair
7	8	–	5	72	3	fair
8	4	–	4	53	2	fair
9	8		4	52	3	fair
10	5	1	5	36	3	fair
11	9	–	5	24	2	fair
12	4	–	7	24	4	fair
13	8	–	6	26	3	fair
15	8	–	6	24	5	fair
16	8	–	4	28	5	fair
18	8	–	4	15	3	fair
19	3	1	4	16	4	poor
20	8	3	7	18	3	fair
22	4	–	4	36	2	fair
23	7	1	3	21	1	fair
24	8	2	4	27	3	fair
25	5	2	6	61	3	fair
26	4	–	6	23	3	fair
28	7	–	5	27	2	fair
29	4	–	5	26	2	fair
30	7	1	6	24	2	fair
31	7	–	5	26	1	fair
Apr. 1	7	–	3	18	1	fair
2	7	–	1	4	–	poor
3	11	–	1	3	1	fair
4	7	–	1	2	1	fair
5	9	–	–	–	–	v. p.
6	8	–	–	–	–	v. p.
7	4	1	1	3	2	fair
8	8	–	–	–	1	fair
9	8	2	2	2	4	fair
10	8	–	1	1	2	fair
Apr. 1	5	4	5	50	3	fair
12	3		1	40	–	poor
13	9	3	7	60	4	fair
14	4	–	7	62	3	fair
15	6	–	7	55	3	fair
16	6	1	8	62	4	fair
17	6	–	8	55	3	fair
18	5	1	8	75	4	fair
19	8	1	9	24	3	poor
20	9	–	8	27		fair
21	16	–	5	27	3	fair
22	8	–	5	27	3	fair
23	7	1	5	68	3	fair
24	5	–	5	22	3	poor
25	6	2	7	42	3	fair
27	6	1	4	30	2	fair
28	7	–	3	16	3	poor
29	7	–	3	13	2	poor
30	7	2	5	27	2	poor
May 1	7	–	5	20	2	poor
2	7	–	5	27	5	fair
3	7	1	4	43	3	fair
4	7	–	2	20	–	poor
6	9	–	2	33	1	poor
7	7	2	4	12	2	poor
8	10	1	5	20	2	fair
9	6	2	7	23	4	good
10	6	–	4	13	2	fair
11	6	1	5	27	3	good
12	5	–	5	33	2	good
13	6	–	5	27	3	fair
14	7	2	7	35	3	good
15	6	–	7	50	3	good
16	6	–	6	45	3	fair
17	8	1	8	78	3	good
18	7	–	8	76	3	good
19	7	–	7	106	3	good
20	5	–	4	68	3	fair
21	7	–	4	80	2	fair
22	7	1	4	67	2	fair
23	2	4	8	52	1	fair
24	7	2	10	70	4	fair
25	9	1	8	62	4	fair
26	6	1	8	68	3	fair
27	7	–	8	66	3	poor

SUNSPOT OBSERVATIONS (Concluded)

1917	Time	New Grs	Total Grs	Total Spots	Fac Grs	Def.	1917	Time	New Grs	Total Grs	Total Spots	Fac Grs	Def.	1917	Time	New Grs	Total Grs	Total Spots	Fac Grs	Def.
May 29	7		8	108	1	fair	June 9	6	2	8	41	5	fair	June 20	6	2	10	48	5	fair
30	7		8	69	2	fair	10	9	2	9	40	4	fair	21	6		6	22	1	fair
31	6	1	8	69	3	fair	11	10		8	30	2	poor	22	6		5	44	4	fair
June 1	2		8	57	2	fair	12	6		8	33	3	poor	23	6	1	6	37	4	fair
2	6	1	9	53	3	poor	13	6		7	63	3	fair	24	6		6	27	4	fair
3	6	1	8	40	3	fair	14	8		6	39	2	poor	25	5	2	8	57	4	fair
4	4	1	7	12	4	fair	15	7	1	6	42	2	fair	26	5	1	9	63	4	fair
5	6	1	7	10	3	fair	16	6	1	7	53	2	fair	27	6		8	42	3	fair
6	7		6	15	4	fair	17	6		7	60	3	fair	28	5		8	44	3	fair
7	11		5	20	3	fair	18	6		7	51	3	fair	29	6		8	37	3	fair
8	6	2	7	31	4	fair	19	6	2	10	62	5	fair	30	6		7	23	4	fair

EPHEMERIS OF THE ASTEROID (471) *PAPAGENA*

By GUSTAF STRÖMBERG

Greenwich Mean Noon.

1917	α App.	δ App.	Log r	Log Δ	m
	h m s	° ′ ″			
Oct. 1	3 58 15.5	+3 11 25	0.3439	0.1618	8.7
5	58 46.8	10 12			
9	58 18.8	9 26	0.3440	0.1420	
13	57 20.2	9 24			
17	55 51.0	10 20	0.3442	0.1249	
21	53 51.8	12 30			
25	51 24.0	16 18	0.3446	0.1110	
29	48 29.9	22 7			
Nov. 2	45 12.7	30 10	0.3452	0.1015	
6	41 36.5	40 36			
10	37 46.1	53 38	0.3460	0.0969	8.4
14	33 47.8	+4 9 21			
18	29 46.5	27 58	0.3470	0.0980	
22	25 48.1	49 19			
26	21 58.1	+5 13 21	0.3481	0.1048	
30	18 21.4	40 15			
Dec. 4	15 2.1	+6 9 31	0.3494	0.1169	
8	12 3.9	40 52			
12	9 30.7	+7 11 12	0.3508	0.1335	
16	7 25.6	49 25			
20	5 50.5	+8 26 13	0.3524	0.1537	
24	4 46.1	+9 4 20			
28	3 4 11.5	+9 43 31	0.3541	0.1761	8.8

The principal terms of the general perturbations of *Jupiter* and *Saturn* are applied and computed from my table in *Astron. Nachr.* 4663. Owing to the exceptional proximity to the *Earth*, the planet is favorable for observation.

Mount Wilson Solar Observatory, 1917, June.

A LIST OF STARS WITH SMALL PROPER-MOTIONS AND LARGE RADIAL VELOCITIES FOR WHICH PARALLAXES ARE DESIRABLE.

By C. D. PERRINE.

In a recent investigation* it was found that if a small proportion of stars having small proper-motions and relatively large radial velocities was rejected, there was strong evidence that the preference for motion in the direction of the ellipsoidal axis was in general confined to the nearer stars. The same criterion indicated that when the sizes of the proper-motions were taken into account there was neither spectral class nor magnitude progression in the radial velocities.

It becomes, therefore, of importance to know whether such stars are distant, as their small proper-motions indicate, or whether they are in fact relatively near, and to be classed with the near stars. A knowledge of the parallactic displacements of these stars would seem to furnish a valuable test of the conclusions referred to above. To this end a list of such stars, 47 from the L. O. and 24 from the Mount Wilson catalogs, was made and is given below in the hope that astronomers engaged in parallax work will put as many of them on their programs as possible. No arbitrary limits were assumed either for proper-motion or velocity on account of the desire for as uniform a distribution among the different spectral types as possible.

It is of interest to note that in the 47 of these stars taken from the L. O. catalogs (the Mount Wilson catalogs being unsymmetrical as to distribution in the sky) 62% are within 20° of the galactic plane and 70% are contained in two regions each about 8000° in area, one of which is not far from the 18^h ellipsoidal vertex, the other from the *antapex* of solar motion. It seems reasonable to conclude, therefore, that, both in distribution and motion, these stars as a whole show a decided preference for the galactic plane and those, at least, which are in the 18^h region, for the ellipsoidal axis.

It is worthy of remark that these stars show no preference for the region of the usually accepted *apex* of solar motion and a preference for a region differing somewhat from the *antapex*. Their small parallactic displacements particularly of the stars grouped between 16^h and 23^h cannot be due primarily, therefore, to solar motion toward the apex at 18^h, $+30°$. It seems probable that these peculiarities are manifestations of "two-stream" effects. It is not impossible that distance will be found not to be the only criterion, but that the region of sky may be equally concerned.

The numbers given are those of the stars in Boss' *Preliminary General Catalog*. v is the star's velocity after correcting for a solar motion of 20 km. toward the apex at 18^h, $+30°$.

* *Astrophysical Journal.*

P. G. C. No.	Mag.	Type	α 1900.0	δ 1900.0	μ	v
			h m	° ′	″	km
315	5.3	*G2*	1 20.2	−42 1	.033	+64
467	5.0	*K*	1 57.7	−45 12	56	−42
478	5.9	*A4*	2 1.7	+57 57	9	−33
488	6.4	*B8*	2 4.5	+57 10	13	−33
493	6.2	*K4*	2 5.5	+25 28	11	−22
660	5.9	*Me*	2 50.2	+17 56	18	+39
826	5.2	*Mb*	3 33.5	+62 54	22	−26
960	5.6	*G6*	4 5.5	− 7 11	10	−28
1130	5.4	*A5*	4 42.9	−59 55	37	−34
1183	6.9	*K5*	4 53.5	+39 30	10	−30
1299	4.2	*K*	5 19.1	− 7 53	43	−36
1301	3.4	*B1*	5 19.4	− 2 29	6	+18
1439	5.0	*Ma*	5 44.2	+37 16	52	+32
1441	5.9	*G4*	5 44.5	+ 9 50	14	+29
1573	6.1	*G4*	6 10.2	+24 0	29	−32
1632	6.0	*K3*	6 22.6	+46 45	7	−51
1698	4.6	*G5p*	6 33.5	−18 9	16	−20
1737	5.0	*K*	6 42.0	+ 8 9	.025	+34

P. G. C. No.	Mag.	Type	α 1900.0	δ 1900.0	μ	r
			h m	° ′	″	km
1847	5.4	*B5p*	6 58.8	−23 41	.009	+30
1846	5.7	*Map*	7 5.6	+54 36	19	−51
1873	6.5	*G1*	7 10.2	+ 0 1	19	−26
1934	2.4	*B5p*	7 20.1	−29 6	10	+22
2003	4.9	*K5*	7 33.2	−52 19	25	+44
2020	5.8	*M6p*	7 36.1	+14 27	15	−28
2056	3.9	*K*	7 43.0	−72 22	9	+35
2236	6.3	*G1*	8 20.6	+45 59	21	−37
2318	4.0	*G5*	8 36.2	−34 57	20	−31
2335	4.7	*G*	8 38.7	− 6 52	7	+19
2426	5.4	*A*	8 56.9	+24 51	5	−21
2467	5.0	*B5*	9 7.4	−44 27	14	+20
2521	4.9	*K*	9 18.5	−61 58	16	+37
2961	5.4	*A*	11 4.1	−61 24	33	−36
3035	5.0	*F8p*	11 27.1	−58 53	18	−24
3036	5.3	*A2p*	11 27.2	−58 58	29	−26
3099	4.7	*K5*	11 43.1	−66 15	30	+30
3446	5.0	*Ma*	13 12.6	+ 6 0	16	−20
3486	5.3	*A*	13 23.3	−50 39	29	−30
3534	6.2	*Ma*	13 36.4	− 8 12	18	−27
3703	6.3	*K1*	14 21.4	+38 51	19	+40
3878	5.2	*F5*	15 9.5	−41 8	18	−24
4034	4.9	*K*	15 46.9	+21 17	49	−45
4313	4.2	*K2*	16 51.6	−53 0	4	+26
4316	6.7	*G5*	16 53.4	+25 30	9	+28
4418	5.7	*A8*	17 20.0	+16 24	44	+30
4492	3.4	*F5p*	17 40.6	−40 5	4	−21
4519	5.0	*K*	17 49.5	−44 19	28	+52
4530	5.7	*K4*	17 51.6	+22 29	2	−23
4635	4.9	*G*	18 15.8	+ 3 20	15	+23
4719	5.8	*A3*	18 32.5	− 0 24	26	+30
4731	4.7	*F*	18 36.8	− 9 9	14	−33
4750	5.8	*F5*	18 44.2	−10 14	5	+25
4832	2.7	*A2*	18 56.3	−30 1	21	+31
4862	4.7	*K*	19 1.3	−40 39	49	+29
4974	5.7	*B8*	19 24.0	+ 1 45	37	+33
5052	3.8	*Map*	19 42.9	+18 17	9	+20
5063	6.2	*G2*	19 45.9	+38 27	16	+29
5147	4.9	*Ma*	19 59.7	−53 10	24	+38
5244	5.0	*F*	20 23.2	−18 9	26	+32
5402	4.7	*G5*	20 55.2	−32 39	9	+23
5430	4.6	*Ma*	21 1.3	−25 24	59	+40
5550	6.2	*A0*	21 32.4	− 0 50	26	+27
5593	Var	*Ma*	21 40.1	−58 19	2	+36
5690	5.4	*K5*	22 2.6	−34 32	56	+12
5695	5.5	*A5*	22 4.4	−34 31	42	−29
5716	5.0	*K*	22 7.9	+71 51	28	+30
5763	4.9	*B5*	22 16.7	+27 50	5	+20
5804	4.6	*K*	22 25.4	+47 11	.021	+66

P. G. C. No.	Mag.	Type	α 1900.0	δ 1900.0	μ	v
			h m	° ′		km
5867	4.9	*K*	22 39.5	−51 1	.025	+36
5884	4.2	*K 5*	22 44.3	−14 7	39	+48
6089	5.1	*Map*	23 38.3	+ 9 17	7	−32
6135	4.8	*F8p*	23 49.1	+56 57	.007	−34

Observatorio Nacional Argentino, Cordoba, August 2, 1917.

OBSERVATIONS OF COMETS,

MADE WITH THE 26-INCH EQUATORIAL OF THE U. S. NAVAL OBSERVATORY.

[Communicated by Rear Admiral T. B. HOWARD, U. S. Navy, Superintendent.]

Date Wash. M. T.	✱	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$ α	log $p\Delta$ δ	Red. to App. Pl. α	Red. to App. Pl. δ
Comet 1916 *b* (WOLF)										
1917 h m s			m s	′ ″	h m s	° ′ ″			s	″
May 9 14 1 35	1	25 , 5	−1 7.84	+5 33.0	20 53 42.10	+14 54 11.1	9.591*n*	0.635	+2.03	−2.5*
19 14 21 14	2	25 , 5	+2 24.21	+4 20.2	21 17 45.38	+17 32 44.2	9.543*n*	0.583	+2.19	−0.8*
24 14 47 22	3	27 , 6	−2 16.52	+0 57.5	21 29 35.64	+18 46 43.4	9.475*n*	0.540	+2.27	+0.6*
June 15 15 0 12	4	27 , 6	+3 19.66	−2 16.2	22 18 31.68	+23 3 27.7	9.331*n*	0.421	+2.71	+5.3†
16 14 48 1	5	25 , 5	−3 44.87	+5 6.6	22 20 34.64	+23 11 47.6	9.370*n*	0.427	+2.71	+5.7*
20 12 17 2	6	25 , 5	−4 7.01	+0 6.0	22 28 27.78	+23 11 2.3	9.652*n*	0.583	+2.81	+6.8*
21 14 25 30	7	49 , 10	−3 23.86	+4 26.0	22 30 36.64	+23 48 16.0	9.415*n*	0.426	+2.84	+7.0†
29 12 31 34	8	25 , 5	+4 4.90	−5 46.0	22 45 31.56	+24 27 15.4	9.618*n*	0.532	+3.02	+8.6*
29 14 16 15	9	12 , 14	+0 1.98	−5 23.4	22 45 39.02	+24 27 30.9	9.394*n*	0.404	+3.01	+8.7†
Comet 1917 *b* (SCHAUMASSE)										
June 4 9 54 49	10	13 , 3	−1 15.64	+5 3.4	8 56 28.97	+29 47 10.1	9.721	0.665	+1.93	−2.6*
13 9 5 55	11	25 , 5	−2 47.83	−5 0.1	9 22 34.64	+19 44 28.3	9.676	0.670	+1.88	−6.3*
16 9 18 49	12	10 , 2	−6 34.37	+0 32.9	9 26 56.35	+17 44 17.0	9.678	0.697	+1.87	−7.2*

Observers: * = H. E. BURTON; † = E. C. BOWER.

All comparisons were made by transits except on June 29 (2d date) when $\Delta\alpha \cos\delta$ and $\Delta\delta$ were measured with driving clock running.

Mean Places of Comparison Stars for 1917.0

✱	α	δ	Authority		✱	α	δ	Authority	
	h m s	° ′ ″				h m s	° ′ ″		
1	20 54 47.91	+14 48 40.6	*A.G. Leipzig I*	8261	7	22 33 57.66	+23 43 43.0	*A.G. Berlin B*	8695
2	21 15 18.98	+17 28 24.8	*A.G. Berlin A*	8688	8	22 41 23.64	+24 32 52.8	*A.G. Berlin B*	8734
3	21 31 49.89	+18 45 45.3	*A.G. Berlin A*	8808	9	22 45 31.03	+24 32 45.6	*A.G. Berlin B*	8761
4	22 15 9.31	+23 5 38.6	*A.G. Berlin B*	8599	10	8 57 42.68	+29 42 9.3	*A.G. Camb.(Eng.)*	4795
5	22 24 16.80	+23 6 35.3	*A.G. Berlin B*	8644	11	9 25 20.59	+19 49 34.7	½[*A. G. Berlin A* 3832 + *A. G. Berlin B* 3770]	
6	22 32 31.98	+23 40 49.5	*A.G. Berlin B*	8685	12	9 33 28.85	+17 43 51.3	*A.G. Berlin A*	3879

NOTES

	Seeing	1916 *b*
May 9.	Poor.	Comet faint. Moonlight. Poor observation.
19.	Fair.	Comet barely visible in 5-inch finder.
21.	Fair.	Haze and clouds.
June 15.	Fair.	Poor observation.
16.	[illegible]	Comet visible in 2-inch finder.
20.	Fair.	
21.	Fair.	Poor observation.
29.	Poor.	Poor observation.
29.	Fair.	Head about 3.5 in diameter.

	Seeing	1917 *b*
June 1.	Poor.	Comet very faint. Moonlight. Barely visible in 5-inch finder. Poor observation.
13.	Fair.	Comet faint.
16.	Fair.	Comet faint. Observation not satisfactory.

THE OBJECT *R. JONCKHEERE 900*.

By E. E. BARNARD.

In *A. N.* 4635 Bd. **191**, 17 Mr. ROBERT JONCKHEERE gives observations of an object discovered by him in the position

$$1920.0 \quad \alpha\ 6^h21^m17^s \quad \delta + 17°50'$$

which he describes as a planetary nebula 3″ in diameter. In this nebula he measured two stellar points that made a double star:

$$\text{P. A. } 118°.0 \quad \text{Dist. } 2''.17 \quad 9^m.8 \quad 9^m.8$$

He also measured a 10.3 magnitude star with respect to the nebula:

$$\text{P. A. } 193°.2 \quad \text{Dist. } 11''.11$$

I examined and measured this object with the 40-inch telescope on 1913 April 14. It is a brightish, ill defined, bluish white disc, possibly a little brighter in the preceding part, and seems to be a planetary nebula. Two settings of the micrometer wires made the diameter 7″.9. There was no central condensation and no trace of the central double star. It was also observed on 1915 October 18 and 21 under good conditions with similar results.

Following are my measures of the estimated center of the nebula and the small star south following observed by Mr. JONCKHEERE:

			Mag. Neb.	Mag. Star
1913.96	P. A. 197°.78	Dist. 11″.68	12	$11\frac{1}{2}$ (3*n*)

On April 14, 1913 the small star was compared direct with Berlin *A. G. C.* 2137:

$$\Delta\alpha + 303''.4\ (2) = +21^s.23 \quad \Delta\delta - 46''.9\ (2)$$

This gives for the star,

$$1913.0 \quad \alpha\ 6^h20^m52^s.49 \quad \delta + 17°50'14''.7$$

and for the nebula,

$$1913.0 \quad \alpha\ 6^h20^m52^s.74 \quad \delta + 17°50'25''.8$$

I cannot account for the entire absence in these observations of the double star seen in the nebula by Mr. JONCKHEERE in 1912. The position shows that the object is correctly identified.

Yerkes Observatory, Williams Bay, Wisconsin, 1917 May 29.

CONTENTS.

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COLLIMATION OF THE OLCOTT MERIDIAN CIRCLE,

By ARTHUR J. ROY.

Throughout several series of observations, it was found that over considerable periods during which the optical adjustments were not disturbed, the various determinations of the OLCOTT Meridian Circle collimation were harmonized by the application of a term varying with the temperature. The coefficient determined during the formation of the Albany *A. G. Zone* $+1°$ to $+5°$ was found applicable to the early observations after the removal of the instrument to the new location in 1892. The thorough investigation of 1897, involving 50 observations (results now in press) exactly confirmed the previous result, $-^s.0040$ per degree centigrade. From 20 determinations in the next 18 months $-^s.0039$ was found, and the extreme temperatures indicated $-^s.0042$.

As the application of this term continued to be satisfactory, no special investigation was made before dismounting the instrument in 1908, but with the enlarged force in San Luis, material accumulated rapidly and three periods furnished sufficient data for a discussion, with the following results: 15 observations from April to September 1909, $-^s.0037$; 24 observations from September 1909 to April 1910, $-^s.0056$; 14 observations from June to November 1910, $-^s.0063$. These are weak compared with the Albany determinations not only numerically but through the relatively small temperature range. The extreme difference in the first group being but 18°C, in the second 18°.5C, in the third 14°C, and for all 25°C — barely half the Albany range — it was assumed that this apparent progressive change in the temperature coefficient was merely an indication of the uncertainty, and a mean value, $-^s.005$, was adopted. (See page 166, *Year Book* 11, 1912, Carnegie Institution of Washington.)

After the remounting of the instrument in Albany, the writer was nonplussed by the failure of the old formula to represent the new observations. Comparisons of extreme temperatures indicated a greatly reduced coefficient, and retrogressions in the seasonal temperatures seemed to have little or no effect, contrasting strongly with the apparently increasing effect in San Luis. One discussion of the material for a temperature gradient term (*A. J.* 659) in which the temperature data lacked precision, showed that there was combined with the negative temperature coefficient a negative gradient term; that is, while it might be expected that the effect would lag behind the temperature, on the contrary, it preceded it. This strange result is consistent with a suggestion made previously by the writer that there exists what is analagous to — and for lack of a better name will be called — a lost motion term. To increase the determinateness of the temperature coefficient, it was always the general practice to make the readings when the seasonal temperature was near the seasonal maximum or minimum with special emphasis on the former in summer and the latter in winter. Furthermore, to largely eliminate the lag effect and the uncertainty as to the temperature applicable and to obtain more stable conditions, the readings were usually made soon after the maximum or minimum of the day when the gradient would be towards the seasonal normal and much of the lost motion effect would appear in any solution of a temperature gradient term.

In April, 1914, the writer inaugurated a campaign to thoroughly investigate the matter. (See page 240, *Year Book* 13, 1914, Carnegie Institution of Washington.) Within a year, seventy-one observations were made and in general so timed as to confirm or refute the lost motion hypothesis and also to largely avoid lag and gradient effects. Two other observations were rejected for reasons other than discordance and repeated immediately. The regular program supplied thirty-eight more observations in the twenty-five months following. In general, each observation con-

sisted of simultaneous determinations by reversal upon mire and nadir, but occasionally one or the other could not be used. The differences, nadir minus mire, due to inequality of pivots, personal equation, etc., appear to fluctuate with evidence both of progressive diminution in the average amount and of variation with the season. Grouping the differences generally under Winter and Summer with divisions near November 1st and May 1st, depending upon extreme temperatures registered, the mean difference with the number of observations are shown chronologically in Table I.

Table I

	Winter	Obs	Summer	Obs.
	+.020	7	+.019	6
	+.023	7	+.026	9
Mean of all			**+.022**	
	+.008	7	+.015	30
	−.004	44	+.022	9
	−.015	6	+.011	8
	.000	6		
Means	**+.002**	**63**	**+.016**	**47**

The half of these mean differences was applied to each and because of a fear of systematic errors in the nadir results, double weight was given to the mire. This procedure was followed to systematically reduce any solitary determination by mire or nadir to the mean of both. In discussing the temperature coefficient in the collimation of the Olcott Meridian Circle, it has always been found best to use the temperature indicated by the thermometer attached to the barometer in the observing room, but the external temperature was also recorded. Since July 6, 1914, the maxima and minima necessary for a solution under the lost motion hypothesis have been obtained from the observatory thermograph installed on that date. Previously they were obtained from the records of the U. S. Weather Bureau Station 1.5 miles distant. These records do not accurately represent the external temperature about the observing room, but all errors are largely eliminated by basing both the solution and application upon the same set of temperatures.

The lost motion hypothesis assumes that the collimation is invariable so long as the temperature fluctuates within the amount of the lost motion and therefore that it is controlled, not by the momentary temperature, but by the last maximum or minimum which established a new range. In rare cases, the momentary temperature would itself be a new maximum or minimum. After some approximating, it was possible to divide the observations into two groups so each observation would form one conditional equation in the form

$$c = x + y\,(T - 0°\mathrm{F}) \qquad (1)$$

or

$$c = x + y\,(T - 70°\mathrm{F}) \qquad (2)$$

in which c is the observed collimation as controlled by some particular temperature; x the collimation at a mean temperature, taken for convenience in (1) at 0°F and in (2) at 70°F; y the temperature coefficient when effective; T a preceding controlling temperature, in (1) a minimum, in (2) a maximum. Assuming that the 109 observations from April 1, 1914 to May 1, 1917 are homogeneous, the solution with (1) and (2) indicates the p. e. (.8453 of the mean residual) of a single observation to be ±.0123, while on the basis of a uniformly varying temperature effect it is ±.0243. The sequences of signs of the outstanding residuals indicate that a division ought to be made at April 1, 1915, and the resulting groups, observations 42 to 112 and 113 to 150, give the following values of c:

a	42	112	$c = +.1684 - .00246\,(T - 0°\mathrm{F})$	29 obs.
b	42	112	$c = +.1427 - .00268\,(T - 70°\mathrm{F})$	42 obs.
c	113	150	$c = +.1801 - .00184\,(T - 0°\mathrm{F})$	10 obs.
d	113	150	$c = +.1681 - .00232\,(T - 70°\mathrm{F})$	28 obs.

Or if we adopt the mean value of y, these respectively become

a′	$c = +.1681 - .00244\,(T - 0°\mathrm{F})$
b′	$c = +.1442 - .00244\,(T - 70°\mathrm{F})$
c′	$c = +.1847 - .00244\,(T - 0°\mathrm{F})$
d′	$c = +.1699 - .00244\,(T - 70°\mathrm{F})$

The p. e. for a single observation is ±.0097 while with the same division but ignoring the lost motion, we [illegible] respective groups:

(e)	42 — 112	$c = +^s.1550 - ^s.00182\ (T_M - 32°F)$
(f)	113 — 150	$c = +\ .1696 - \ .00107\ (T_M - 32°F)$

in which T_M is the momentary temperature and the p. e. from all is $\pm^s.0214$.

To further test the homogeniety of the material, groups (a) and (b) were subdivided with the following results:

(a_1)	$c = +^s.1671 - ^s.00217\ (T - \ \ 0°F)$	11 obs.	Extreme temperatures.
(a_2)	$c = +\ .1669 - \ .00262\ (T - \ \ 0°F)$	15 obs.	Moderate temperatures.
(b_1)	$c = +\ .1427 - \ .00279\ (T - 70°F)$	17 obs.	Extreme temperatures.
(b_2)	$c = +\ .1433 - \ .00260\ (T - 70°F)$	25 obs.	Moderate temperatures.

(a_1) and (b_1) comprise respectively the observations in (a) and (b) for which the momentary temperatures are within 15°F of the preceding minimum or maximum, and (a_2) and (b_2) those taken under intermediate temperatures. If we ignore the lost motion a similar division gives:

(e')	$c = +^s.1415 - ^s.00129\ (T_M - 32°F)$	31 obs.	Extreme temperatures
(e'')	$c = +\ .1657 - \ .00240\ (T_M - 32°F)$	40 obs.	Moderate temperatures

The respective p. e.'s are $\pm^s.029$ and $\pm^s.015$, the discordance in the temperature coefficients contrasts strongly with the agreement above. But if we largely avoid the lost motion by subdividing the extreme temperatures into high and low, we get:

(e'_1)	$c = +^s.2203 - ^s.00284\ (T_M - 32°F)$	15 obs. High extreme
(e'_2)	$c = +\ .1149 - \ .00248\ (T_M - 32°F)$	16 obs. Low extreme

The p. e. is now reduced from $\pm^s.029$ to $\pm^s.011$ and we have a satisfactory agreement among the temperature coefficients, including (e'') but the constants differ radically, showing the lack of homogeneity. It might also be noted that the mean c and T_M for the 15 observations of the high extreme are respectively $+^s.128$ and 64°F and for the low extreme $+^s.114$ and 32°F, a difference of $+^s.014$, which to correspond to the difference in temperature should be about $-^s.080$.

The amount of the lost motion is deduced by a comparison of (a') with (b') where it is $70°F + (+^s.1681 - ^s.1412 \div -^s.00244) = 59°F$; the amount adopted in this solution. Also from (c') and (d') we find 64°F. Similarly from observations 1 to 23 it is found to be 35°F and from 24 to 41, 52°F.

Observations 42 to 112 were discussed for a gradient effect by adding another term to (1) and (2) as follows:

$$(3)\qquad c = x + y\,(T - \ \ 0°F) + z\Delta T'$$

$$(4)\qquad c = x + y\,(T - 70°F) + z\Delta T'$$

in which $\Delta T'$ is the hourly rate of change in temperature as indicated by the attached thermometer on the centigrade scale. The resulting values of z were $+^s.0008$ and $-^s.0009$, the first equal to its p. e. and the second one-half greater, but the combined result somewhat less than its p. e., the term y was unchanged and the results for x were $+^s.1675$ and $+^s.1438$, differing from (a) and (b) above by $-^s.0009$ and $+^s.0011$.

Table II exhibits typical results under the different hypotheses and classifications for the year in which the collimation was most frequently determined. The respective columns contain, — first, the chronological number of each observation; second, the astronomical date; third, the reading of the attached thermometer; fourth, the residuals from solution (e) when the lost motion is ignored; fifth, the residuals from (a') and (b') with lost motion 59°F (67 to 95 being under control of minimum temperatures with the p. e. from all $\pm.0080$); sixth, seventh, and eighth, the residuals respectively from (e'_1), (e'_2), and (e''). The residuals in the last three columns from observations taken under high, low, and moderate seasonal temperatures respectively are to be contrasted with those in the fourth column where all are combined.

Mr. VARNUM has made a tentative solution of the collimation from November, 1911 to April, 1915, assuming that it varies uniformly with the temperature, and attempting to avoid the lost motion by a division into small groups. This method fairly well represents

the observed values but it involves in the application the labor of ascertaining the momentary temperature for each observation and what is more important, introduces considerable errors if the application temperatures differ materially from the mean T_M as used in (e). Detailed comparisons of the values prepared for application, combined into groups as they occur Clamp E and Clamp W, indicate errors of groups varying from $-^s.035$ to $+^s.037$ Clamp E and from $^s.000$ to $+^s.036$ Clamp W. The average error from all is $\pm^s.017$ and the mean errors W and E respectively are $+^s.017$ and $-^s.009$.

Conclusion

In the ideal instrument, the collimation would be invariable under all conditions. Lacking such constancy, it is highly desirable that the variation for a year or more can be represented by a simple formula, that some of the systematic errors in fundamental or semi-fundamental work dependent upon declination may not vary with the right ascension and vary irregularly.

The writer concludes that since February, 1912, the determined collimations of the Olcott Meridian Circle cannot be well represented over any considerable period by a constant plus a temperature term; that they can be sufficiently well represented by the addition of the lost motion hypothesis and their homogeneity is established; that to obtain a good representation with a constant and a temperature term by division into relatively small periods would sometimes introduce important errors varying from period to period and more or less recurring with the season; that there is no good evidence in the material to date of even a minute gradient term; that the lost motion is increasing; that while it is disappointing to have another term in the collimation formula, there is a decided compensation in the simplicity of application, c seldom varying within a series and at times constant for several weeks; that agreement of the present temperature coefficient, $-^s.00244$ per 1°F or $-^s.0044$ per 1°C, with the previously determined values precludes the existence of any appreciable lost motion term in previous series.

Table II

No.	1914	T	O − C Unit $^s.001$ No. L. M.	L. M. 59° F	Temperatures High	Low	Mod.
42	Apr. 6.0	+ 1.9°C	− 9	−23	...	....	−18
43	" 27.2	+12.7	+32	+10			+35
44	May 18.2	+21.3	+21	− 8	+ 1		..
45	26.8	+27.2	+29	+12	+14		...
46	27.4	+30.5	+35	+ 7	+26		....
47	31.9	+23.0	+ 1	+ 1			+17
48	June 21.9	+20.8	− 9	− 5		...	+ 2
49	July 19.9	+19.7	− 8	0		.	+ 2
50	Aug. 10.4	+31.6	+23	− 8	+16		...
51	11.4	+24.0	− 7	−13	−28		....
52	13.9	+17.8	−18	− 4			−10
53	26.2	+22.2	+ 4	+ 3			+16
54	31.2	+26.5	− 7	−22	−24		...
55	Sept. 8.4	+15.5	−33	−12			−28
56	8.9	+10.4	−58	−20	− 5		.
57	9.4	+16.2	−23	− 4			−17
58	11.9	+10.4	−36	+ 2		+17	
59	14.9	+16.6	−34	−16			−27
60	15.2	+22.8	+20	+17			+33
61	15.9	+14.0	−52	−16		+ 1	
62	22.2	+30.9	+ 7	−22	− 1	.	.
63	23.9	+8.5	31	−22			−25
64	24.2	+16.5	25	− 7			−18

TABLE II. — *Cont.*

No.	1914	T	O — C Unit ".001 No. L. M.	L. M. 59 F	Temperatures High	Low	Mod.
65	Sept. 25.9	+ 8.3°C	−44	+ 5		+ 6	
66	28.5	+ 3.8	−58	+ 2		−13	
67	29.9	+12.2	−24	+ 6			−22
68	Oct. 5.1	+25.8	+ 2	−12	−15		
69	6.1	+15.6	−27	− 8			−21
70	13.5	+ 4.0	−60	− 3		−15	
71	21.2	+21.4	0	0			+12
72	26.9	+ 1.1	−41	+25		0	
73	27.9	+ 3.2	−41	+12		+ 3	
74	Nov. 8.9	+ 1.6	−53	+ 5		−11	
75	9.9	− 3.2	−60	− 2		−21	
76	17.9	− 6.6	−37	+23		− 4	
77	23.1	− 4.8	−38	+ 6		− 3	
78	24.1	− 2.1	−27	+ 1		+11	
79	26.1	+ 7.8	− 1	− 6			− 1
80	26.9	+ 7.5	−13	−17			−16
81	27.1	+ 8.6	− 5	−12			− 7
82	Dec. 1.1	+11.0	+ 4	−11			+ 5
83	2.1	+14.1	+ 2	−23	−37		
84	2.9	+10.1	− 3	−15			− 3
85	3.1	+10.6	+ 3	−11			+ 3
86	15.1	− 6.1	−34	+ 5		− 1	
87	21.0	− 2.8	− 9	+ 9			−23
88	21.2	+ 0.2	− 7	+ 1			−18
89	24.2	− 7.4	−15	+ 6			−34
90	27.9	− 9.2	+27	+ 5			+ 7
91	28.0	− 6.4	+30	− 1			+13
92	28.1	− 3.8	+49	+ 9			+31
93	29.9	+ 2.0	+72	+13	+10		
94	30.1	+ 4.8	+67	− 1	+10		
	1915						
95	Jan. 5.1	− 0.3	+49	− 2	−17		
96	6.9	+10.2	+67	− 1	+21		
97	7.2	+ 6.4	+58	+ 7	+ 4		
98	8.9	− 1.4	+39	+14			+27
99	10.9	− 6.8	+20	+12			+ 2
100	21.9	−10.0	+11	+14			−10
101	28.9	−12.2	− 2	+ 8		+24	
102	29.9	−17.2	− 4	+22		+15	
103	Feb. 5.2	− 0.3	+47	+18			+36
104	8.2	− 1.4	+32	+ 7			+20
105	15.9	+ 2.1	+34	− 3			+25
106	22.0	+ 0.8	+32	− 1			+22
107	26.9	− 8.3	+ 2	− 1			−17
108	Mar. 3.9	− 8.7	+ 3	+ 1			−17
109	21.9	+ 2.2	+33	− 4			+25
110	25.1	+11.2	+64	+ 4	+19		
111	25.9	− 0.1	+31	+ 8			+20
112	26.8	− 8.8	0	+ 6			−20

Dudley Observatory, July, 1917.

OBSERVATIONS OF THE DISTANT COMPANION OF *PROCYON*.

By E. E. BARNARD.

	Date	P. A.	Dist.	H. A.	Remarks
				h m	
1914.123	Feb. 14	356.38	71.42	0 00	Seeing poor.
.126	15	356.24	70.79	W 0 30	Seeing fair.
.184	Mar. 8	356.16	71.18	W 2 30	Seeing fair.
1914.144		356.26	71.13		
1915.963	Dec. 18	357.13	73.55	W 0 30	Seeing very bad.
1916.127	Feb. 16	357.37	72.97	0 00	Through clouds.
1916.045		357.25	73.26		
1916.826	Oct. 28	357.65	73.75	E 2 00	
.839	Nov. 2	357.50	72.74	E 0 50	
1916.833		357.57	73.30		

THE DISTANT COMPANION AND A SMALL STAR

1915.804	Oct. 21	156.30	37.84	0 00	
1916.826	Oct. 28	156.05	36.84	E 1 45	Seeing 2-3.
1916.315		156.17	37.34		

For previous measures of this small star see *A. J. 27*, p. 107.

SCHAEBERLE's close companion to PROCYON was not seen in these observations, though it was frequently looked for.

Yerkes Observatory, Williams Bay, Wisconsin, 1917, May 25.

OBSERVATIONS OF THE SATELLITE OF *NEPTUNE*.

By E. E. BARNARD.

The following observations of the satellite of *Neptune* are continued from those in *A. J. 30*, p. 2.

[illegible]	C. S. Time	P. A.	Dist.	Cps.	Remarks
Sept. 22	16 31 11	182.33		5	Faint, but well seen occasionally.
	16 39 21		10.47	8	
Oct. 3	16 11 37	248.50		5	Seen only momentarily.
	16 19 29		11.58	10	Seeing very bad.

SATELLITE OBSERVATIONS. — *Cont.*

1916	C. S. Time	P. A.	Dist.	Cps.	Remarks
	h m s	°			
Oct. 10	16 22 22	155.86	. .	7	Very faint. Seeing very poor.
	16 28 25	.	12.73	8	
21	15 37 30	221.85	.	6	Seeing excessively bad.
	15 46 15		10.24	10	
26	16 45 14	279.39	. . .	7	Faint. Seeing = 2.
	16 51 49	. .	14.41	10	
31	15 34 11	316.77	. . . .	6	
	15 43 14	.	15.74	10	
Nov. 2	16 26 33	193.11	. . .	5	Very faint in clouds.
	16 33 4		10.51	12	
18	15 4 56	303.34	. .	5	Very difficult.
	15 10 23	. .	16.54	9	
25	16 15 17	240.67	. .	6	Very difficult.
	16 20 22		11.45	2	Stopped by clouds.
27	15 46 12	113.36	. .	6	Glimpsed through passing clouds.
	15 57 14		16.69	10	
Dec. 2	15 51 46	150.93	. .	5	Very faint in very thick sky. Seeing = 1-2.
	15 58 24		14.65	10	
9	14 14 35	105.83	. . . .	7	14 mag. Very faint and difficult.
	14 21 50	. . .	16.22	10	
16	16 40 18	20.10	. . .	5	Seeing = 2-3.
	16 47 5	.	10.12	10	
27	12 26 15	92.48	. . . .	5	Faint; seeing bad, but observation fair.
	12 31 38	. . . .	14.86	10	
30	14 36 20	265.05	. .	5	Seeing = 2.
	14 41 23	.	13.38	10	
1917					
Jan. 6	10 25 7	184.46		5	Very faint. Seeing 1-2.
	10 31 47	.	11.02	10	
13	12 8 59	119.53	. . .	6	Very faint. Seeing very bad.
	12 17 58	.	17.02	10	
18	15 16 28	153.83	. .	8	
	15 23 25	. .	14.22	11	
	15 30 21	153.37		5	Better seen.
23	9 35 41	245.75		6	Image jumping very badly.
	9 42 1		11.58	10	Seeing 1-2.
25	14 21 52	107.01	. .	6	Seeing excessively bad.
	14 27 19	.	16.49	10	
27	14 37 5	325.42	. .	5	Faint in thick sky.
	14 41 44		14.88	10	Seeing 2-3.
Feb. 8	14 1 35	313.45	.	7	Very difficult. Seeing excessively bad.
	14 8 47		16.10	10	
10	11 2 30	205.67		6	
	11 9 3		10.40	10	
13	9 7 51	30.47		5	Faint in foggy sky.
	9 12 33		10.64	10	Seeing = 3.
17	10 56 37	131.78		7	Very faint. Seeing very bad.
	11 2 57	.	16.38	10	

SATELLITE OBSERVATIONS. — *Cont.*

1917	C. S. Tm.	P. A.	Dist.	Cps.	Remarks
Feb. 20	9 51 21	311.99		5	
	9 56 14		15.99	10	
27	8 8 26	266.51		6	Seeing = 3.
	8 13 30		13.87	10	
Mar. 17	12 33 56	226.51		6	Very difficult from wind. Seeing
	12 39 1		10.78	8	2-3.
20	8 38 37	56.43		7	Seeing = 3.
	8 44 10		11.24	10	
22	9 30 28	285.83		5	Faint in misty sky.
	9 36 17		15.95	11	Seeing very bad.
	9 42 19	285.65		5	
24	10 37 40	144.84		5	
	10 41 7		15.02	8	
27	8 19 36	326.39		7	
	8 25 24		14.17	10	
Apr. 10	7 47 6	199.00		5	Seeing = 2. Very high wind shak-
	7 52 13		10.25	10	ing everything.
May 1	8 17 47	342.41		7	Faint in clouds.
	8 22 27		12.15	10	
8	8 39 34	288.82		7	Seeing very poor = 2.
	8 44 44		15.34	10	
1916			*Neptune* AND A STAR		
Dec. 9	14 37 20	315.21	....	5	13 magnitude.
	14 42 13	....	14.93	10	

Yerkes Observatory, Williams Bay, Wisconsin, 1917 May 23.

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